S-72.341 Coding Methods (3 cr) P spring 2004

Home Assignment 2: Block Codes

This assignment consists of three mandatory tasks. Depending on your student number, **solve only one instance of each task**. Let your student number be *abcde*F ($0 \leq a, b, c, d, e \leq 9$, F a letter). For the first task, solve instance i, where $i \equiv e \pmod 5$. For the second task, solve instance j, where $a+b+c+d+e \equiv j \pmod 4$. For the third task, solve instance k, where $k \equiv d \pmod 4$. All answers must be motivated; if the answer is just yes/no or a number, it will not be accepted.

Task 1. We consider the ternary code (that is, we consider GF(3) and operate on $\{0, 1, 2\}$ modulo 3) that has the following parity-check matrix:

$$\boldsymbol{H} = \begin{bmatrix} 1 & 0 & 0 & 0 & 0 & 2 & 0 & 2 & 1 & 1 & 2 \\ 0 & 1 & 0 & 0 & 0 & 2 & 2 & 0 & 2 & 1 & 1 \\ 0 & 0 & 1 & 0 & 0 & 2 & 1 & 2 & 0 & 2 & 1 \\ 0 & 0 & 0 & 1 & 0 & 2 & 1 & 1 & 2 & 0 & 2 \\ 0 & 0 & 0 & 0 & 1 & 2 & 2 & 1 & 1 & 2 & 0 \end{bmatrix}.$$

(0) What is the dimension of the code and how many errors it may detect?
(1) How many codewords does the code have?
(2) This code has minimum distance 5. You are not asked to prove that the minimum distance is exactly 5, but you need to find a linear combination of columns of $\boldsymbol{H}$ that proves that the minimum distance is ≤ 5.
(3) Is the code perfect? (Hint: you may consider the number of codewords and the minimum distance.)
(4) Give a generator matrix $\boldsymbol{G}$ for the code.

Task 2. Let C_1 be the binary cyclic code of length 15 generated by $g(x) = x^5 + x^4 + x^2 + 1$. Compute the polynomial in C_1 and the associated code word for the following message polynomials using systematic technique:
(0) $x^7 + x^3 + x$; (1) $x^8 + x^7 + x^6 + x^5 + x^4$; (2) $x^9 + x^4 + x + 1$; (3) $x^2 + x + 1$.

Task 3. Find the highest-degree generator polynomial for the binary cyclic code containing the following code words:
(0) 1111111 (1) 1101001 (2) 1100101 (3) 1010101

Return your solutions into the box underneath the bulletin-board of the course no later than March 30, 2004. Remember to include all your personal data. Co-operation is not allowed.