S-72.340 INFORMATION THEORY

Exercise 8, November 8, 2000

1. The so-called Z channel is a binary channel with the following transition probabilities $P(y|x)$:

$$P(0|0) = 1 - \epsilon\,, \quad P(1|0) = \epsilon\,, \quad P(1|1) = 1\,.$$

   Find the capacity of the channel. (This problem, like the other problems of this exercise, is from R.G. Gallager's book *'Information Theory and Reliable Communication'.*)

2. Compute

$$g_n(s) = \sum_{y_n} P(y_n|0)^{1-s} P(y_n|1)^s$$

   for the Z channel, and minimize $g_n(s)$ over $0 \leq s \leq 1$. Use the result to provide an upper bound to the probability of maximum likelihood decoding error $P_{e,m}$, $(m = 1, 2)$ attained with a code of two code words x_1 (a sequence of N zeros) and x_2 (a sequence of N ones).

3. Repeat Problem 2 for the binary erasure channel defined in Problem 5 of Exercise 4.

4. Find the exact expressions for $P_{e,1}$ and $P_{e,2}$ for the Z channel. Evaluate the expressions (using the Stirling approximation $N! \approx \sqrt{2\pi N}(N/e)^N$ where necessary) for $N = 25$ and $\epsilon = 0.1$ and compare with the bound in Problem 2.

5. Repeat Problem 4 for the binary erasure channel.

Try to solve as many problems as you can. Return your solutions by the time that the exercise begins. You can bring them to the exercise or put them into the 'return box' below the information board of our laboratory on the 2nd floor.