HELSINKI UNIVERSITY OF TECHNOLOGY
SMARAD Centre of Excellence

Parameter Estimation of Double Directional Radio Channel Model

S-72.4210 Post-Graduate Course in Radio Communications

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Outline

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2. Channel sounding – HUT Sounder
3. Double Directional Radio Channel model
4. Parameter estimation
    - Maximum likelihood principle
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Acronyms and abbreviations

AWGN	additive white gaussian noise
DMC	dense multipath component
EKF	Extended Kalman Filter
i.i.d.	independent identically distributed
IR	impulse response
MIMO	multiple input multiple output
PDF	Probability Density Function
PDP	Power Delay Profile
RIMAX	parameter estimation method
Rx	receiver
Tx	transmitter

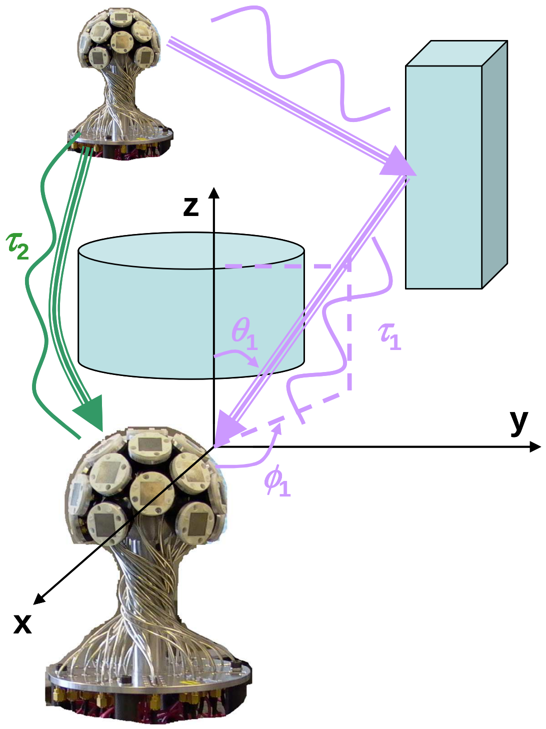
f	frequency
$G_R(f)$	frequency response of the receiver
$G_T(f)$	frequency response of the transmitter
M_R	number of receive antennas
M_T	number of transmit antennas
M_f	number of frequency (delay) domain samples
$\mathbf{R}$	covariance matrix
$\mathbf{s}(\boldsymbol{\theta}_{sp})$	observation vector modeling propagation paths
t	time
$\mathbf{x}$	measured observation vector
$\boldsymbol{\theta}_{dmc}$	parameters of dense multipath component
$\boldsymbol{\theta}_{sp}$	parameters of concentrated propagation paths

Introduction

- Future wireless MIMO communication systems
  - Exploit the spatial and temporal diversity of the radio channel
  - Require new complex models for simulations
  - Studying and comparing different transceiver structures
- Models are found through radio channel sounding measurements
  - Measurements are fitted to double directional channel models
  - Signal processing used for parameter estimation
  - Influence of measurement equipment is removed

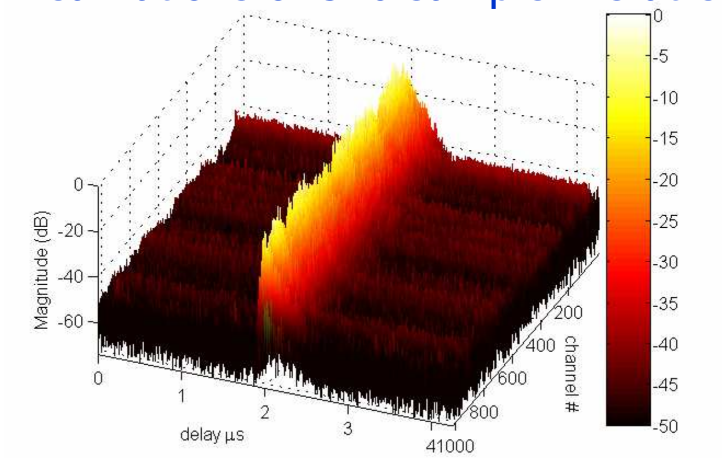
Channel sounding

- Sequential channel measurement from between each Tx and Rx ports
- TKK 5.3 GHz MIMO setup
  - 32 x 32 channels (16 dual polarized elements in arrays at both ends)
  - Length of each impulse response (IR) is 510 samples (120 MHz sampling rate)
  - Observation ("snapshot") separation 8.7 ms
- What sounder produces?
  - Complex array of 32 x 32 x 510 elements for each snapshot



Sounder output (single snapshot)

- 32 x 32 realizations of 510 sample IRs at every 8.7 ms



- Parameter estimation fits data to a channel model
  - Compresses the channel information using model parameters
  - Remove measurement antenna influence
  - Later the channel model parameters can be plugged into any antenna/transceiver configuration
  - Or parameters can be used to find out model statistics

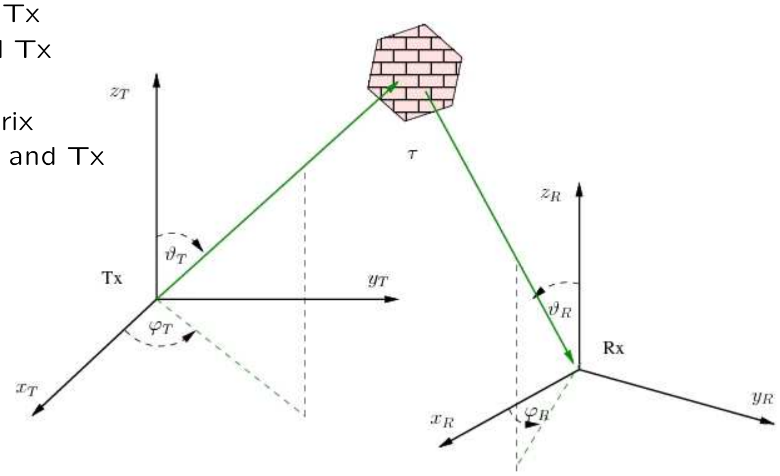
Double directional channel model

- Channel frequency response (Fourier transform of IR) at time t constructed with discrete propagation paths

$$\mathbf{H}(f,t) = G_{R_f}(f) G_{T_f}(f) \sum_p \left\{ \underbrace{\mathbf{B}_R(\varphi_{R,p}, \vartheta_{R,p})}_{\in \mathcal{C}^{M_R \times 2}} \mathbf{\Gamma}_p \underbrace{\mathbf{B}_T(\varphi_{T,p}, \vartheta_{T,p})}_{\in \mathcal{C}^{M_T \times 2}}{}^T e^{-j2\pi f \tau_p} \right\}$$

φ_R, φ_T	azimuth angle at Rx and Tx
ϑ_R, ϑ_T	elevation angle at Rx and Tx
τ	time delay of arrival
$\mathbf{\Gamma}$	complex path weight matrix
$\mathbf{G}_{R_f}, \mathbf{G}_{T_f}$	frequency response of Rx and Tx

$$\mathbf{\Gamma}_p = \begin{bmatrix} \gamma_{HH,p} & \gamma_{VH,p} \\ \gamma_{HV,p} & \gamma_{VV,p} \end{bmatrix} \in \mathcal{C}^{2\times 2}$$



Sampled double directional channel model

- In practice discrete samples of $\mathbf{H}(f,t)$ are measured
- Sampled model for the observation consists of two parts:

$$\mathbf{x} = \mathbf{s}(\boldsymbol{\theta}_{sp}) + \mathbf{d}_{dmc} \in \mathcal{C}^{M_R M_T M_f \times 1}$$

Dense multipath component:

$$\mathbf{d}_{dmc} \sim \mathcal{N}_C(\mathbf{0}, \mathbf{R}(\boldsymbol{\theta}_{dmc}))$$

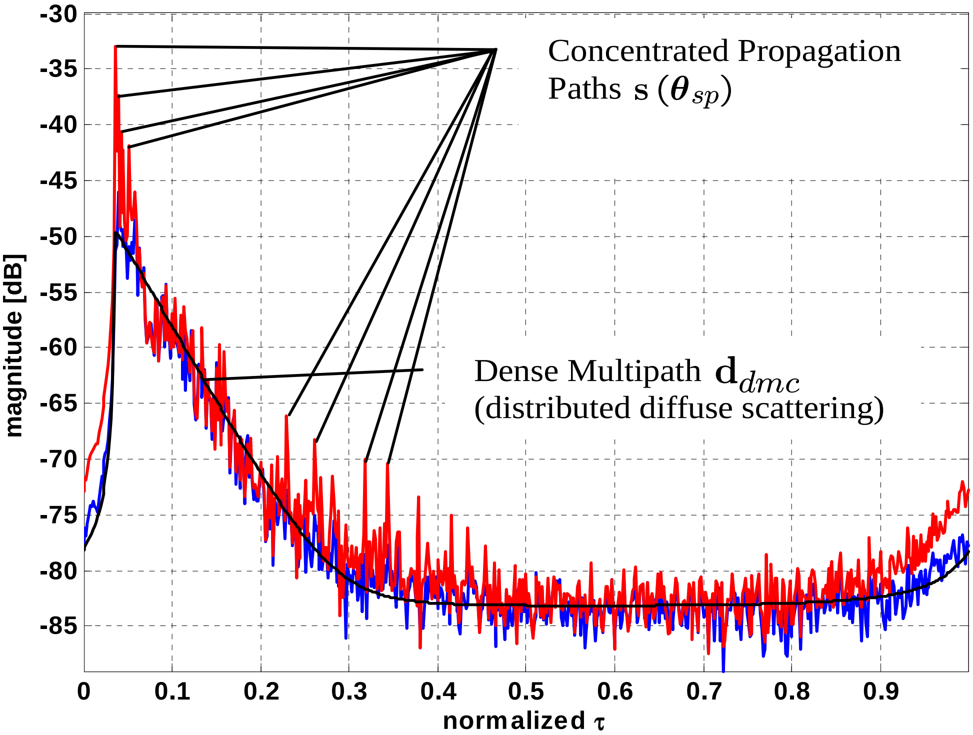
Specular propagation paths:

$$\boldsymbol{\theta}_{sp} = \{\boldsymbol{\tau}, \boldsymbol{\varphi}_T, \boldsymbol{\vartheta}_T, \boldsymbol{\varphi}_R, \boldsymbol{\vartheta}_R, \boldsymbol{\gamma}\}$$

$$\mathbf{s}(\boldsymbol{\theta}_{sp}) = \left(\mathbf{B}_{R_H} \diamond \mathbf{B}_{T_H} \diamond \mathbf{B}_f\right) \cdot \boldsymbol{\gamma}_{HH} + \left(\mathbf{B}_{R_V} \diamond \mathbf{B}_{T_H} \diamond \mathbf{B}_f\right) \cdot \boldsymbol{\gamma}_{HV} + \left(\mathbf{B}_{R_H} \diamond \mathbf{B}_{T_V} \diamond \mathbf{B}_f\right) \cdot \boldsymbol{\gamma}_{VH} + \left(\mathbf{B}_{R_V} \diamond \mathbf{B}_{T_V} \diamond \mathbf{B}_f\right) \cdot \boldsymbol{\gamma}_{VV}$$

where $\diamond$ denotes the Khatri-Rao (columnwise Kronecker) product

Illustration of specular paths vs. DMC



Parameter estimation techniques

- Subspace techniques
  - ESPRIT (Estimation of Signal Parameters via Rotational Invariance Techniques)
  - MUSIC (MUltiple SIgnal Classification)
  - RARE (RAnk Reduction Estimator)
- Maximum likelihood estimators
  - SAGE (Space-Alternating Generalized Expectation maximization)
  - RIMAX (iterative maximum likelihood)
- State-Space Methods
  - Extended Kalman Filter

Parameter estimation example: Maximum Likelihood Estimation (1)

- The Observation $\mathbf{x}$ is assumed i.i.d. Gaussian

$$\mathbf{x} \sim \mathcal{N}_C\Big(\underbrace{\mathbf{s}(\boldsymbol{\theta}_{sp})}_{\text{mean}}, \underbrace{\mathbf{R}(\boldsymbol{\theta}_{dmc})}_{\text{covariance}}\Big)$$

- The Likelihood function, i.e., the pdf of $\mathbf{x}$:

$$\begin{aligned} l(\boldsymbol{\theta}, \mathbf{x}) &= p\big(\mathbf{x}|\boldsymbol{\theta}\big) \\ &= \frac{1}{\pi^M \det\big(\mathbf{R}(\boldsymbol{\theta}_{dan})\big)} e^{-(\mathbf{x}-\mathbf{s}(\boldsymbol{\theta}_{sp}))^H \cdot \mathbf{R}^{-1}(\boldsymbol{\theta}_{dan}) \cdot (\mathbf{x}-\mathbf{s}(\boldsymbol{\theta}_{sp}))} \end{aligned}$$

- The maximum likelihood estimates are the parameters θ_{sp} and θ_{dmc} that maximize this function

Maximum Likelihood (2)

- Usually the log-likelihood $L(\theta,\mathbf{x})$ is preferred

$$L(\boldsymbol{\theta}, \mathbf{x}) = \ln(l(\boldsymbol{\theta}, \mathbf{x})) = \ln\left(\frac{1}{\pi^M \det(\mathbf{R})}\right) - (\mathbf{x}-\mathbf{s}(\boldsymbol{\theta}_{sp}))^H \mathbf{R}^{-1} (\mathbf{x}-\mathbf{s}(\boldsymbol{\theta}_{sp}))$$

- Let us assume that $\mathbf{R}=\mathbf{R}(\theta_{dmc})$ is known. Then the maximum of $L(\theta,\mathbf{x})$ is found by minimizing the last term

$$\hat{\boldsymbol{\theta}}_{sp,ML} = \arg\min_{\theta_{sp}} \Big((\mathbf{x} - \mathbf{s}(\boldsymbol{\theta}_{sp}))^H \mathbf{R}^{-1} (\mathbf{x} - \mathbf{s}(\boldsymbol{\theta}_{sp})) \Big)$$

- The minimum is found by evaluating zeros of the gradient (first order derivatives) of this term

Maximum Likelihood (3)

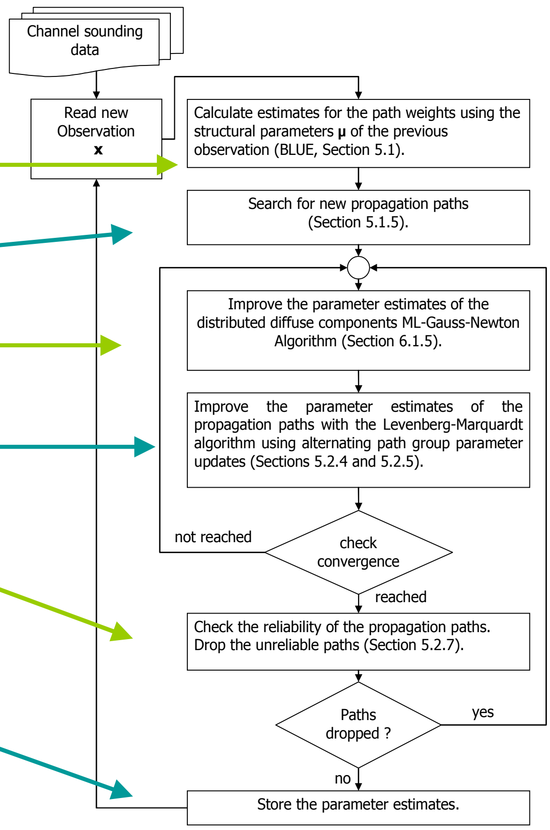
- The first order derivatives of $\left(\mathbf{x}-\mathbf{s}(\boldsymbol{\theta}_{sp})\right)^H\mathbf{R}^{-1}\left(\mathbf{x}-\mathbf{s}(\boldsymbol{\theta}_{sp})\right)$ are given by the score function:

$$\mathbf{q}\left(\mathbf{x}|\boldsymbol{\theta},\mathbf{R}\right) = 2\cdot\Re\left\{\mathbf{D}^H(\boldsymbol{\theta})\mathbf{R}^{-1}(\mathbf{x}-\mathbf{s}(\boldsymbol{\theta}))\right\}, \quad \mathbf{D}(\boldsymbol{\theta}) = \frac{\partial}{\partial\boldsymbol{\theta}^T}\mathbf{s}(\boldsymbol{\theta})$$

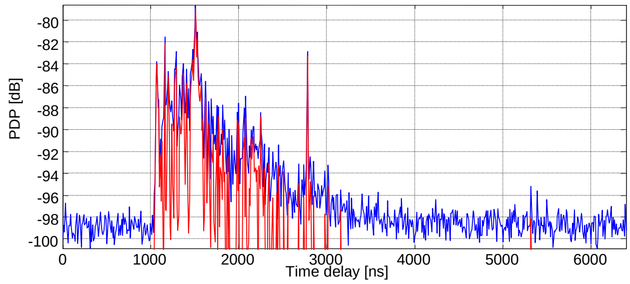
- The score function $\mathbf{q}(\mathbf{x}|\theta,\mathbf{R})$ has typically several zeros
  - Global search or other initialization (estimates from previous snapshot) required
  - Iterative (e.g. Gauss-Newton or Levenberg-Marquardt) method can be used to reach the optimal parameter estimates

Outline of the RIMAX structure [1]

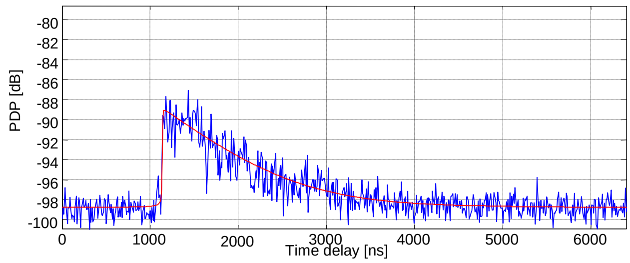
1. Read new snapshot $\mathbf{x}$
2. Use previous estimates as initial values
3. Search for new propagation paths
4. Estimate the DMC component
5. Use iterative maximum likelihood method to improve propagation path estimates
6. Check reliability of propagation paths based on estimation error variance
7. Store the estimates and proceed to the next snapshot



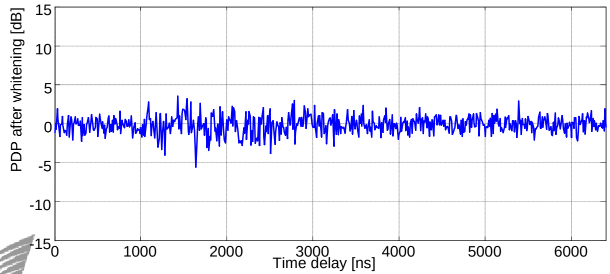
Example of succesful parameter estimation [1]



- Example for the PDP of a measured impulse response (blue) and of the estimated concentrated propagation paths (red).



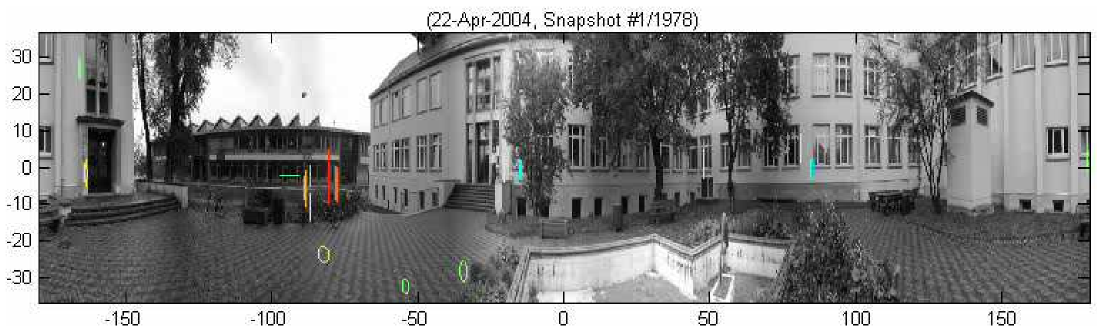
- Example for the PDP of the remainder (blue) of a measured impulse response after removing the estimated concentrated propagation paths. Red line is the estimated PDP of the DMC.



- Example for the PDP of the remainder of a measured impulse response after removing the estimated concentrated propagation paths and whitening (removing the DMC).

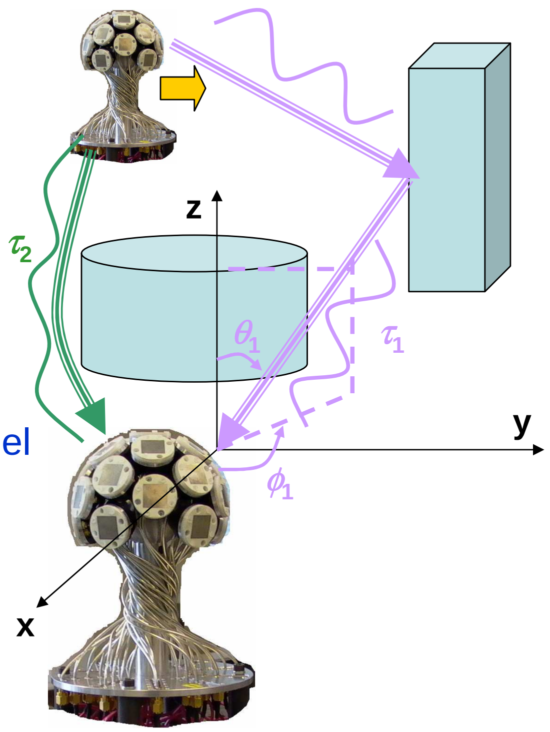
Illustration of estimation results

- Panoramic (full 360°) view at courtyard of Technical University of Ilmenau
  - Rx at the middle of the courtyard (at point where the photo has been taken)
  - Tx going around the courtyard



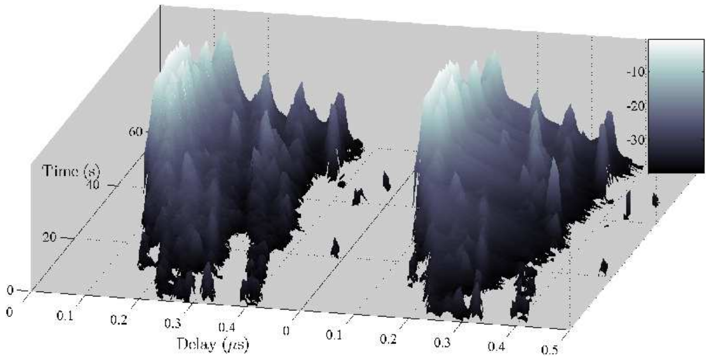
Alternative approach: Tracking of the propagation path parameters

- Propagation path parameter estimation as a multi-target tracking problem
  - Number of (reliable) paths *P* represent multiple targets
  - Large number of parameters for each target
  - Linear vs. Nonlinear motion model
  - Nonlinear Measurement model
  - Modeling the noise process



State-space methods

- State transition (possibly nonlinear): $\mathbf{x}_{k+1} = \mathbf{f}_k(\mathbf{x}_k, \mathbf{q}_k)$
- Measurement equation (nonlinear): $$\mathbf{y}_k = \mathbf{h}(\mathbf{x}_k, \mathbf{r}_k)$$

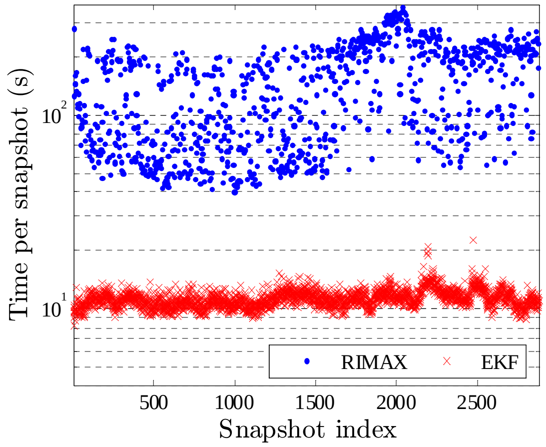


Measured PDP over time compared to PDP of EKF estimates.

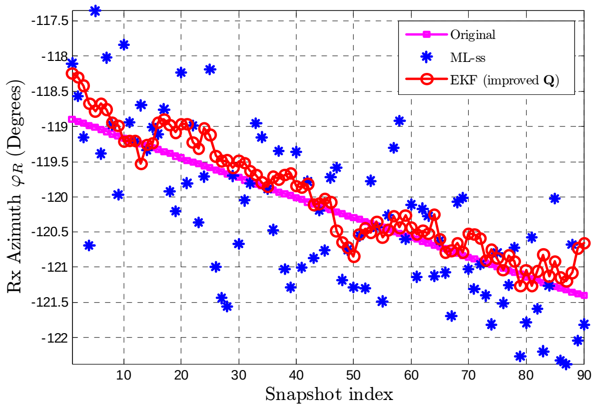
- Extended Kalman Filter (EKF)
  - EKF assumes Gaussian distribution
  - Linearizes state transition and measurement equations through Taylor series approximation
  - Tracks the parameters over time using recursive ”filtering” (Kalman filters are popular in radar applications)
  - Low computational complexity compared with iterative maximum likelihood
  - Initialization using e.g. RIMAX
  - Reliable tracking requires some statistics of the behavior of the parameters

RIMAX vs. EKF

- EKF is computationally lighter than RIMAX



- Simulation results show how EKF filters the parameters resulting in lower estimation error variance



Conclusions

- Parameter estimation fits measured data to a channel model
  - Compresses the channel information to model parameters
  - Removes measurement antenna influence
  - Later the channel model parameters can be used for
    1. Statistical analysis of the channel parameters
    2. Simulations with arbitrary antenna/transceiver configurations
- Most popular classes of parameter estimation techniques are subspace and maximum likelihood
- State-space methods are under research for revealing and utilizing the time-dependt properties of the radio propagation environments

References

[1] A. Richter, "Estimation of radio channel parameters: Models and algorithms," Ph. D. dissertation, Technische Universität Ilmenau, Germany, 2005, [Online]. Available: www.db-thueringen.de

[2] A. Richter, M. Enescu, V. Koivunen, "State-Space Approach to Propagation Path Parameter Estimation and Tracking", in Proc. 6th IEEE Workshop on Signal Processing Advances in Wireless Communications, New York City, June 2005.

[3] J. Salmi, Statistical Modeling and Tracking of the Dynamic Behavior of Radio Channels, Master's Thesis, Helsinki University of Technology, Espoo, Finland, June 2005.

[4] V-M. Kolmonen, J. Kivinen, L. Vuokko, P. Vainikainen, 5.3 GHz MIMO radio channel sounder, in Proc. 22nd Instrumentation and Measurement Technology Conference, IMTC'05, Ottawa, Ontario, Canada, May 16-19 2005, pp.1883-1888.

Homework

Maximum likelihood estimation of mean and variance

Consider a discrete-time received signal $r(k) = \mu + w(k), \quad k = 0, 1, \ldots, N-1$
where μ is a constant mean and $w(k) \sim \mathrm{N}(0, \sigma^2)$ is AWGN with variance σ^2.

The PDF (likelihood) of the observation vector $\mathbf{r}$ is thus given by

$$p\left(\mathbf{r}|\mu, \sigma^2\right) = \frac{1}{(\sqrt{2\pi\sigma^2})^N} e^{-\frac{1}{2\sigma^2}\sum_{k=0}^{N-1}(r(k)-\mu)^2}$$

Derive the maximum likelihood estimates for both the mean μ and the variance σ^2.

HINT: Differentiate the log-likelihood function with respect to both parameters and set the derivatives to zero.