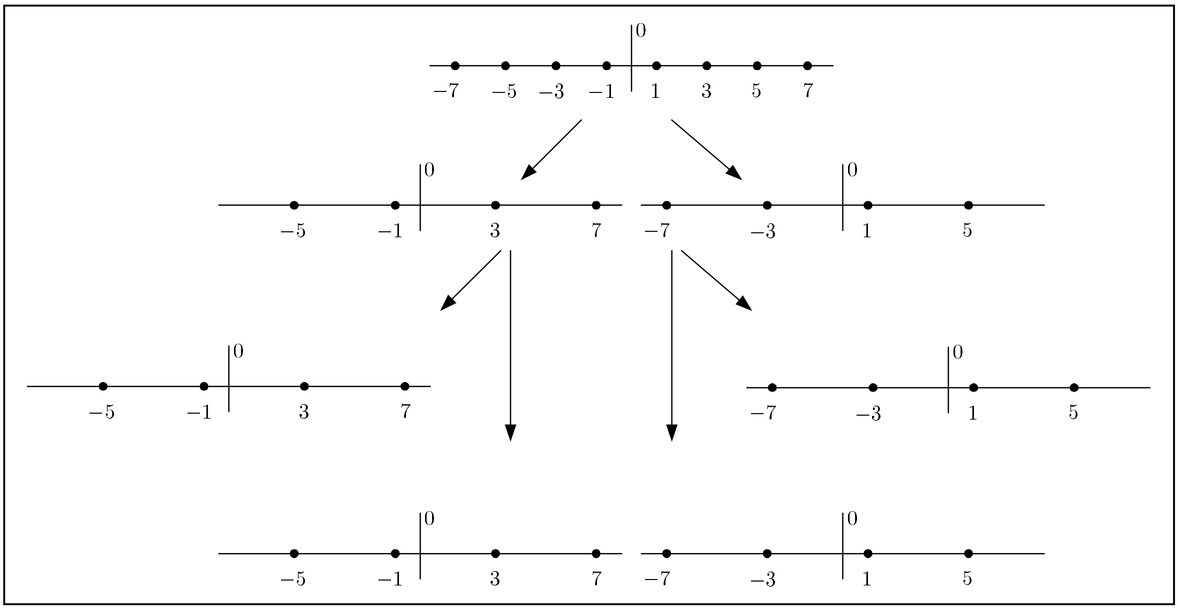
S-72.341 CODING METHODS

Tutorial 10, Solutions

1. (Wicker, problem 15.1)

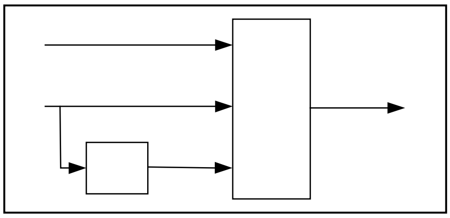
The goal of constellation partitioning: each partition should product subconstellations with increased minimum distance. The partition is given on the figure below. Note that there are two equally valid options for partition at level 3.



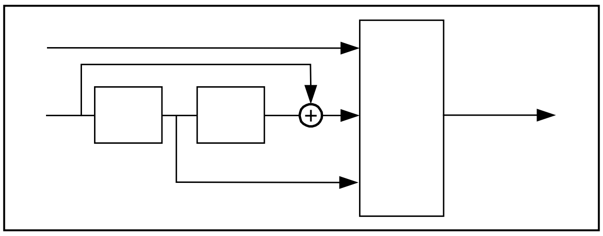
2.

a) We are limited only with amount of the states in the trellis diagram. Amount of these states is equal to the memory length of encoder, otherwise the encoder may be selected arbitrarily.

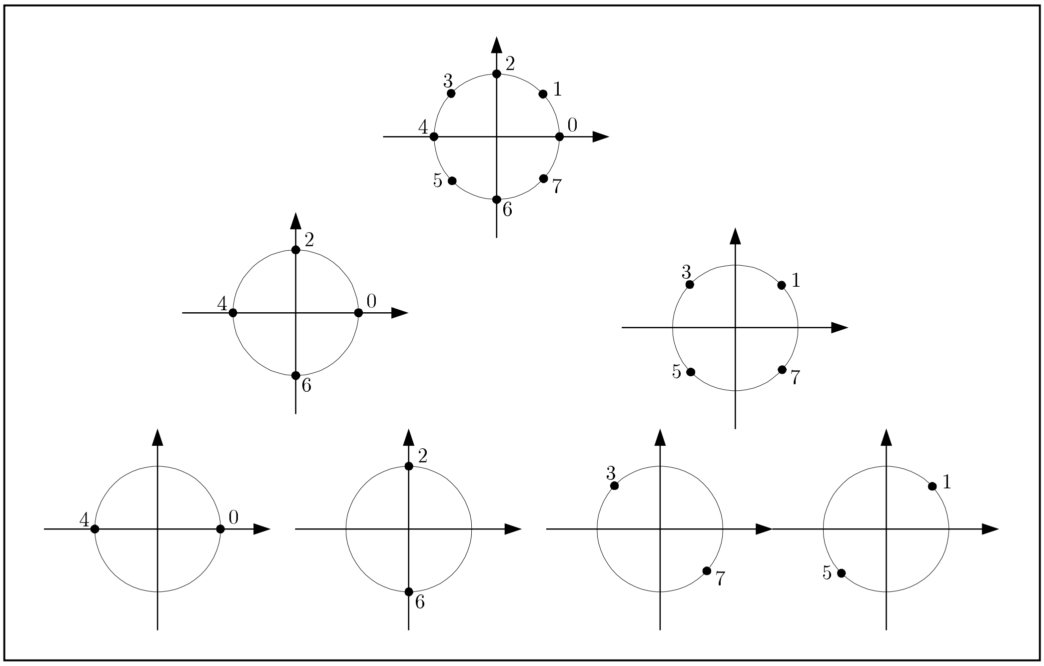
A trellis with two states means that the encoder contains an one memory element. A simplest such decoder is just an delay element as seen in the figure below.



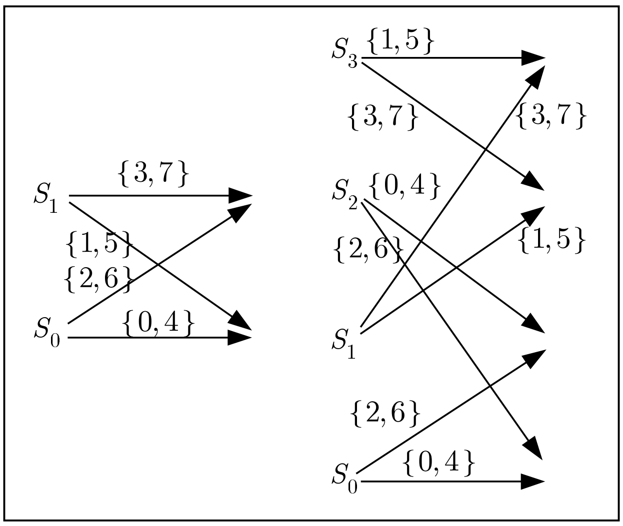
A trellis with four states requires two memory elements. We select a encoder where one output has one delay element and other sum of input signal and signal delayed by two elements. (See figure below.)



a) The state of the trellis is determined by the content of the memory elements. The output of the decoder specifies from which partition the output symbol is selected and non coded bit tells us which symbol from particular partition is transmitted.



We use a following notation $C_0 = \{0,4\}, C_1 = \{2,6\}, C_2 = \{1,5\}, C_3 = \{3,7\}$



b)

Two state trellis.

The code can be calculated as distance between the all zero path and path from state 0 to state 1 and back to 0. The sequence having nearest distance to all zero transmitted symbols in this transition is sequence $\{2,4\}$. By assuming the signal amplitude 1 we get.

$$d_{free} = d^2(0,2) + d^2(0,1) = 2 + 4\sin^2\frac{\pi}{8} = 2.586\,.$$

The asymptotic coding gain compared to the QPSK is

$$\gamma = \frac{2.586}{2} = 1.293 \Rightarrow 1.1\,dB\,.$$

Four state trellis

In case the error is dominated by the distance of the signal points in constellation.

$$d_{free} = d^2(0,4) = 4\,.$$

$$\gamma = \frac{4}{2} = 2 \Rightarrow 3\,dB\,.$$

3. By using both encoders from previous exercise:

   b) Code information bit sequence $[1,0,0,1,1,0,1,1,1,0]$.

We split the sequence into two streams, code one of them, and map the coded sequence into corresponding partitions. The uncoded bits define the symbol that is selected from the partition.

Two state TCM.

Splitting: each first bit goes into first sequence every second to second

$$\begin{matrix}[1,0,1,1,1]\\ [0,1,0,1,0]\end{matrix}$$ after coding the lower sequence we get $[0,0,1,0,1]$.

Mapping results in following sequence of partition sets.

$$\begin{matrix} 0 & 1 & 0 & 1 & 0 \\ 0 & 0 & 1 & 0 & 1 \\ C_0 & C_1 & C_3 & C_1 & C_3 \end{matrix}$$

We select by bit 0 a first symbol in the partition set and by 1 the second one. The transmitted sequence is $\{4,2,5,6,5\}$.

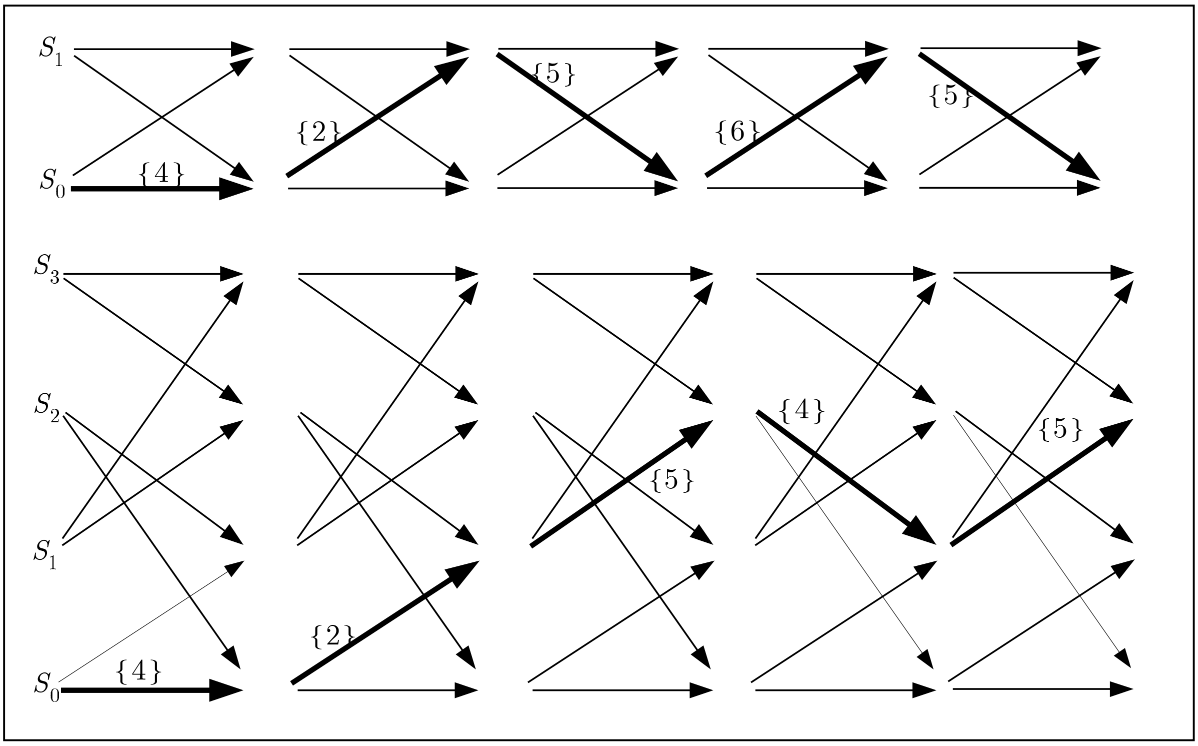
Four state TCM.

The splitting of the input bit sequence gives same input streams as in the two state case. By coding one of those and mapping to the partition sets we get

$$0\quad 1\quad 0\quad 1\quad 0 \Rightarrow \begin{matrix} 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 1 \end{matrix} \Rightarrow C_0 \quad C_1 \quad C_3 \quad C_0 \quad C_3\,.$$

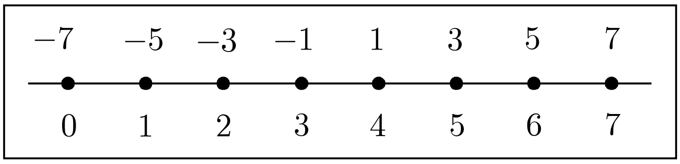
By using the other bit sequence for selecting element from the set we get a sequence $\{4,2,5,4,5\}$.

c) The path trough the trellis for both encoders.



4. (Wicker, problem 15.4)

a) Label the signals, and compute the squared inter-signal distances and average signal energy.



Squared intersignal distances = {4,16,36,64,100,144,164}

Average signal sum over all the symbols where symbol energy ε is weighted with symbol probability p.

$$S = \sum_i \varepsilon(i)\, p(i)$$

$$S = \frac{1}{4}(1 + 9 + 25 + 49) = 21$$

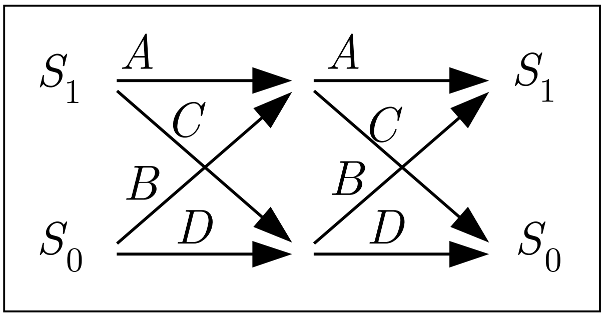
b) Determine the appropriate partitions for the signal constellation.

See exercise 1 above and select the second level from partitioning.

$$A = \{0,4\} \quad C = \{2,6\}$$
$$B = \{1,5\} \quad D = \{3,7\}.$$

c) Construct and label a trellis diagram for the system.

We refer to page 365 of course book (Wicker). From there we know that the amount of states in trellis diagram correspond to the amount of the memory elements in the convolutional encoder. The transitions are corresponding outputs to given input. Each output is mapped to the corresponding signal label and assigned to the corresponding transition branch.



d) Determine the squared minimum free distance and asymptotic coding gain for the system relative to an uncoded 4-AM system.

The minimum distance in an uncoded 4-AM system is the minimum distance between any pair of symbols in the constellation. By normalising the signal amplitude to be 1 we get for the 4-AM

$$d_{free\,/\,uncoded} = 2\,,\; S = 5\,.$$

The minimum distance for trellis codes is calculated as minimum of distances of any pairs of paths trough the trellis and any parallel transitions.

$$d_{free\,/\,parallel} = 4\,,$$
$$d_{free\,/\,nonparallel} = \sqrt{2^2 + 2^2} = \sqrt{8}\,,$$
$$d_{free} = \min\left(d_{free\,/\,parallel}, d_{free\,/\,nonparallel}\right) = \sqrt{8}\,.$$

Asymptotic coding gain γ is the performance improvement of the coded system relative to the uncoded system.

$$\gamma = \left(\frac{S_{uncoded}}{S_{coded}}\right)\left(\frac{d^2_{free\,/\,coded}}{d^2_{free\,/\,uncoded}}\right) = \gamma_C \cdot \gamma_D$$

where

$S_{\#}$ stands for normalised average received energy,

$d_{\#}$ stands for minimum free distance.

$$\gamma = \left(\frac{5}{21}\right)\left(\frac{8}{4}\right) = 0.4 \Rightarrow -3.19\, dB\ .$$

We can conclude that the one state encoder does not provide enough coding gain to make up for expanded constellation.