S-72.341 CODING METHODS

Tutorial 4

1. Generator matrix of a linear block code is

$$\mathbf{G} = \begin{bmatrix} 0 & 0 & 1 & 1 & 1 & 0 & 1 \\ 0 & 1 & 0 & 0 & 1 & 1 & 1 \\ 1 & 0 & 0 & 1 & 1 & 1 & 0 \end{bmatrix}.$$

   a) Express the generator matrix in systematic form $\mathbf{G} = \left[\mathbf{I} \mid \mathbf{P}\right]$.

   b) What is the parity-check matrix $\mathbf{H}$?

   c) Construct the syndrome table.

   d) What is the minimum Hamming distance of the code $d_{\min}$?

   e) Show that the code word corresponding to the message $\mathbf{m} = \begin{bmatrix} 1 & 0 & 1 \end{bmatrix}$ is orthogonal with $\mathbf{H}$.

2. (Exam 6.9.2001) Show that the binary code C of all *palindromes* of length n, that is, words that are the same read forwards or backwards, is linear. Describe the code words $(x_1, x_2, \ldots, x_n)$ using equations, give a parity check matrix for C, and determine the number of errors it detects.

3. (Wicker, problem 4.14) Compute the weight distribution for a (7,4) Hamming code.

4. (Wicker, problem 4.17) Construct a parity check matrix for a (13,10) 3-ary Hamming code [i.e. with code symbols taken from GF(3)].

5. (Wicker, problems 5.1 - 5.5) Let C_1 be the binary cyclic code of length 15 generated by $g(x) = x^5 + x^4 + x^2 + 1$

   a) Compute the parity-check polynomial for C_1 and show that $g(x)$ is a valid generator polynomial.

   b) Determine the dimension of C_1 and compute the number of code words in C_1.

   c) Construct the parity check and generator matrices for C_1.

d) Compute the code polynomial in C_1 and the associated code word for the following message polynomials using the polynomial multiplication encoding technique: i) x^2 ii) $x^9 + x^4 + x^2 + 1$

e) Compute the code polynomial in C_1 and the associated code word for the message polynomials of the previous part using the systematic encoding technique. Verify that the messages have been systematically encoded.

6. (Wicker, problem 5.6) Given the code C_1 and the parity-check polynomial $h(x)$ derived in the previous problem, compute the syndrome $s(x)$ for the following received polynomials.

   a) x^{10}

   b) $x^3 + x^2$

   c) $x^{14} + x^{10} + x^5 + x^3$

   d) $x^8 + x^6 + x + 1$