S-72.341 CODING METHODS

Tutorial 8

1. (Wicker, problem 11.1) Prove that the convolutional encoder in Figure 1 below generates a linear code.

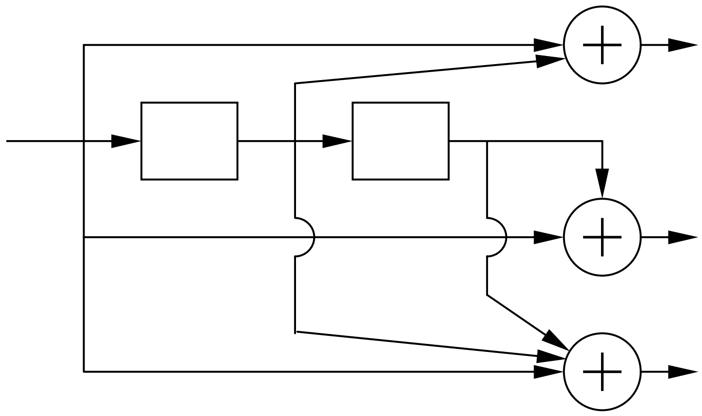

Figure 1. A rate 1/3 convolutional encoder.

2. (Wicker, problem 11.2)

   a) Find the impulse response for the encoder in Figure 1.

   b) Find the transfer function matrix for the encoder in Figure 1.

   c) Use the transfer function matrix to determine the code word associated with the input sequence $\mathbf{x}$=(11101).

   d) What is the code rate?

3. (Wicker, problem 11.4) Construct an encoder for the convolutional code represented by the following transfer function matrix.

$$\mathrm{G}(D) = \begin{bmatrix} 1+D+D^2 & 1+D+D^3 & 1+D^2+D^3 \\ 1+D^3 & 1+D^2 & 1+D+D^3 \end{bmatrix}$$

4. (Wicker, problem 11.7) Construct a state diagram for the encoder in Figure 1.

5. (Wicker, problem 11.11)

   a) Prove that the encoder with the following impulse responses generates a catastrophic convolutional code. $\mathbf{g}^{(0)} = (0111)$, $\mathbf{g}^{(1)} = (1001)$.

   b) Find an infinite weight input sequence $\mathbf{x}$ that corresponds to a finite weight output sequence $\mathbf{y}$ for this encoder.

6. Consider the (3,1,2) code with $G(D) = \begin{bmatrix} 1+D^2 & 1+D+D^2+D^3 \end{bmatrix}$

   a) Find the GCD of its generator polynomial.

   b) Draw the encoder state diagram.

   c) Find an infinite-weight information sequence that generates a code word of finite weight.

   d) Is this code catastrophic of non-catastrophic?

7. The generator polynomials of a convolutional code, given in octal form, are $(4,5,7)$.

   a) Sketch the encoding circuit.

   b) Is the code systematic?

   c) Find the transfer function $T(D)$ and the minimum free distance d_{free} of the code.

8. An encoded message, transmitted over a BSC using the code of the previous problem, is 101001011110111 at the output of a hard decision detector. Decode the message using the Viterbi algorithm. The shift register values of the encoder are initially 000, and the message is terminated with a stream of zero bits.

9. An encoded message, transmitted over a BSC using the code of the problem 7, is y = [1.1139 1.2993 1.0568 -1.0944 -1.8377 1.2943 0.0662 -0.1643 2.0667 -1.8236 -0.7580 0.0334 -2.5940 0.2510 -0.4236] at the output of a soft decision detector. Decode the message using the Viterbi algorithm for both soft and hard decision decoding. The shift register values of the encoder are initially 000, and the message is terminated with a stream of zero bits. What is the signal to noise ratio at the decoder output if to assume that the transmitted signal was 111001011000000 and average signal energy per bit is normalised to be 3?