S-72.340 INFORMATION THEORY

Exercise 9, November 15, 2000

1. Repeat Problem 4 from Exercise 8 for a binary symmetric channel (BSC) with bit error probability ϵ.

2. For the Z channel defined in Problem 1 of Exercise 8, both upper-bound and evaluate $P_{e,m}$, $(m = 1, 2)$ for a two-word code in which $\mathbf{x_1}$ is $N/2$ zeros followed by $N/2$ ones and $\mathbf{x_2}$ is $N/2$ ones followed by $N/2$ zeros.

3. Suppose that two code words of length N are chosen for the channel shown in Figure 1 by selecting each letter of each codeword independently with the probability assignment $Q(0) = Q(1) = Q(2) = 1/3$. Find the ensemble average probability of error, assuming maximum likelihood decoding. In case of ambiguity, assume that the decoder makes an error. *Hint:* Let $\mathbf{x_1}$ be transmitted and $\mathbf{y}$ received. Find how many channel input sequences can lead to $\mathbf{y}$ and find $P_N(\mathbf{y}|\mathbf{x_1})$ for each of them. Then find the probability that code word $\mathbf{x_2}$ has been chosen in this set.

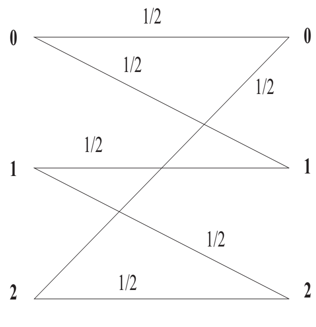

Figure 1: A channel model

4. Let the two-word code consist of the code words $\mathbf{x_1} = (0, 0, 0, \ldots, 0)$ and $\mathbf{x_2} = (1, 1, 1, \ldots, 1)$. Using the same decoding procedure and channel as in Problem 3, find the probability of error for this code.

5. Use the Chernoff bound to show that

$$Q(x) \leq e^{-x^2/2}\,,$$

where

$$Q(x) = \frac{1}{\sqrt{2\pi}} \int_x^\infty e^{-t^2/2}\,dt\,, \quad x \geq 0.$$

Try to solve as many problems as you can. Return your solutions by the time that the exercise begins. You can bring them to the exercise or put them into the 'return box' below the information board of our laboratory on the 2nd floor.