S-72.4210 PG Course in Radio Communications

Orthogonal Frequency Division Multiplexing

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Outline

- OFDM History
- OFDM Applications
- OFDM Principles
- Spectral shaping
- Synchronization
- Interleaving and channel diversity
- Real channel estimation and imperfect interleaving
- Summary
- References
- homework

OFDM History

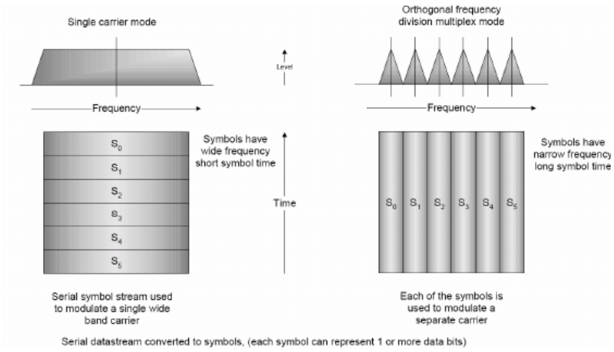
- A principle for multi-channel transmission over a band-limited channel was proposed in 1966 [3]
- An analysis on this parallel system was done in 1967 [4]
  - Crosstalk between adjacent channels
  - The individual channels' performance.
- DFT being used in baseband modulation/demodulation in 1971 [5]
  - Guard Interval (GI) and raised-cosine window being utilized.
  - Suffering from the loss of orthogonality between channels
- Cyclic Prefix (CP) being used in 1980 [6]
  - Orthogonality problem being solved

OFDM Applications

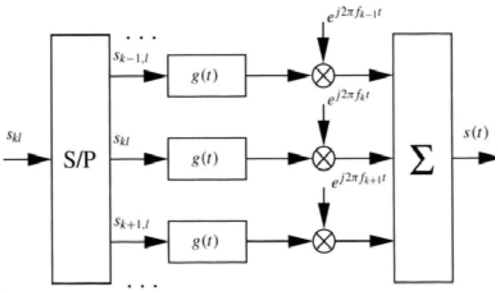
- Broadcasting
  - DAB (Digital Audio Broadcasting)
  - DVB (Digital Video Broadcasting)
- WLAN (Wireless Local Area Networks)
  - IEEE 802.11a,
  - HiperLAN/2
- WirelessMAN (Wireless Metropolitan Area Networks)
  - IEEE 802.16 (WiMAX)

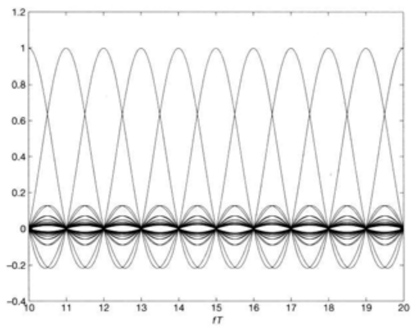
OFDM Principles

- Motivation
  - To overcome frequency-selective fading environment
  - Simpler channel equalization



OFDM Principles





Spectral efficiency

OFDM Principles

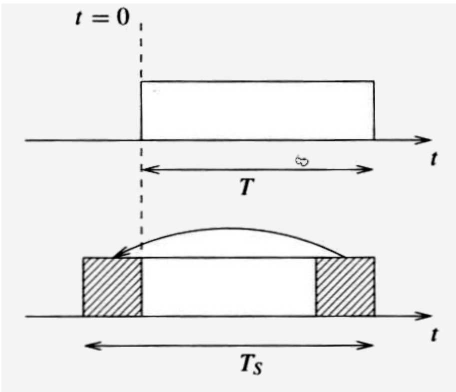
- Bandwidth W being divided into K sub-carriers, leading to carrier spacing Δf, satisfying $\Delta f = \frac{W}{K}$
- Symbol duration T satisfying $T = \frac{1}{\Delta f}$
- To avoid ISI: $\tau_m << T$

  τ_m : delay spread

  $f_c = \frac{1}{\tau_m}$: coherence bandwidth
- Due to time incoherence of channel: $T << \frac{1}{\upsilon_{\max}}$
  - $\upsilon_{\max}$: doppler frequency
  - $t_c = \frac{1}{\upsilon_{\max}}$: coherence time

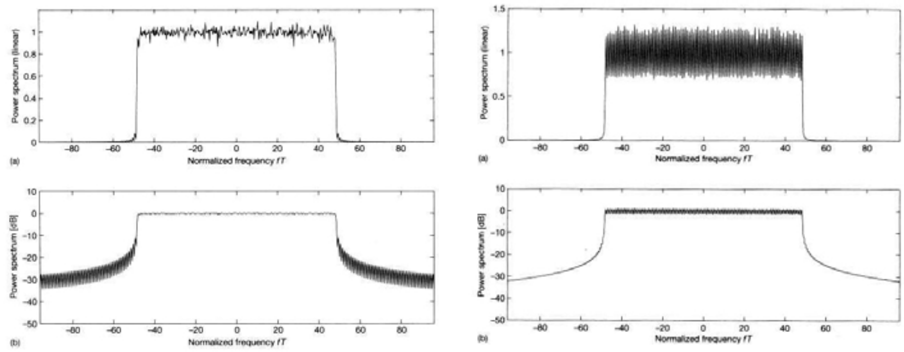
OFDM Principles

- Cyclic prefix (CP): combat ISI and ICI
- CP transforms a linear-convolution channel to a cyclic convolution channel
- Orthogonality over dispersive channel if CP is long enough.
- CP introduces a loss in SNR to mitigate interference.



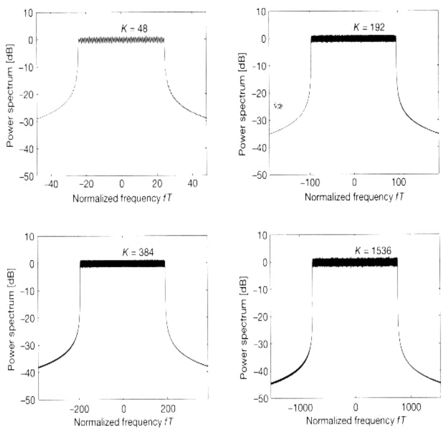
OFDM Principles

- Spectral property of OFDM



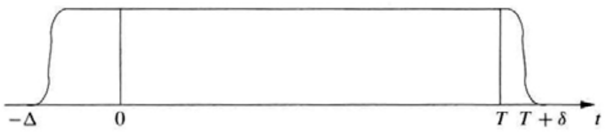
Spectral shaping

- For the same bandwidth W, the number of sub-carriers K affects the side-lobes of power density spectra.
- Side-lobes decay faster for a larger K.
- Side-lobe reduction in the order of -70 dB for broadcasting system.



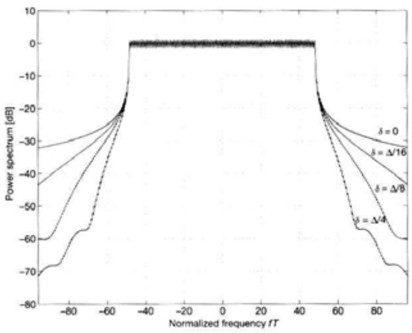
Spectral shaping

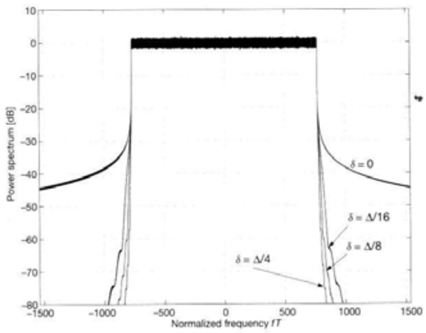
- Spectral filtering requires high computational complexity.
- Pulse shaping method is preferred.
  - 1 for $-T_{cp}+\delta < t < T$
- Effective CP is reduced by δ



Spectral shaping

- An example for K=96, raised-cosine pulse shape
- An example for K=1536, raised-cosine pulse shape (one mode in DAB, -71dB at fT=970)



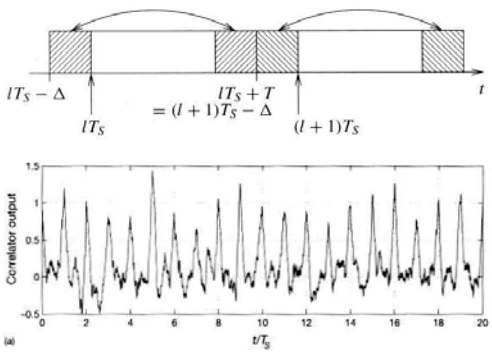


Time synchronization

- OFDM is robust to time sync. error due to GI
  - e.x. $\triangle = T_s/5 = T/4$ allows timing uncertainty of 20% at most
- In DAB, each transmission frame starts by a *null symbol*
- Cross-correlation by training sequence
  - Subcarriers are modulated with known symbols
  - Local copy of the known symbols
  - Cross-correlating the received symbols and the local copy symbols
- Autocorrelation by 2 identical symbol component
  - One OFDM symbol of twice the symbol time
  - Two identical symbol components in sequence
  - Autocorrelating the OFDM symbol
  - Used in IEEE 802.11a and HIPERLAN/2

Time synchronization

- Autocorrelation based on GI
  - Cyclically extended part of an OFDM symbol occurs twice
  - Autocorrelating s(t) and s(t+T) in an interval of $\triangle$

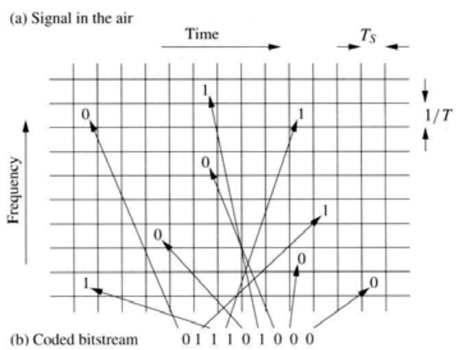


Frequency synchronization

- Sub-carrier spacing $\triangle f = 1/T$ is much smaller than the total bandwidth W due to long T.
- A residual frequency offset δf causes:
  - Corrupted orthogonality between sub-carriers
  - Time-variant phase rotation of the received symbols
- In wireless LAN systems, frequency synchronization is achieved by subspace method
  - A special OFDM symbol at the beginning of a burst is defined
  - Only 12 sub-carriers are modulated
  - Minimizing the energy in the null subcarriers

Interleaving and channel diversity

- A time-domain equalizer may provides path diversity
- OFDM allows frequency-domain coding
- Diversity gain of coding can be achieved utilizing interleaving



Interleaving and channel diversity

- Diversity degree of coding can be achieved if diversity degree of channel is larger.
- For a GWSSUS channel, the channel samples:

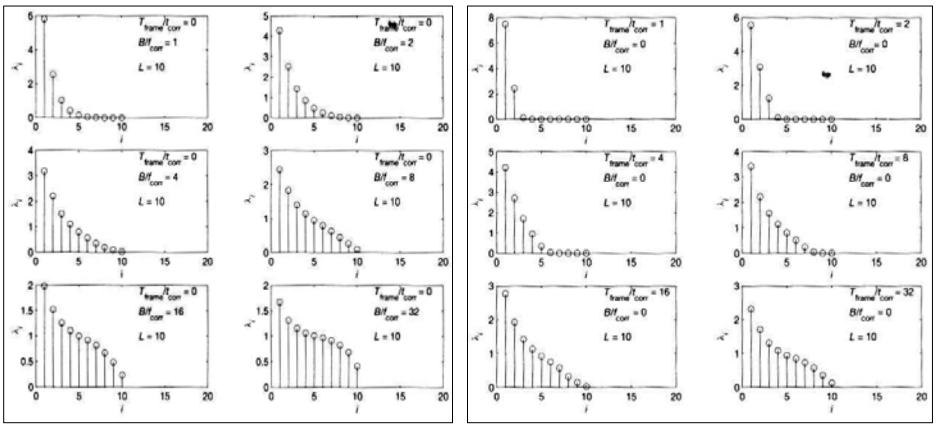
  $c_i = H(f_i, t_i),\ \ i = 1, 2, \ldots, K$

  $\mathbf{R} = E\{\mathbf{c}\mathbf{c}^H\}$

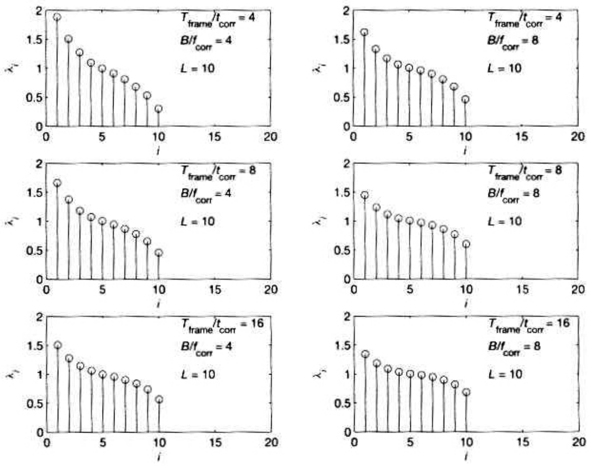
  $\mathbf{U}\mathbf{R}\mathbf{U}^H = \mathbf{D}$

- Eigenvalue plot for K=64, exponential delay power profile , isotropic Doppler spectrum and normalized bandwidth $X = W\tau_m$ normalized frame length $T_{frame}\,\nu_{max}$

Interleaving and channel diversity



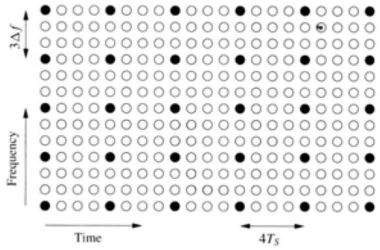
Interleaving and channel diversity



Real channel estimation and imperfect interleaving

- Powerful interleaving
  - An sufficient decorrelation of symbols
  - An incoherent channel is required
- Coherent demodulation
  - An coherent enough channel is required
- An convolutionally coded QAM with real channel estimation and imperfect interleaving:
  - $T_s = T + \triangle$, $\triangle = T/4$ (adopted by most OFDM system)
  - Convolutional code with generators $(133, 171)_{oct}$, $R_c = 1/2$
  - K=1536
  - Pilot insertion

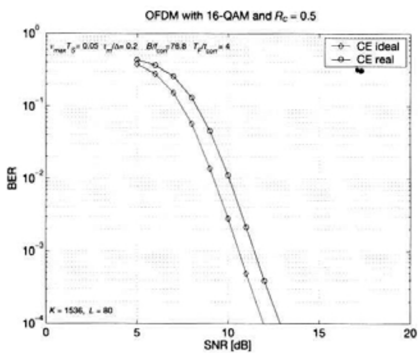
Real channel estimation and imperfect interleaving

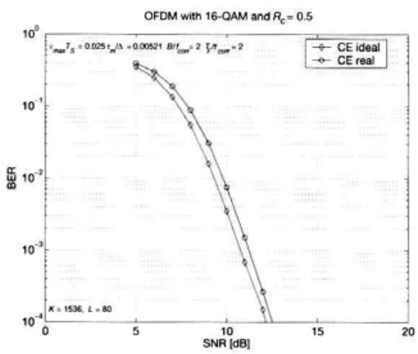


- Rayleigh fading channel with isotropic Doppler spectrum (ν_{max}) and exponential delay power spectrum (τ_m)
- 1-D*1-D Channel estimator by Wiener filter
- Time variance $\nu_{max}T_s$=0.05, frequency selectivity $\tau_m = \triangle/5$
- Time interleaver T_F=4 t_c , frequency interleaver W=76.8 f_c

Real channel estimation and imperfect interleaving

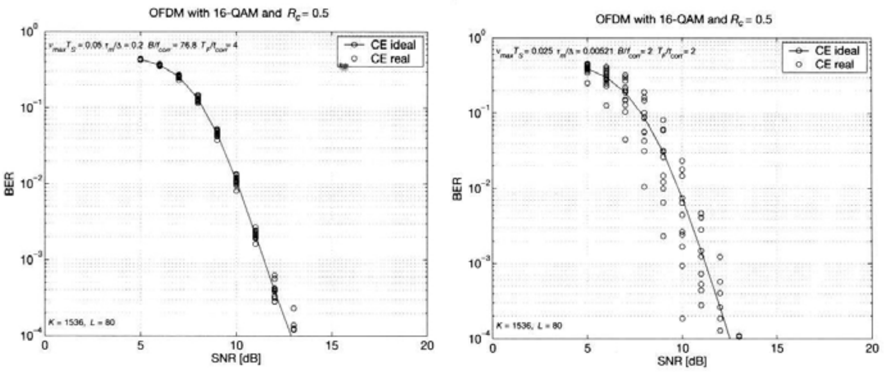
- $\nu_{max}T_s$=0.025, smaller delay spread W=2 f_c
  - typical urban situation





Real channel estimation and imperfect interleaving

- Plots for individual frames



Summary

- OFDM is robust against multi-path propagation.
- Its typical applications are in tough radio environments.
- High peak-to-average-power ratio and synchronization are two main obstacles for using OFDM
- OFDM is applicable to both coherent and differential modulation
- Interleaving allows symbols to be spread in both time and frequency

References

[1]H. Schulze, C. Lüders, "*The Theory and Applications of OFDM and CDMA*", John Wiley & Sons, 2005..

[2]Ove Edfors etc., "*An Introduction to Orthogonal Frequency Division Multiplexing*", Luleå University of Technology, http://epubl.luth.se/avslutade/0347-0881/96-16/index.html.

[3]R. W. Chang, " Synthesis of band-limited orthogonal signals for multichannel data transmission", *Bell System Tech. J.*, 45:1775-1796, Dec. 1966.

[4]B. R. Saltzberg, "Performance of an efficient parallel data transmission system", *IEEE Trans. Commun.*, COM-15(6): 805-811, Dec. 1967.

[5]S. B. Weinstein, P. M. Ebert, "Data transmission by frequency-division multiplexing using the discrete Fourier transform", IEEE Trans. Commun., COM-19(5): 628-634, Oct. 1971.

[6] A. Peled, A. Ruiz, "Frequency domain data transmission using reduced computational complexity algorithms", *Proc. IEEE Int. Conf. Acoust., Speech, Signal Processing*, pages: 964-967, Denver, CO, 1980.

Homework

1. OFDM signals in the frequency domain look very similar to band-limited white noise. Please argue that the same is true in time domain. (i.e., OFDM signals in time domain is complex Gaussian process)
2. In some systems, the pilot symbols are boosted. Considering the rectangular pilot grid shown in page 20, what negative effects it would have if the pilot symbols are boosted. How would you improve it?