S-72.341 CODING METHODS

Tutorial 1

1. The International Standard Book Number (ISBN) is a 10-digit codeword. For example, the text by Wicker has ISBN 0-13-200809-2. Here the first digit indicates the language (0 or 1 for English, 951 would be Finnish); the next two digits specify the publisher (13 for Prentice-Hall, 672 for Otatieto); the next six digits are the book number assigned by the publisher; and the final digit is chosen to make the entire number $x_1 x_2 \ldots x_{10}$ satisfy the check equation

$$\sum_{i=1}^{10} i x_i = 0 \bmod 11 .$$

The value $x_{10} = 10$ is represented by the letter X. The language, publisher, and book number fields are variable length, so hyphens may appear in different locations.

a) Calculate the check digit for the ISBN whose first nine digits are 0-471-06259.

b) The ISBN code can detect any single digit error. Show that the ISBN code can detect the transposition of any two digits (not necessarily consecutive).

c) The sixth digit in the ISBN 0-13-28[illegible]796-X was smudged by a frustrated student. Find the missing digit.

2. Identity and inverse elements of a group.

a) (Wicker, problem 2.1) Prove that the identity element e in a group G is unique.

b) (Wicker, problem 2.2) Prove that the inverse a^{-1} of an element a in a group G is unique.

3. Construct the addition and multiplication tables for GF(5).

4. (Exam 6.9.2001) Dual spaces.

   a) (Wicker, problem 2.17) Find a basis for the dual space to the vector space over GF(2) spanned by $\{(11100),(01110),(00111)\}$.

   b) (Wicker, problem 2.18) Find a basis for the dual space to the vector space over GF(5) spanned by $\{(12322),(14312),(41233)\}$.

5. For the Galois field GF(11), find all primitive elements and determine the multiplicative orders of the other nonzero elements. Express all nonzero elements as powers of a single element in GF(11).