S-72.341 CODING METHODS

Tutorial

1. Let $\mathbf{a}$, $\mathbf{b}$ ja $\mathbf{c}$ be binary words of length n. Show that the triangle inequality, $d(\mathbf{a},\mathbf{b}) + d(\mathbf{a},\mathbf{c}) \geq d(\mathbf{b},\mathbf{c})$, holds in GF(2). $d(\mathbf{x},\mathbf{y})$ is the Hamming distance between words $\mathbf{x}$ and $\mathbf{y}$.

2. (Exam 9.1.2002) An $(8,4)$ binary linear block code C has parity-bit equations

$$\begin{aligned} v_0 &= u_0 + u_1 + u_2 \\ v_1 &= u_0 + u_1 + u_3 \\ v_2 &= u_0 + u_2 + u_3 \\ v_3 &= u_1 + u_2 + u_3 \end{aligned}$$

   where u_0, u_1, u_2, u_3 are message bits and v_0, v_1, v_2, v_3 are parity-check bits. Codewords are 8-tuples of the form $(v_0, v_1, v_2, v_3, u_0, u_1, u_2, u_3)$.

   a) Find the generator matrix and the parity-check matrix for this code.

   b) What is the minimum distance of the code? How many errors can it detect? How many errors can it correct?

3. (Exam 15.5.2001) Consider the binary code that has the following parity-check matrix:

$$\mathbf{H} = \begin{bmatrix} 1 & 0 & 0 & 1 & 1 & 0 & 1 \\ 0 & 1 & 0 & 1 & 0 & 1 & 1 \\ 0 & 0 & 1 & 0 & 1 & 1 & 1 \end{bmatrix}.$$

   Give well-motivated answers to the following questions.

   a) What is the dimension of the code?

   b) How many codewords does the code have?

   c) What is the minimum distance of the code?

   d) Is 1111111 a codeword? What about 1011011?

   e) How many errors is the code able to correct? Detect? If any of the words in the previous question (1111111, 1011011) is in error, correct it/them.

f) Give a generator matrix **G** for the code and encode the word 0101.

4. (Wicker, problems 4.4, 4.5) Find the lower and upper bound on required redundancy for the following codes.

   a) A single-error correcting binary code of length 7.

   b) A single-error correcting binary code of length 15.

   c) A triple-error correcting binary code of length 23.

   d) A triple-error correcting 4-ary code of length 23.

   e) A triple-error correcting 16-ary code of length 23.

5. (Wicker, problem 4.8) Find the length, dimension, and minimum distance for the linear codes defined by the following parity-check matrices.

   a) $\mathbf{H} = \left[\begin{array}{cccc|ccc} 1 & 0 & 0 & 0 & 1 & 0 & 1 \\ 0 & 1 & 0 & 0 & 1 & 1 & 1 \\ 0 & 0 & 1 & 0 & 1 & 1 & 1 \\ 0 & 0 & 0 & 1 & 0 & 1 & 1 \end{array}\right]$ (binary)

   b) $\mathbf{H} = \begin{bmatrix} 1 & 0 & 1 & 0 & 0 & 0 & 1 \\ 1 & 0 & 0 & 1 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 1 & 1 \\ 1 & 1 & 0 & 1 & 0 & 0 & 1 \end{bmatrix}$ (binary)

   c) $\mathbf{H} = \begin{bmatrix} 0 & 1 & 1 & 1 \\ 1 & \alpha & \alpha^2 & 0 \end{bmatrix}$ α primitive in GF(4).

6. (Wicker, problem 4.9) Consider the 4-ary code C defined by the following parity-check matrix, α is primitive in GF(4).

$$\mathbf{H} = \begin{bmatrix} \alpha & \alpha^2 & 1 & 1 \\ \alpha^2 & \alpha & 1 & 0 \end{bmatrix}$$

   a) Write out the 16 code words in C.

   b) Does C achieve the Gilbert bound on redundancy?

   c) Does C exceed the Hamming bound on redundancy?

   d) Does C achieve the Singleton bound?

7. Find a necessary condition on the code word length n so that a binary code with $d_{\text{min}} = 3$ is perfect. What is the condition for a perfect q-ary code ($d_{\text{min}} = 3$)?