S-72.341 CODING METHODS

Tutorial 9

1. (Wicker, problem 12.9) Assume that the encoder from Problem 12.1 (see the book) is used over a symmetric memoryless channel. The bit metrics are as follows.

y/r	$\underline{0}$	0	1	$\underline{1}$
$y=0$	0	1	3	6
$y=1$	6	3	1	0

   Find the maximum-likelihood code word corresponding to the following received sequence. $\mathbf{r} = (1\underline{0}, 01, \underline{1}\,\underline{0}, \underline{1}\,\underline{1}, 00, \underline{1}\,\underline{0}, \underline{1}1, 0\underline{0})$.

2. (Wicker, problem 15.1) Hamming code is used in a pure ARQ protocol over a binary symmetric channel with crossover probability $p = 0.05$. Each packet consists of a single code word. The application has a 3 km, free space, line-of-sight channel. The transmitter has a 10^6 bit-per-second transmission rate, and the receiver processing delay is negligible. The feedback channel is assumed to be error-free. Determine the throughput for the following three protocols.

   a) SW-ARQ b) GBN-ARQ c) SR-ARQ.

3. (Wicker, problem 15.2) Determine the reliability performance for the system described in Problem 2.

4. (Wicker, problem 15.4) Repeat Problem 2, but this time assume that each packet consists of four code words. What is the probability of retransmission as a function of channel crossover probability when each packet consists of n code words?

5. (Wicker, problem 15.5) Repeat Problem 2 but this time assume that the feedback channel has an error rate of $P_{AKC \to RQ} = P_{RQ \to AKC} = 0.2$.

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Tutorial 9, April 4, 2003, **Solutions**

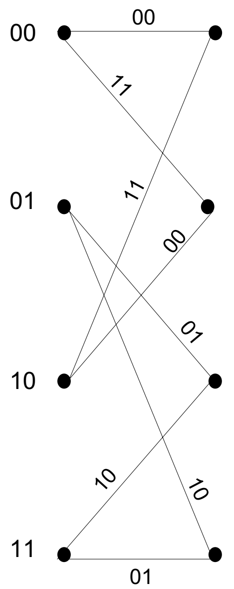
6. (Wicker, problem 12.9) Assume that the encoder from Problem 12.1 (see the book) is used over a symmetric memoryless channel. The bit metrics are as follows.

y/r	$\underline{0}$	0	1	$\underline{1}$
$y=0$	0	1	3	6
$y=1$	6	3	1	0

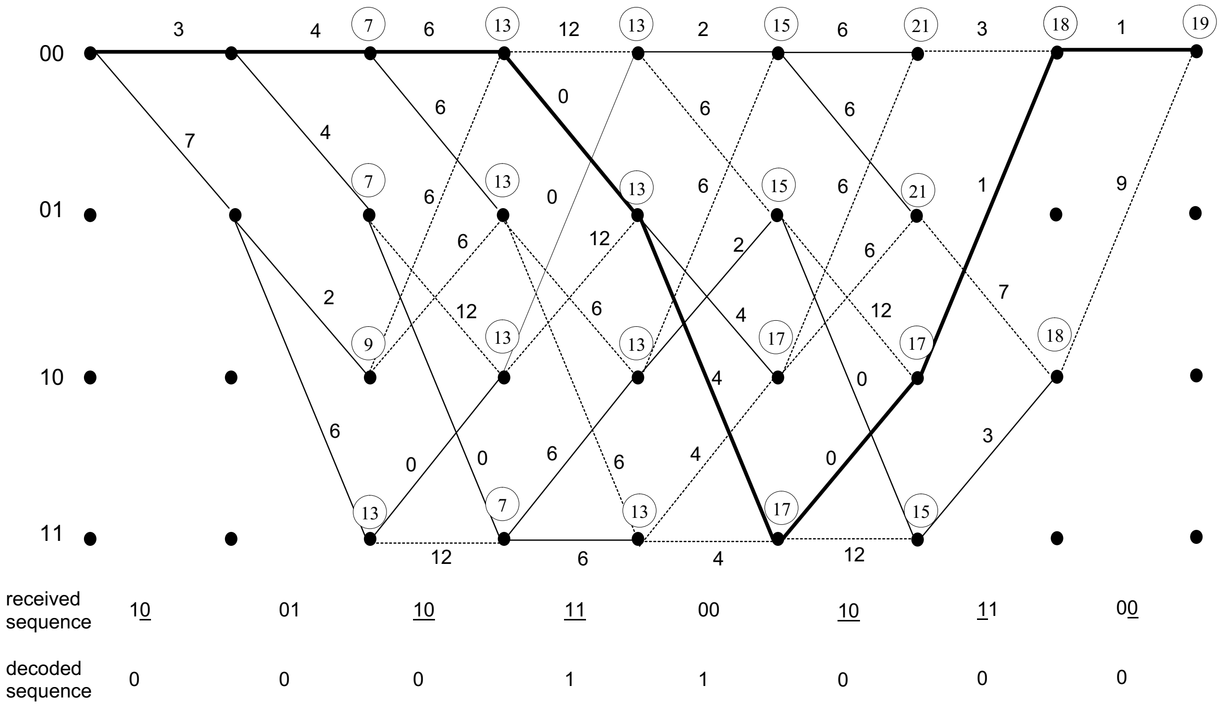
Find the maximum-likelihood code word corresponding to the following received sequence.

$$\mathbf{r} = (1\underline{0}, 01, \underline{1}\,\underline{0}, \underline{1}\,\underline{1}, 00, \underline{1}\,\underline{0}, \underline{1}1, 0\underline{0}).$$

One stage of the trellis and the branch outputs are shown below.



The decoding procedure is illustrated in the following figure (check it!). Dashed lines and solid lines indicate terminating branches and surviving branches, respectively. The survivor path is indicated with bold solid line. In case of ties, upper transition was chosen.



7. (Wicker, problem 15.1) Hamming code is used in a pure ARQ protocol over a binary symmetric channel with crossover probability $p = 0.05$. Each packet consists of a single code word. The application has a 3 km, free space, line-of-sight channel. The transmitter has a 10^6 bit-per-second transmission rate, and the receiver processing delay is negligible. The feedback channel is assumed to be error-free. Determine the throughput for the following three protocols.

   a) SW-ARQ b) GBN-ARQ c) SR-ARQ.

   The throughput (η) for an ARQ error control system is the average number of encoded data packets accepted by the receiver in the time it takes the transmitter to send a single k-bit data packet. A packet is accepted if the receiver does not detect any error. This can happen if the received packet is error free or the receiver could not detect errors. The undetected error probability is P_e. The packet is retransmitted as long as any error is detected. The detected error probability is P_r. The error remains undetected if the received code word is transformed to another allowed code word. The error patters that generate such code words are given by the weight distribution. The probability of detected error is simply the probability that one or more bit errors occurs in the received word, subtracting the probability that the resulting pattern is undetectable. We assume that the system uses a (7,4) Hamming code. The code rate is $R = 4/7$. The weight enumerator function for this code is $1 + 7x^3 + 7x^4 + x^7$ by equation (4-12). Thus,

$$P_e = 7p^3(1-p)^4 + 7p^4(1-p)^3 + p^7 = 7.5 \times 10^{-4}$$

$$P_r = 1 - (1-p)^7 - 7.5 \times 10^{-4} = 0.30\,.$$

$$\text{Roundtrip propagation delay } = \lambda_1 + \lambda_2\,.$$

$$\Rightarrow \frac{2 \cdot 3000\,\mathrm{m}}{3 \cdot 10^8 \,\frac{\mathrm{m}}{\mathrm{s}}} = 2 \times 10^{-5}\, s$$

The parameters Γ and N are

$$\Gamma = \left(1 \cdot 10^6\right) \cdot \left(2 \cdot 10^{-5}\right) = 20\,\text{bits}$$
$$N = 2\,.$$

Γ is the amount of bits that could have been transmitted during the idle time. N is the amount of packets that should be retransmitted in the case of error.

The equations for throughput are derived in the chapter 15.1.

a) From equation 15-5

$$\eta_{SW} = R\left(\frac{1 - P_r}{1 + \Gamma / n}\right) = \frac{4}{7}\left(\frac{1 - 0.3}{1 + \frac{20}{7}}\right) = 0.104\,.$$

b) From equation 15-7

$$\eta_{GBN} = R\left(\frac{1 - P_r}{1 + P_r(N-1)}\right) = \frac{4}{7}\left(\frac{1 - 0.3}{1 + 0.3(2-1)}\right) = 0.31$$

c) From equation 15-8

$$\eta_{SR} = R(1 - P_r) = \frac{4}{7}(1 - 0.3) = 0.40\,.$$

8. (Wicker, problem 15.2) Determine the reliability performance for the system described in Problem 2.

The reliability $P(E)$ is computed by summing the events when the packet can be accepted. The packet is accepted if it has no errors during first transmission, or had an error in first transmission but no errors during first retransmission, and so on.

$$
\begin{aligned}
P(E) &= P_e + P_r P_e + P_r^2 P_e + P_r^3 P \dots \\
&= P_e \sum_{k=0}^{\infty} P_r^k \\
&= \frac{P_e}{1 - P_r} \\
&= \frac{7.5 \cdot 10^{-4}}{1 - 0.3} = 1.07 \cdot 10^{-3}
\end{aligned}
$$

9. (Wicker, problem 15.4) Repeat Problem 2, but this time assume that each packet consists of four code words. What is the probability of retransmission as a function of channel crossover probability when each packet consists of n code words?

If a packet contains more than one code word, packet error is equal to probability that any of those words contains an error. Let P_1 denote the probability that a reserved code word contains detectable errors.

In problem 3 probability of one packet error is $P_1 = 0.3$. Here it is extended to the case of n packet errors:

$$P_r = 1 - (1 - P_1)^4 = 0.76.$$

In general, $P_r = 1 - (1 - P_1)^n$ for packets with n code words.

We repeat calculations in problem 3 with this new probability P_r.

a) From equation 15-5

$$\eta_{SW} = R\left(\frac{1 - P_r}{1 + \Gamma / n}\right) = \frac{4}{7}\left(\frac{1 - 0.76}{1 + \frac{20}{7}}\right) = 0.036.$$

b) From equation 15-7

$$\eta_{GBN} = R\left(\frac{1 - P_r}{1 + P_r(N - 1)}\right) = \frac{4}{7}\left(\frac{1 - 0.76}{1 + 0.76(2 - 1)}\right) = 0.0018.$$

c) From equation 15-8

$$\eta_{SR} = R(1-P_r) = \frac{4}{7}(1-0.76) = 0.14.$$

10. (Wicker, problem 15.5) Repeat Problem 2 but this time assume that the feedback channel has an error rate of $P_{AKC\to RQ} = P_{RQ\to AKC} = 0.2$.

The throughput of the system has to consider also the error due to the feedback. This is done in the chapter 15.2 of the coursebook.

a) From the equations 15-13 to 15-14 we can calculate the trhoughput as:

$$E_{SW} = \frac{(1-\Gamma/n)\left(1-P_rP_{ACK\to RQ} - P_{RQ\to ACK} + P_rP_{RQ\to ACK}\right)}{\left(1-P_{ACK\to RQ}\right)(1-P_r)\left(1-P_{RQ\to ACK}\right)}$$

$$\eta_{SW} = \left(\frac{k}{n}\right)\left(\frac{1}{E_{SW}}\right) = R\left(\frac{\left(1-P_{ACK\to RQ}\right)(1-P_r)\left(1-P_{RQ\to ACK}\right)}{(1+\Gamma/n)\left(1-P_rP_{ACK\to RQ} - P_{RQ\to ACK} + P_rP_{RQ\to ACK}\right)}\right)$$

$$\eta_{SW} = \left(1-P_{ACK\to RQ}\right)\eta_{SW}^{NOISE-FREE\ Feedback} = 0.08$$

b) From the equations 15-16 and 15-17 we can calculate the trhoughput as:

$$E_{GBN} = \frac{\left[\begin{array}{l}1-P_{RQ\to ACK} - P_rP_{ACK\to RQ} + P_rP_{RQ\to ACK} \\ +(N-1)\left(\begin{array}{l}P_{ACK\to RQ} + P_r - P_{ACK\to RQ}P_{RQ\to ACK} \\ -P_rP_{RQ\to ACK} - 2P_rP_{ACK\to RQ} + 2P_rP_{ACK\to RQ}P_{RQ\to ACK}\end{array}\right)\end{array}\right]}{\left(1-P_{ACK\to RQ}\right)(1-P_r)\left(1-P_{RQ\to ACK}\right)}$$

$$\eta_{GBN} = \left(\frac{k}{n}\right)\left(\frac{1}{E_{GBN}}\right) = 0.18$$

c) From the equations 15-19 and 15-20 we can calculate the trhoughput as:

$$E_{SW} = \frac{1-P_rP_{ACK\to RQ}}{(1-P_r)\left(1-P_{ACK\to RQ}\right)}$$

$$\eta_{SW} = \left(\frac{k}{n}\right)\left(\frac{1}{E_{SW}}\right) = R\frac{(1-P_r)\left(1-P_{ACK\to RQ}\right)}{1-P_rP_{ACK\to RQ}} = 0.34$$