S-72.423 Telecommunication Systems

Overview to Pulse Coded Modulation

Overview to Pulse Coded Modulation

- Sampling
  - Ideal
  - Practical sampling with
    - chopper sampler
    - bipolar sampler
    - flat-top sampler (PAM)
- Line coding techniques
  - HDB-3
  - Manchester coding
- Quantization
  - Uniform
  - μ - law - quatization
  - quantization noise
- PCM and channel noise
- Time division multiplexing (TDM) and frequency division multiplexing (FDM) systems compared

TDM: Time Division Multiplexing
FDM: Frequency Division Multiplexing
PAM: Pulse Amplitude Modulation
PCM: Pulse Coded Modulation
HDB: High Density Bipolar code

Why to apply digital transmission?

- Digital communication withstands channel noise and distortion better that analog system. For instance in PSTN inter-exchange STP-links NEXT (Near-End Cross-Talk) produces several interference. For analog systems interference must be below 50 dB whereas in digital system 20 dB is enough. With this respect digital systems can utilize lower quality cables than analog systems
- Regenerative repeaters can be used. Note that generally cleaning of analog-signals repeatedly is not very successful
- Digital HW implementation is straight forward
- Circuits can be easily reconfigured by DSP techniques (an application: software radio)
- Digital signals can be coded to yield very low error rates
- Digital communication enables efficient exchanging of SNR to BW-> easy adaptation into different channels
- The cost of digital HW continues to halve every two or three years

STP: Shielded twisted pair

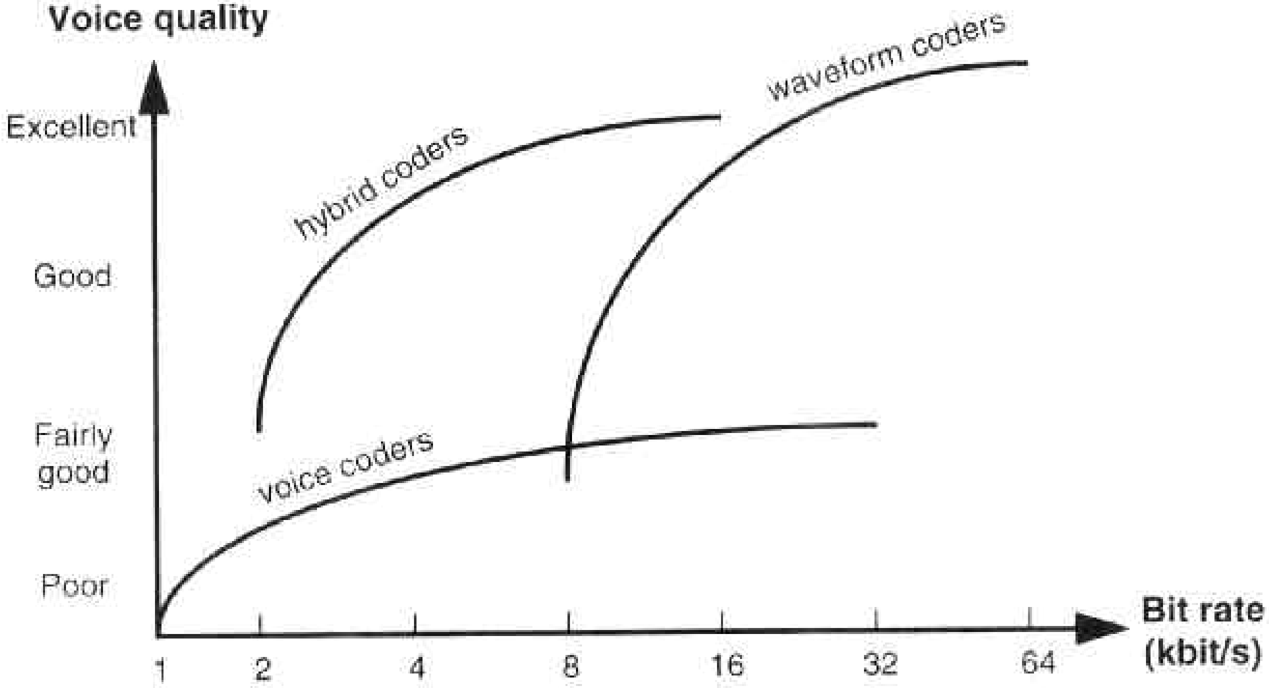
Some important ITU-T speech/video coding standards

Standard no. Name	Description	Current status
G.711	Pulse code modulation (PCM) of voice frequencies (64 kbit/s)	Adopted 1984
G.722, G.725	7 kHz audio-coding within 64 kbit/s	Adopted 1988
G.726	16/24/32/46 kbit/s adaptive differential pulse code modulation (ADPCM)	Adopted 1990
G.728	16 kbit/s speech coding with excited linear prediction	Adopted 1992
G.729	8 kbit/s speech coding	Adopted 1996
H.221	Frame structure for a 64 to 1920 kbit/s channel in audiovisual teleservices	Adopted 1990
H.230	Control and indication signals for audiovisual systems	Adopted 1990
H.231, H243	Multipoint videoconferencing	Adopted 1993
H.233	Encryption / Privacy systems	Adopted 1993
H.261	Video codec for audiovisual teleservices at p x 64 kbit/s	Adopted 1993
H.263	Video coding for low bit rate communication	Adopted 1996
MPEG1	Stored motion video stored at <2 Mbit/s	Adopted 1993
MPEG2	Stored/live motion video at 5–60 Mbit/s	Adopted 1994
MPEG4	Low bit rate (<64 kbit/s) coding of motion video	
JPEG	Still-frame graphics for multimedia	Adopted 1991

MPEG = Motion picture expert group. JPEG = Joint photograph expert group

JPEG= Joint photograph expert group

A voice coder classification



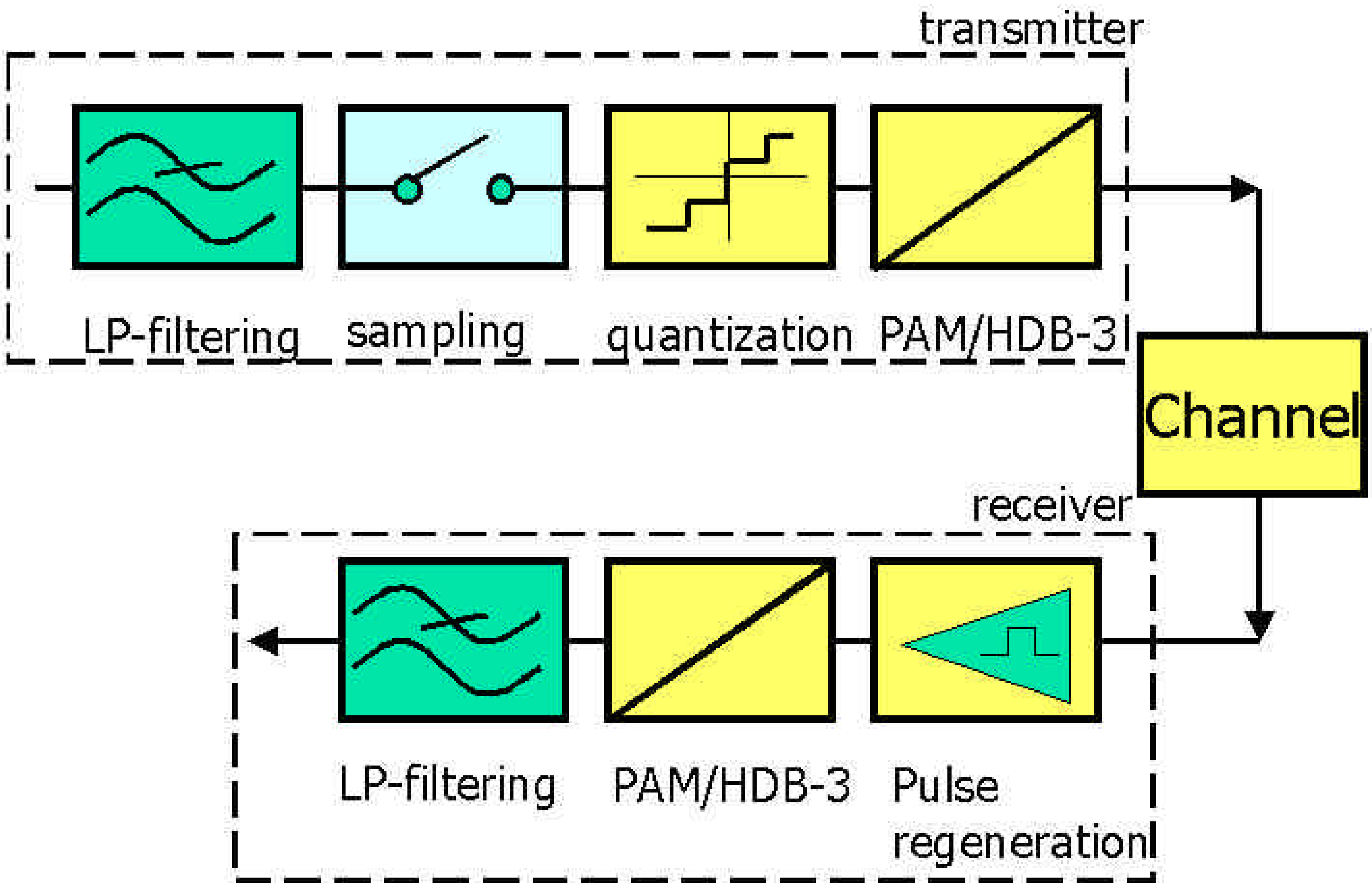
- Waveform coders (as PCM) describe the signal by numbered values, very precise operation but requires many bits
- Voice coders parameterize speech by counting on a system model that produces the signal. Only model parameters are transmitted and updated. Very low rate can be obtained but quality may suffer
- Hybrid coder is a compromise used for instance in PLMN apps

Short history of pulse coded modulation

- A problem of PSTN analog techniques was that transmitting multiple channels was difficult due to nonlinearities resulting channel cross-talk
- 1937 Reeves and Delorane ITT labs. tested TDM-techniques by using electron-tubes
- 1948 PCM was tested in Bell Labs
- TDM was taken into use in 1962 with a 24 channel PCM link
- The first 30-channel PCM system installed in Finland 1969

Pulse Coded Modulation (PCM)

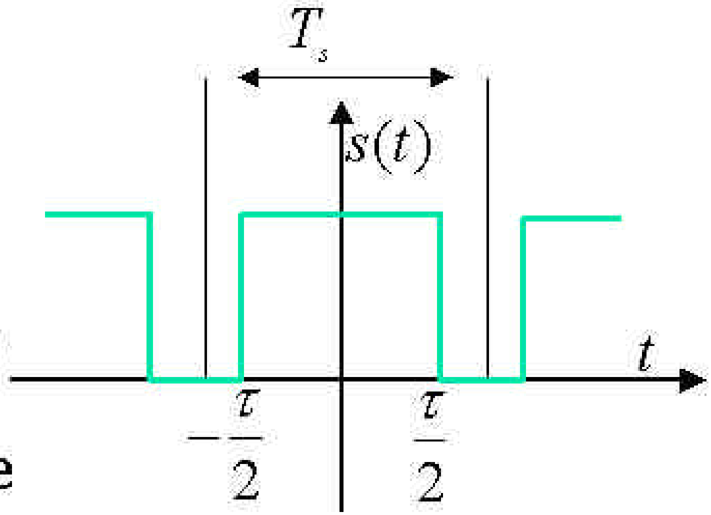
- PCM is a method by which an analog message can be transformed into numerical format and then decoded at the receiver



PCM principles: Ideal sampling

- The rectangular pulse train

$$s(t)=\sum_{k=-\infty}^{\infty}\Pi\left(\frac{t-kT_s}{\tau}\right)$$



- The ideal sampling function

$$s_\delta(t)=\lim_{\tau\to 0}\left[\frac{1}{\tau}s(t)\right]=\sum_{k=-\infty}^{\infty}\delta(t-kT_s)$$

- The ideal sampled signal is a pulse train of weighted impulses

$$x_\delta(t)=x(t)s_\delta(t)=x(t)\sum_{k=-\infty}^{\infty}\delta(t-kT_s)$$
$$=\sum_{k=-\infty}^{\infty}x(kT_s)\delta(t-kT_s)$$

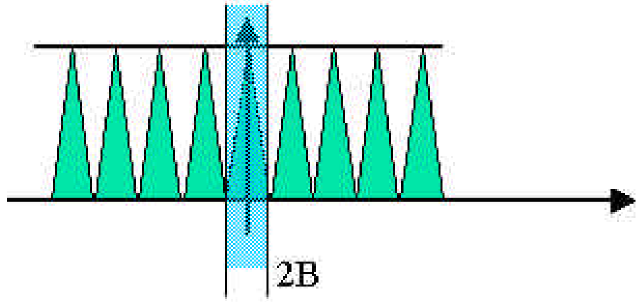
- Translation Fourier tables: $\mathcal{F}[\sum_{k=-\infty}^{\infty}\delta(t-kT_s)]=f_s\sum_{n=-\infty}^{\infty}\delta(f-nf_s)$

the ideally sampled signal is then $X_\delta(f)=f_s\sum_{n=-\infty}^{\infty}X(f-nf_s)$

Ideal sampling: reconstruction

- Reconstruction is obtained by lowpass filtering. Assume the ideal lowpass filter with

$$H(f) = K\Pi\left(\frac{f}{2B}\right)\exp(-j\omega t_d)$$



- Due to the translations

$$\operatorname{sinc} 2Wt \leftrightarrow \frac{1}{2W}\Pi\left(\frac{f}{2W}\right) \qquad v(t-t_d) \leftrightarrow V(f)\exp(-j\omega t_d)$$

the respective impulse response is therefore

$$h(t) = 2BK\operatorname{sinc} 2B(t-t_d)$$

- In ideal sampling reconstruction weighted impulse train (representing the sampled signal) is applied to this filter and the output is $y(t) = h(t)\otimes x_s(t)$

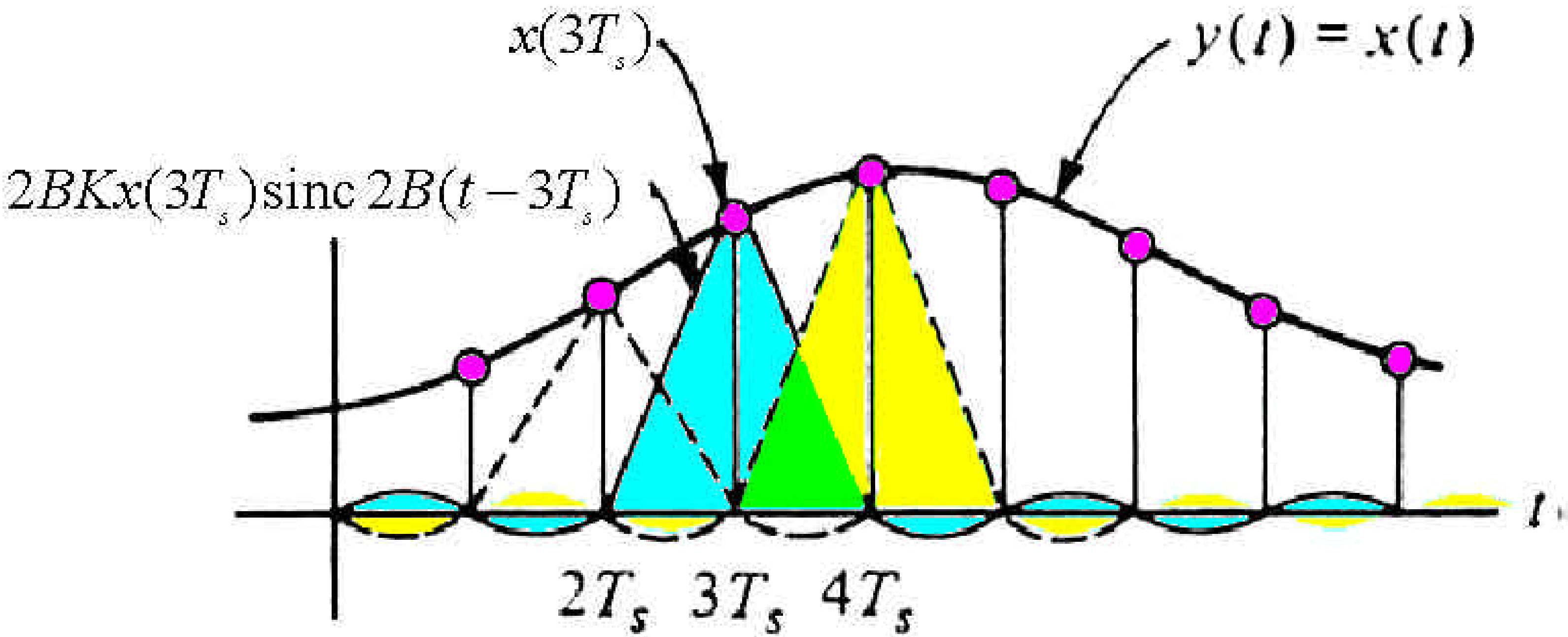
$$= 2BK\operatorname{sinc} 2B(t-t_d)\otimes \sum_{k=-\infty}^{\infty} x(kT_s)\delta(t-kT_s)$$

$$= 2BK\sum_{k=-\infty}^{\infty} x(kT_s)\operatorname{sinc} 2B(t-t_d-kT_s)$$

Reconstructed signal consists of interpolated sinc-functions

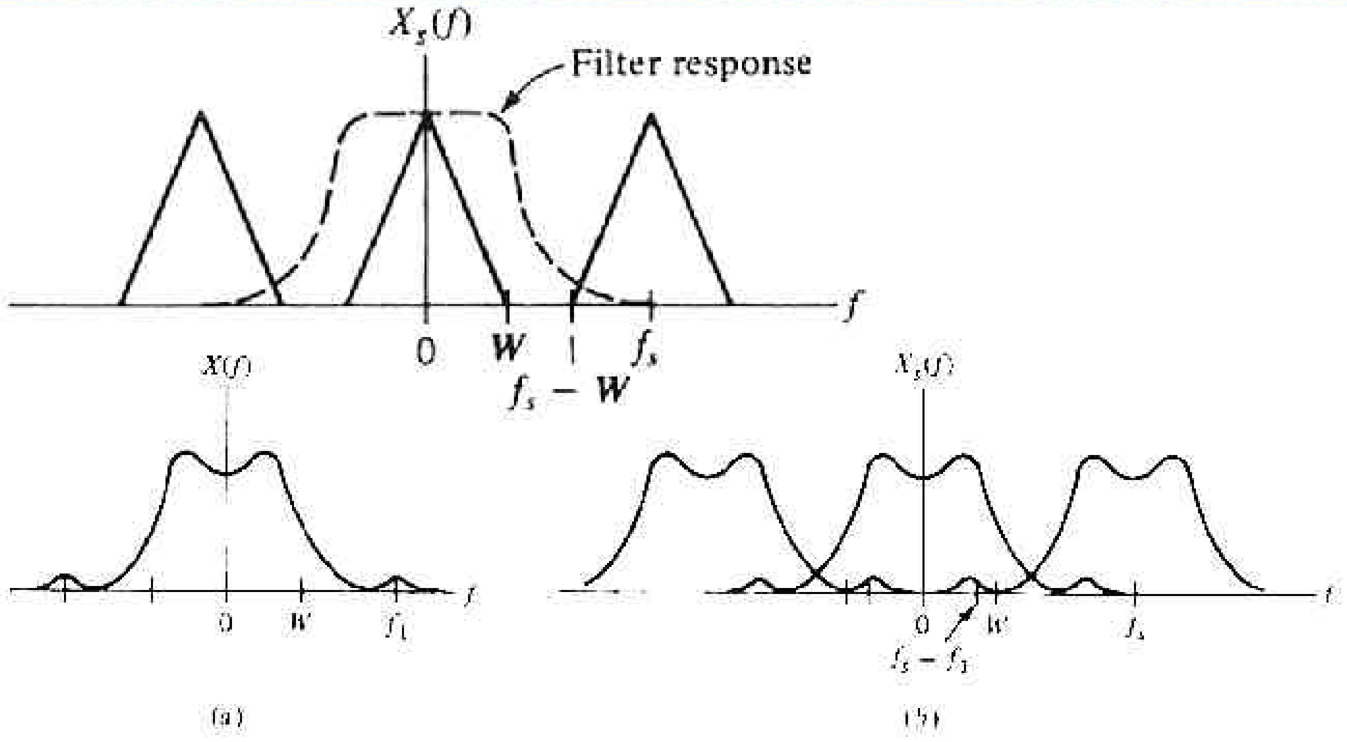
- At the sample instances all but one sinc functions are zero
- Therefore all band limited signals can be expressed as the sinc-series:

$$2BK\sum_{k=-\infty}^{\infty} x(kT_s)\,\mathrm{sinc}\,2B(t-t_d-kT_s)$$



Unperfect reconstruction: spectral folding

1. Sampling wave pulses have finite duration and risetimes
2. Reconstruction filters are not ideal lowpass filters
3. Sampled messages are time limited and therefore their spectra is not frequency limited



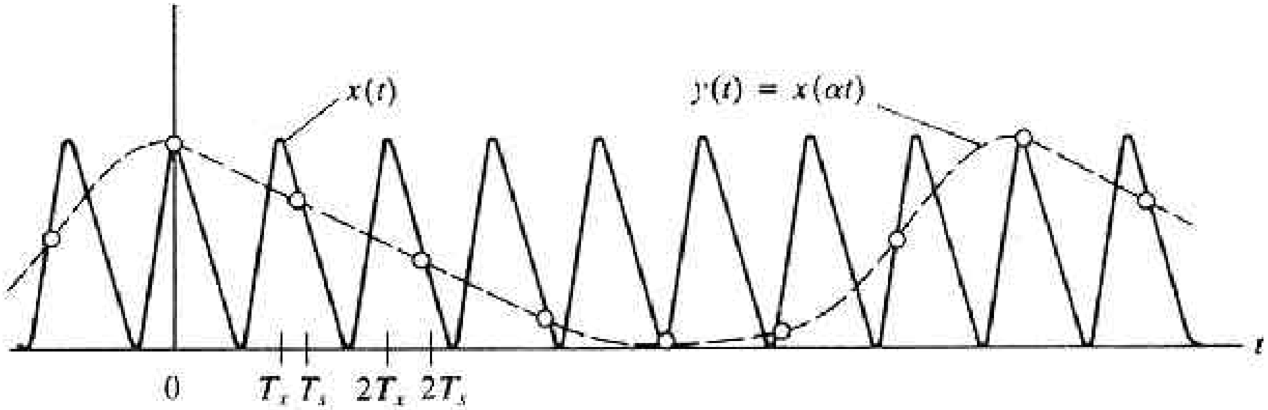
Aliasing and sampling theorem

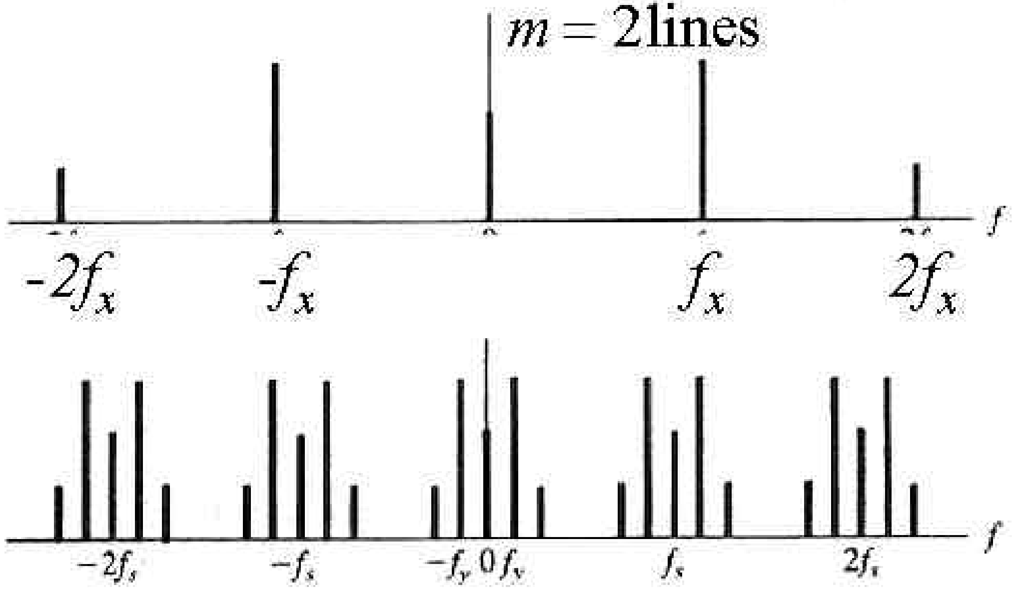
- Nyqvist sampling theorem:

> If a signal contains no frequency components for $|f| \geq W$ it is completely described by instantaneous uniformly spaced time samples having period $T_s \leq 1/2W$. The signal can hence been reconstructed from its samples by an ideal LPF of bandwidth B such that $W \leq B \leq f_s - W$.

- Note: If the signal contains higher frequencies than twice the sampling frequency they will also be present at the sampled signal! An application of this is the sampling oscilloscope:

Sampling oscilloscope





$$f_S = (1-\alpha) f_X, 0 < \alpha < 1$$

$$\pm f_Y = \pm |f_X - f_S| = \pm \alpha f_X$$

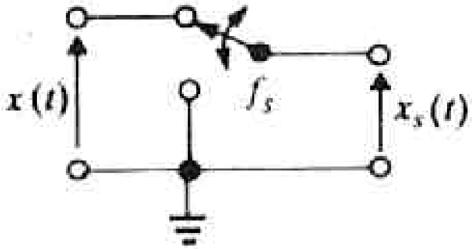
$$m f_Y < f_S / 2$$

$$m\alpha f_X < (1-a) f_X / 2$$

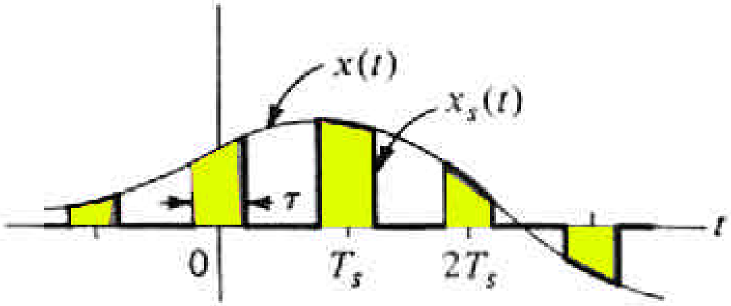
$$2m\alpha < 1-\alpha, \alpha(2m+1) < 1$$

$$\alpha < 1/(2m+1)$$

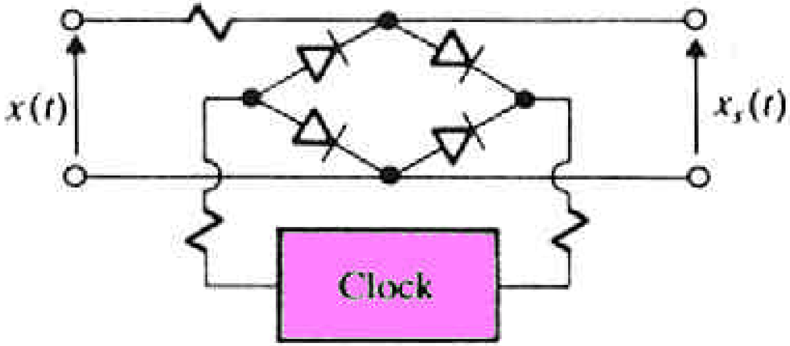
Chopper sampling



(a)



(b)



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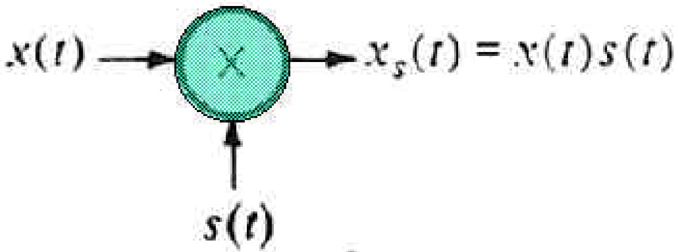
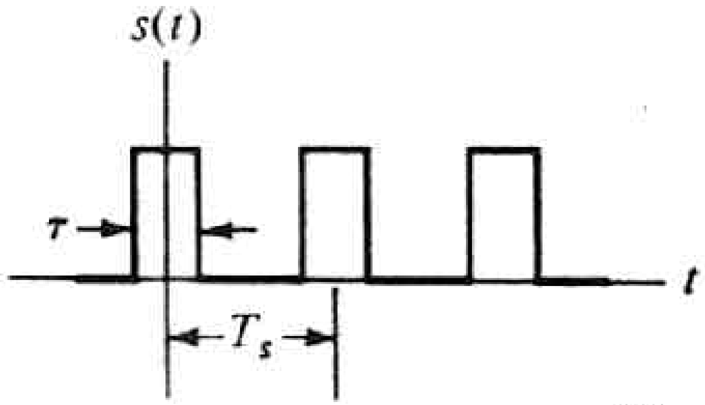
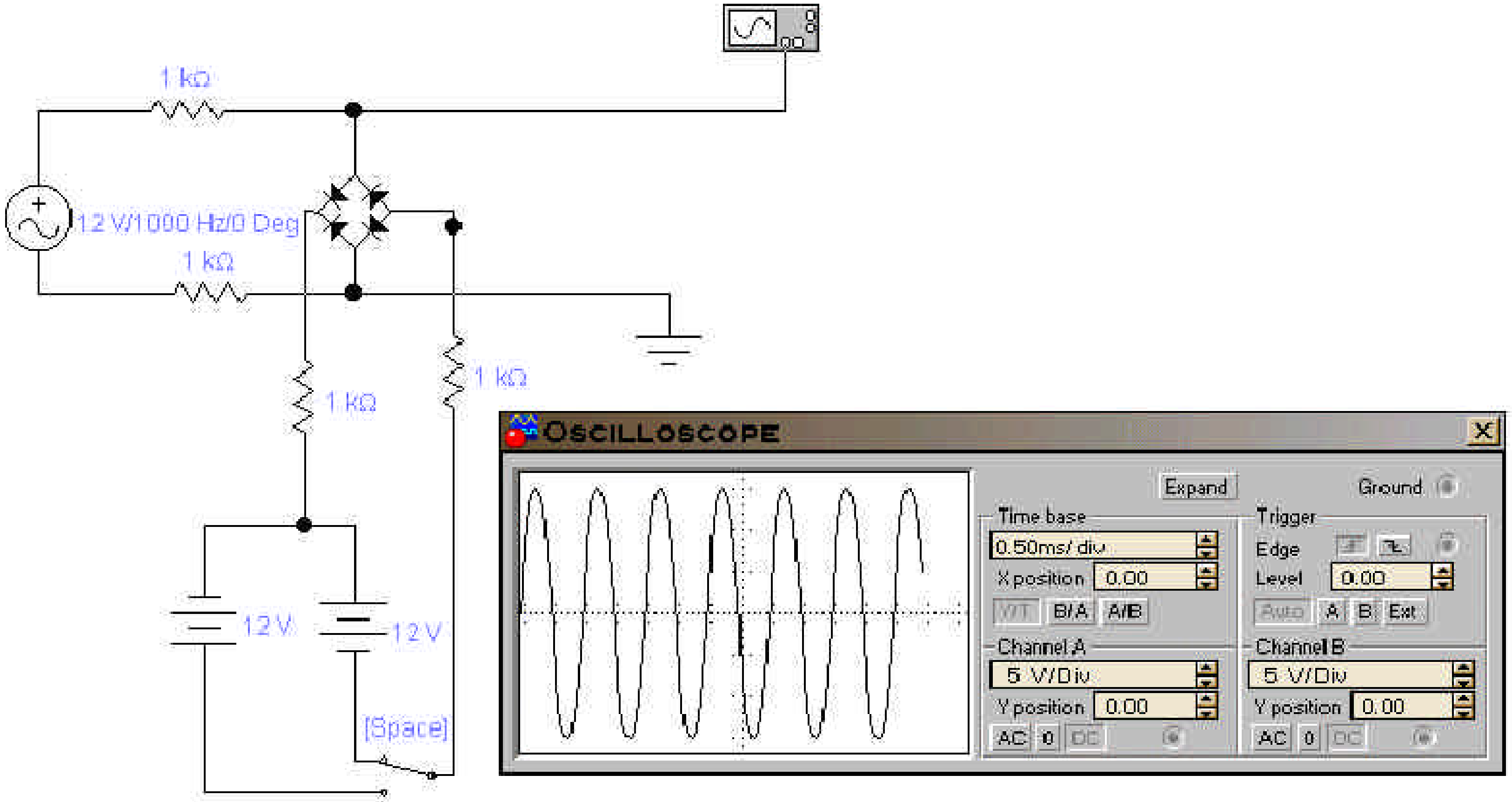
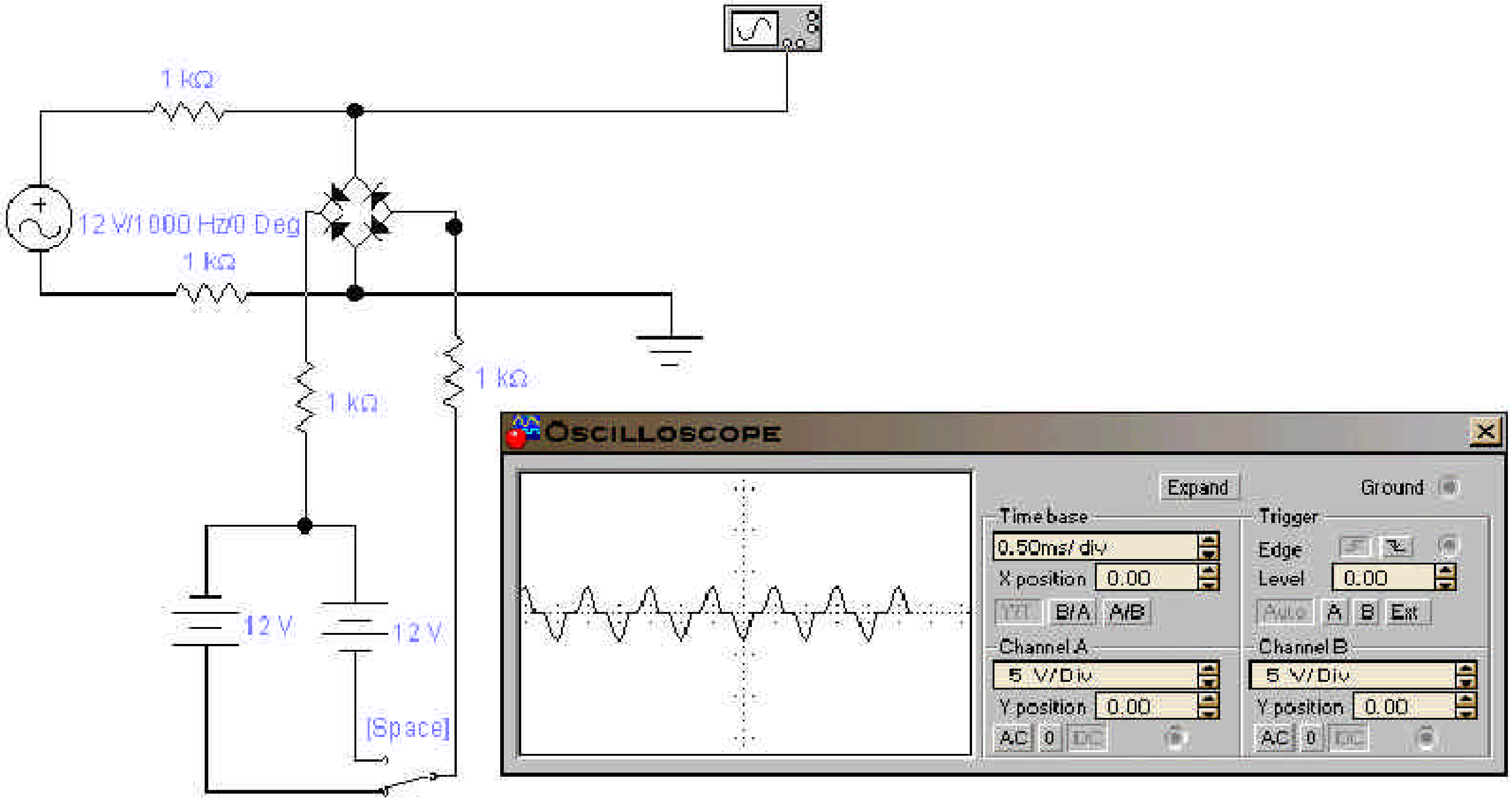




Modeling Chopper sampler in EWB: 1/2



Modeling Chopper sampler in EWB: 2/2



The chopper sampler waveforms

- Sampling wave consists of a periodic pulse train whose duration is τ and period is T_o

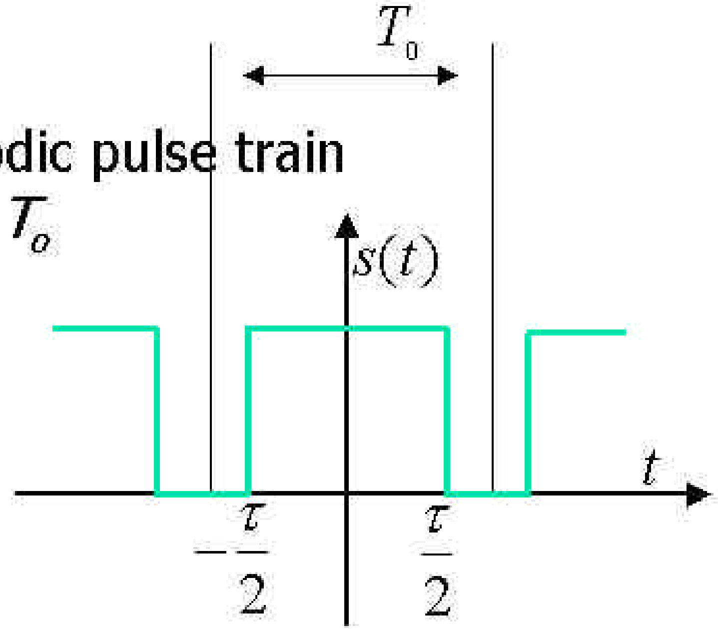
$$c_n = \frac{1}{T_0}\int_{T_0} v(t)\exp(-2\pi t / T_0)dt$$

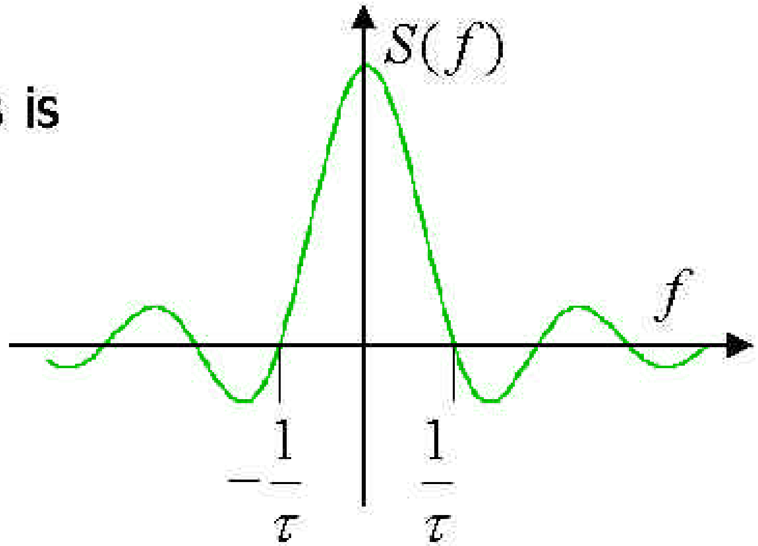
$$c_n = \frac{A\tau}{T_o}\text{sinc}(nf_0\tau)$$

$$s(t) = \sum_{n=-\infty}^{\infty} c_n \exp(j2\pi n f_0 t)$$

The Fourier series for real signals is

$$s(t) = c_0 + \sum_{n=1}^{\infty} 2c_n \cos n\omega_s t$$





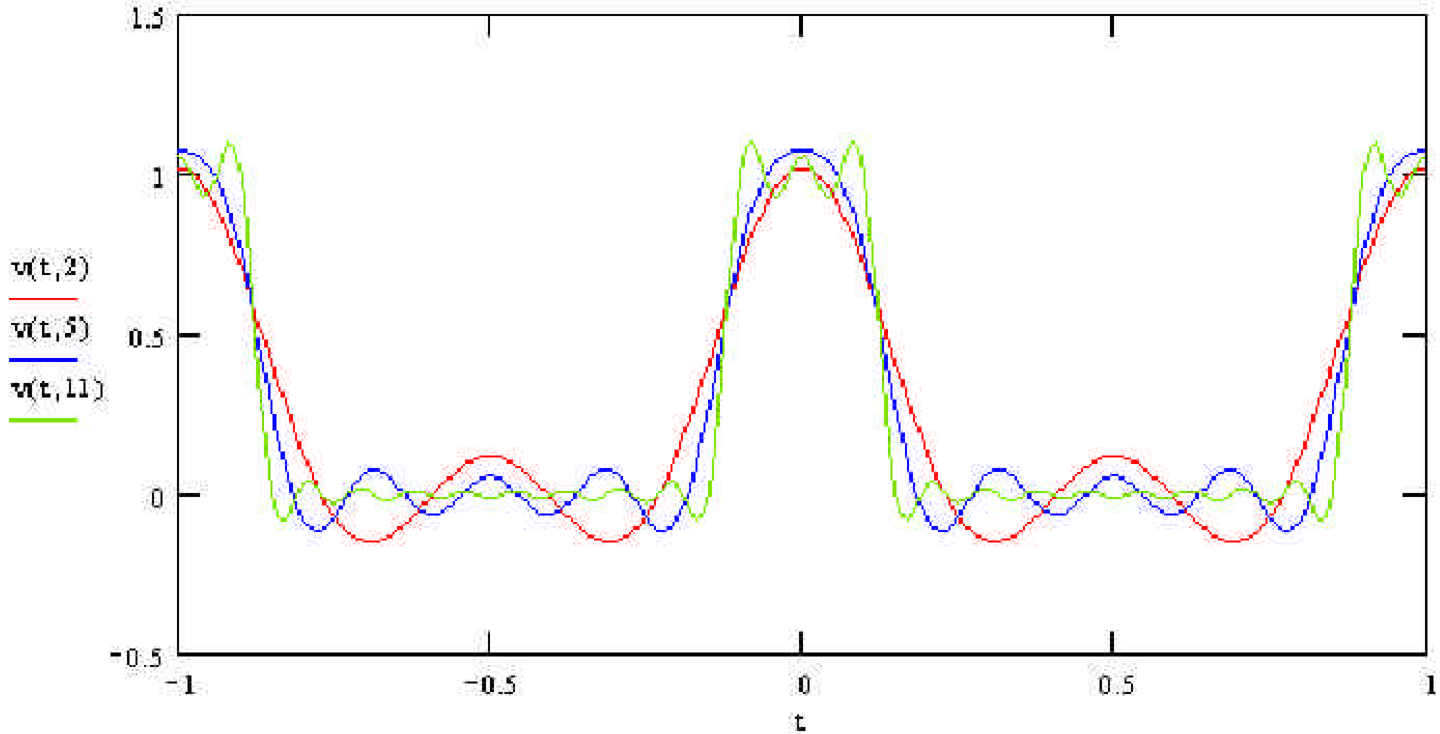
Inspecting Fourier series of the sampling wave by Mathcad

$$\mathrm{sinc}(x) := \mathrm{if}\left(x = 0, 1, \frac{\sin(\pi \cdot x)}{\pi \cdot x}\right) \qquad A := 1 \qquad To := 1 \qquad \tau := \frac{1}{4}$$

$$c(n) := A \cdot \frac{\tau}{To} \cdot \mathrm{sinc}\left(n \cdot \frac{\tau}{To}\right)$$

$$v(t,k) := \sum_{n=-k}^{k} c(n) \cdot \exp\left(i \cdot 2 \cdot \pi \cdot n \cdot \frac{t}{To}\right)$$

$$t := -1, -0.99 .. 1$$



Chopper sampler: sampled spectra

- Consider the sampled signal from the chopper sampler by term-by-term multiplication

$$x_s(t) = x(t)s(t)$$
$$= x(t)\left[c_0 + \sum_{n=1}^{\infty} 2c_n \cos n\omega_s t\right]$$

$$x_s(t) = x(t)s(t)$$
$$= x(t)c_0 + 2c_1 x(t)\cos\omega_s t + 2c_2 x(t)\cos 2\omega_s t \ldots$$

Remember the modulation theorem:

$$v(t)\cos(\omega_c t + \phi) \leftrightarrow ½[V(f - f_c)\exp(j\phi) + V(f + f_c)\exp(-j\phi)]$$

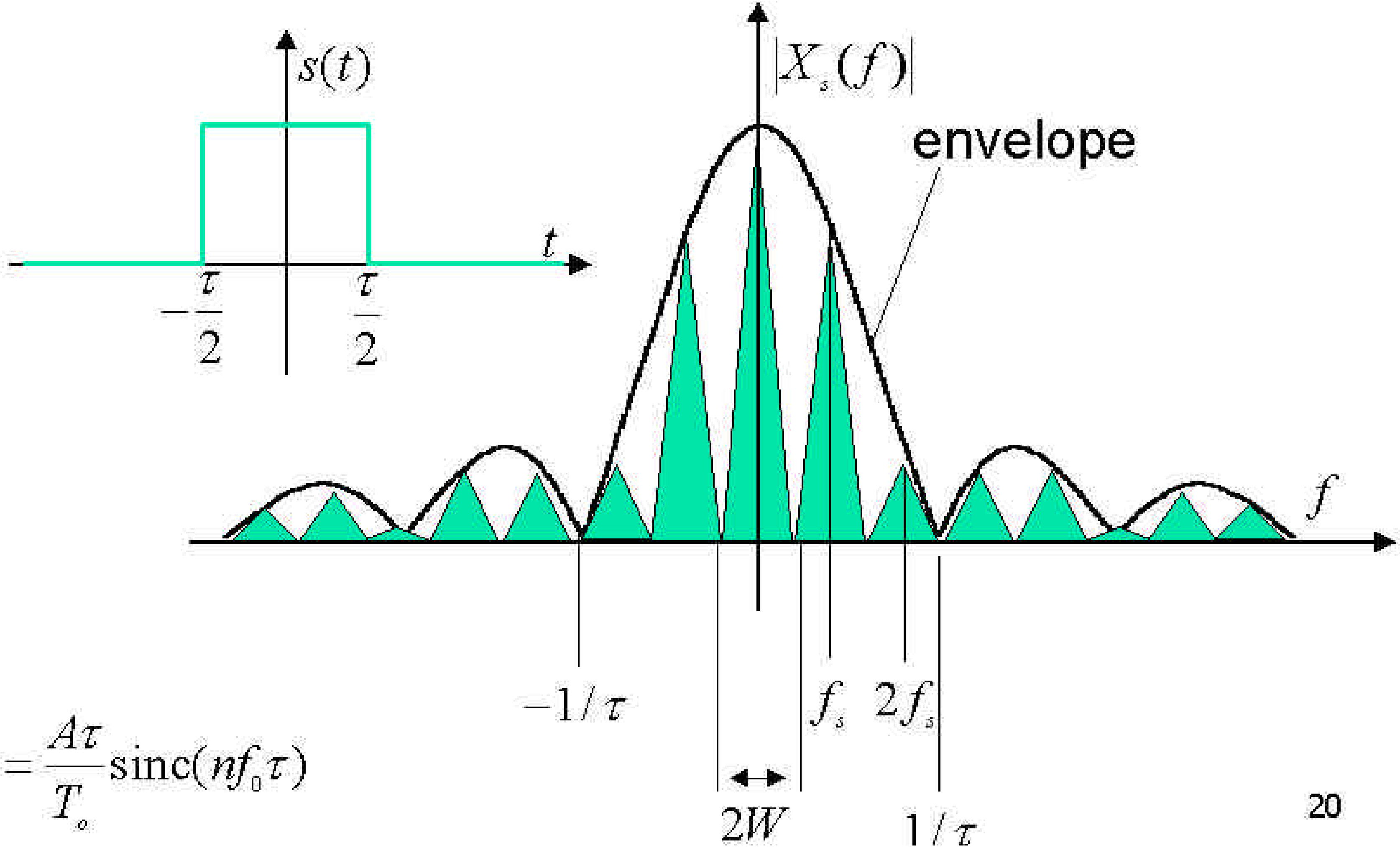
- Therefore the sampled signal is in frequency domain

$$X_s(f) = c_0 X(f) + c_1[X(f - f_s) + X(f + f_s)]$$
$$+ c_2[X(f - 2f_s) + X(f + 2f_s)] \ldots$$

$$s(t) = c_0 + \sum_{n=1}^{\infty} 2c_n \cos n\omega_s t$$

Chopper sampler spectra and its envelope

$$X_s(f) = c_0 X(f) + c_1[X(f - f_s) + X(f + f_s)]$$
$$+ c_2[X(f - 2f_s) + X(f + 2f_s)] \ldots$$



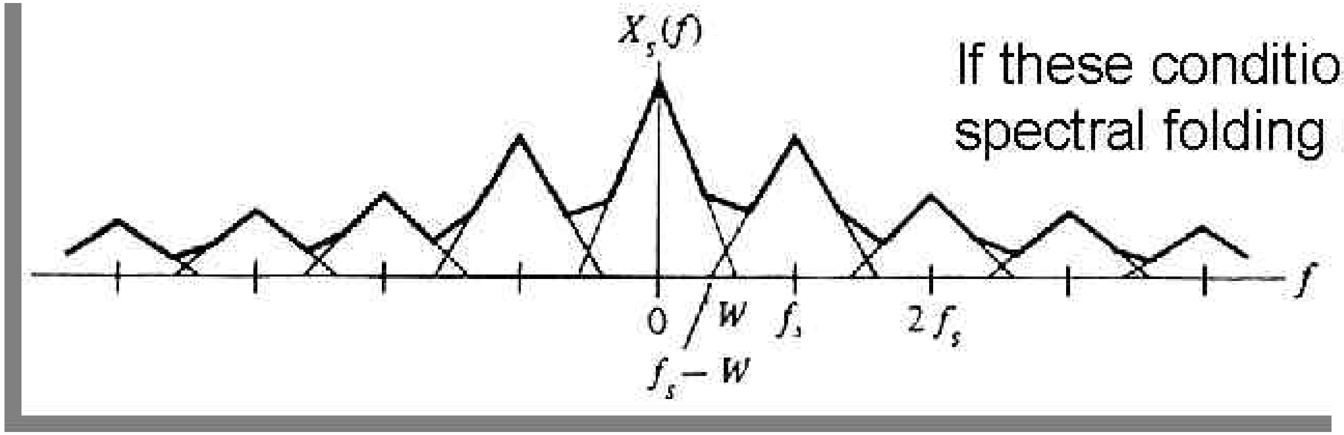
$$c_n = \frac{A\tau}{T_o}\,\mathrm{sinc}(n f_0 \tau)$$

Observations on chopper sampling

- Resulting spectra
  - has the envelope of the sampling waveform
  - has the sampled signal repeated at the integer multiples of the sampling frequency
- Therefore the sampled signal can be reconstructed by filtering provided that

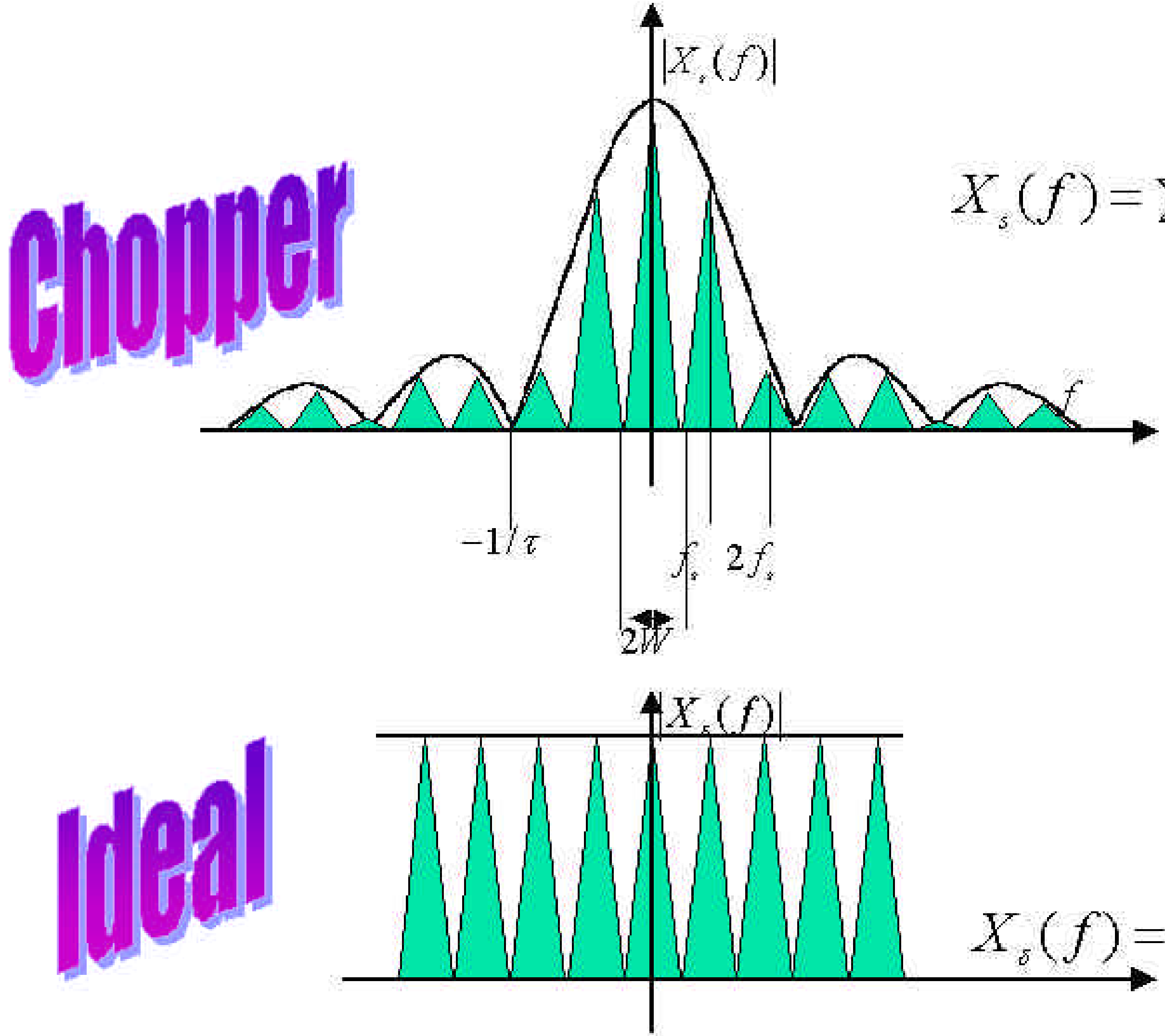
$X(f) = 0, |f| > W$ Sampled signal is band limited

$f_s \geq 2W$ Sampling rate is high enough



If these conditions are not met, spectral folding results:

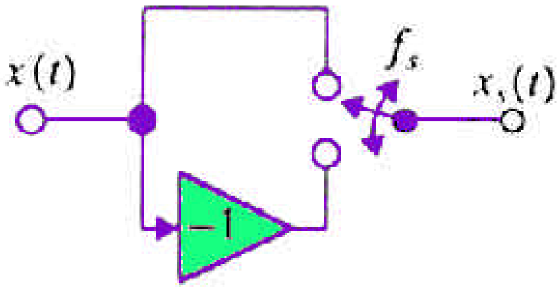
Ideal sampling and chopper sampling compared

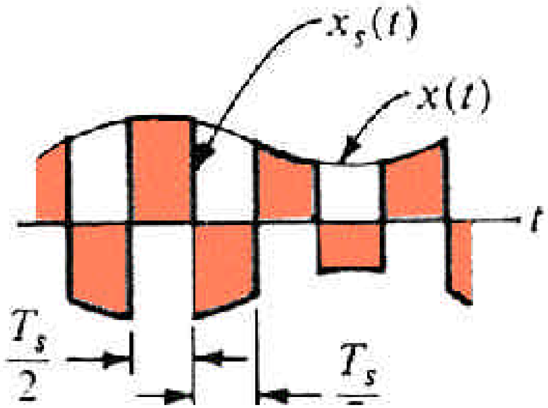


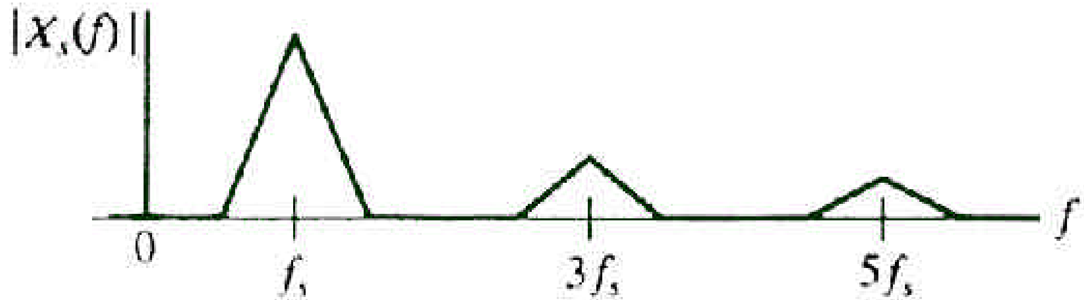
$$X_s(f) = \sum_{n=-\infty}^{\infty} c_n X(f - nf_s)$$

$$X_\delta(f) = f_s \sum_{n=-\infty}^{\infty} X(f - nf_s)$$

Bipolar sampling







$$x_s(t) = \frac{4}{\pi} x(t)\cos\omega_s t - \frac{4}{3\pi} x(t)\cos 3\omega_s t + \frac{4}{5\pi} x(t)\cos 5\omega_s t - \ldots$$

Bipolar sampling waveform

- Note that for the square wave having odd-symmetry, eg, for a period

$$A = \begin{cases} +1, 0 < t < T_0/2 \\ -1, 0 > t > -T_0/2 \end{cases}$$

  Fourier coefficients are

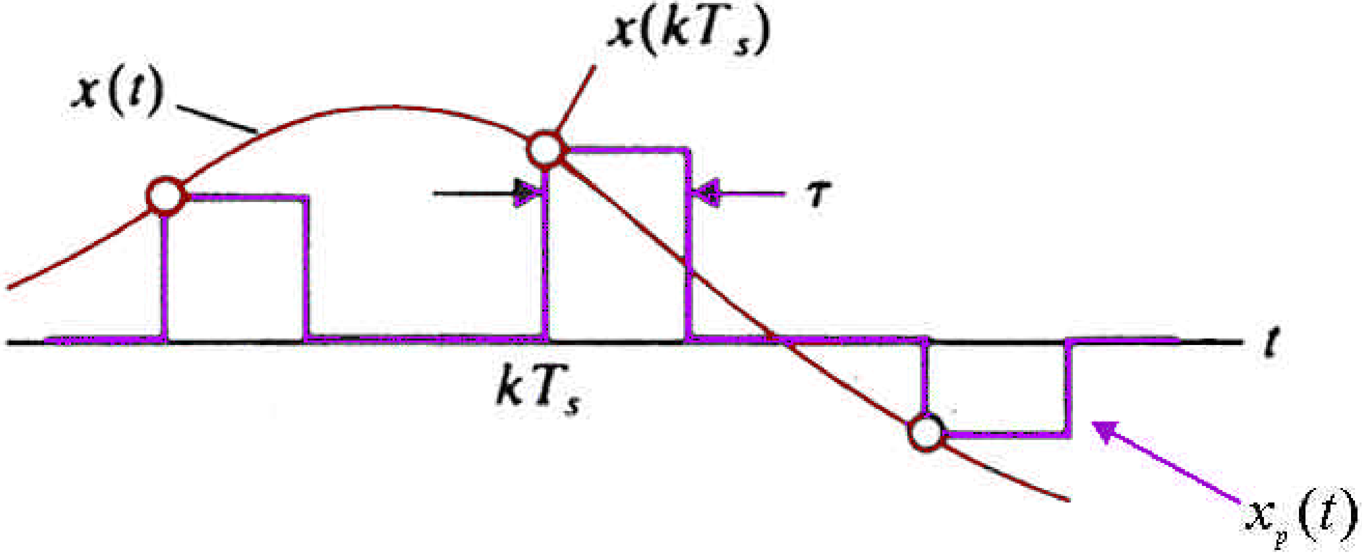
$$c_n = \begin{cases} 0, n \text{ is even} \\ -2j/(\pi n), n \text{ is odd} \end{cases}$$

- and therefore

$$x_s(t) = x(t)s(t)$$
$$= |c_0|x(t) + 2|c_1|x(t)\cos\omega_s t + 2|c_2|x(t)\cos(2\omega_s t + \arg(c_n))\ldots$$
$$= \frac{4}{\pi}x(t)\cos\omega_s t - \frac{4}{3\pi}x(t)\cos 3\omega_s t + \frac{4}{5\pi}x(t)\cos 5\omega_s t - \ldots$$

- Applications: DSB modulators, DSB, SSB demodulators (output lowpass filtered)

Pulse amplitude modulation (PAM) (Flat-top sampling)

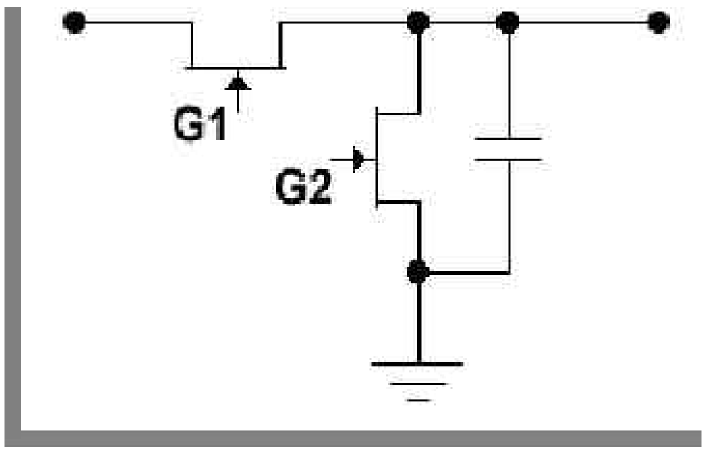


$$x_p(t) = \sum_k x(kT_s)\, p(t - kT_s)$$

$$x_p(t) = p(t) \otimes \sum_k x(kT_s)\delta(t - kT_s)$$

$$x_p(t) = p(t) \otimes x_\delta(t)$$

$$X_p(f) = P(f)\left[f_s \sum_n X(f - nf_s) \right]$$



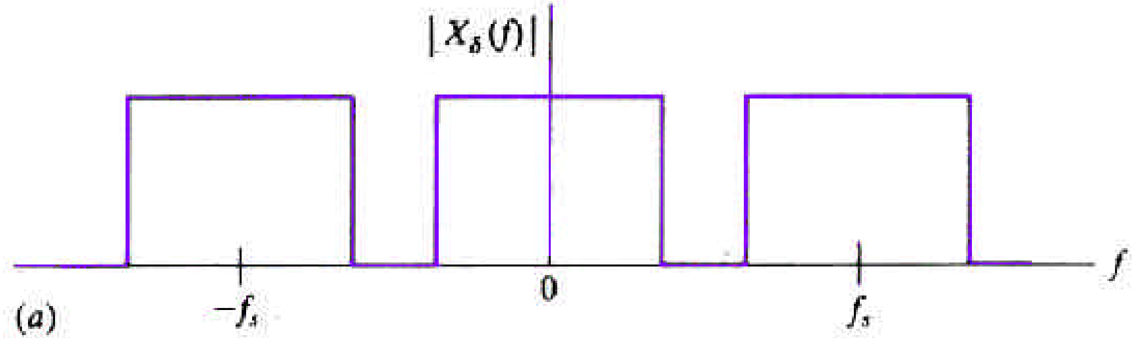
PAM spectra

- Linear spectral distortion can be alleviated by inverse filter

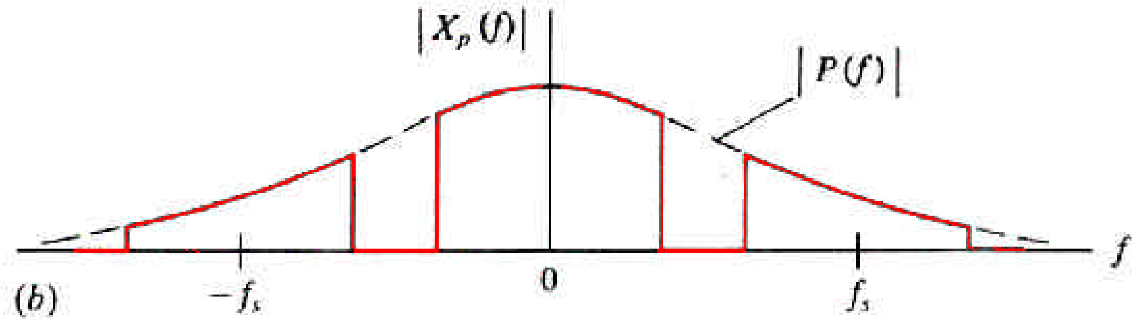
$$H_{eq}(f) = K\exp(-j\omega t_d)/P(f)$$

- Note that little or no equalization is required if $\tau / T_s \ll 1$

$$X_p(f) = P(f)\left[f_s \sum_n X(f - nf_s)\right]$$



Ideal sampled spectra



Flat-top sampled spectra

Line coding waveforms

Unipolar RZ and NRZ

Polar RZ and NRZ

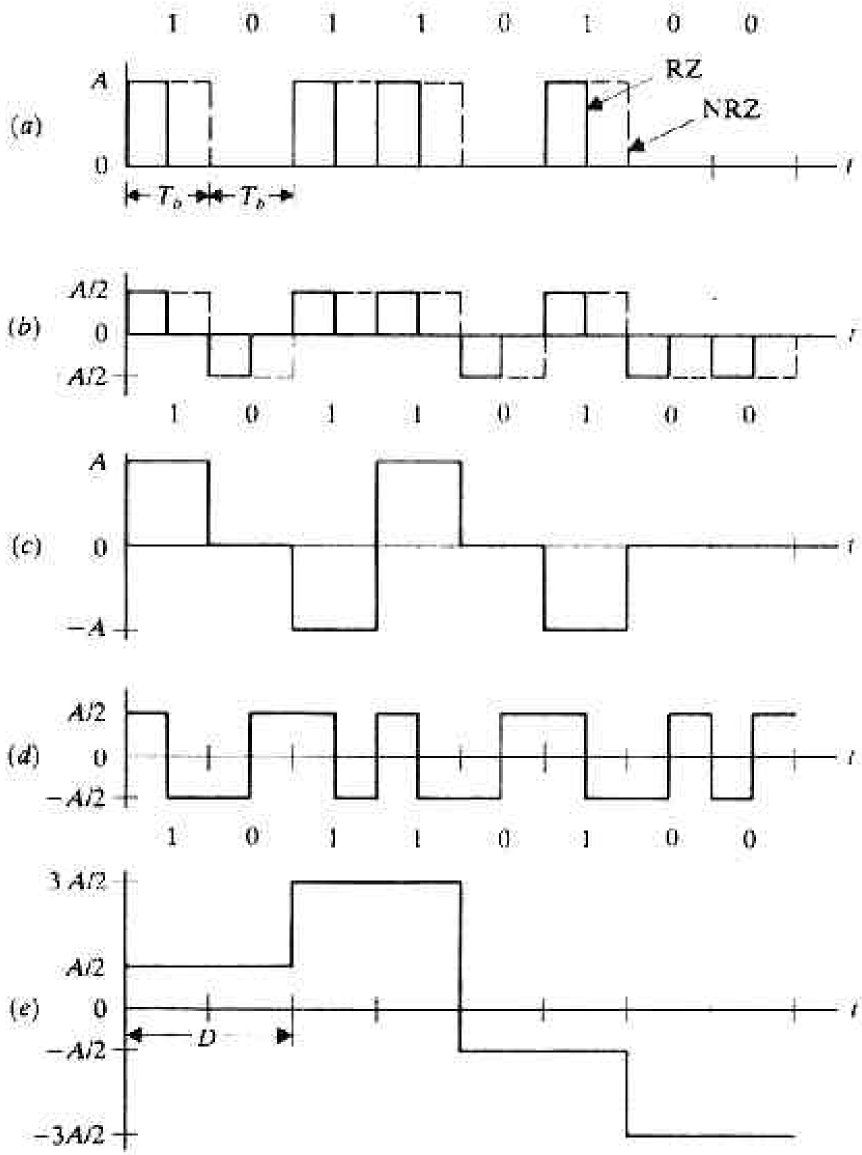
Bipolar NRZ

Split-Phase Manchester

Split-Polar quaternary NRZ

$$M = 2^n$$

$$r_{be} = r / n = r / \log_2 M$$



Line coding waveforms (cont.)

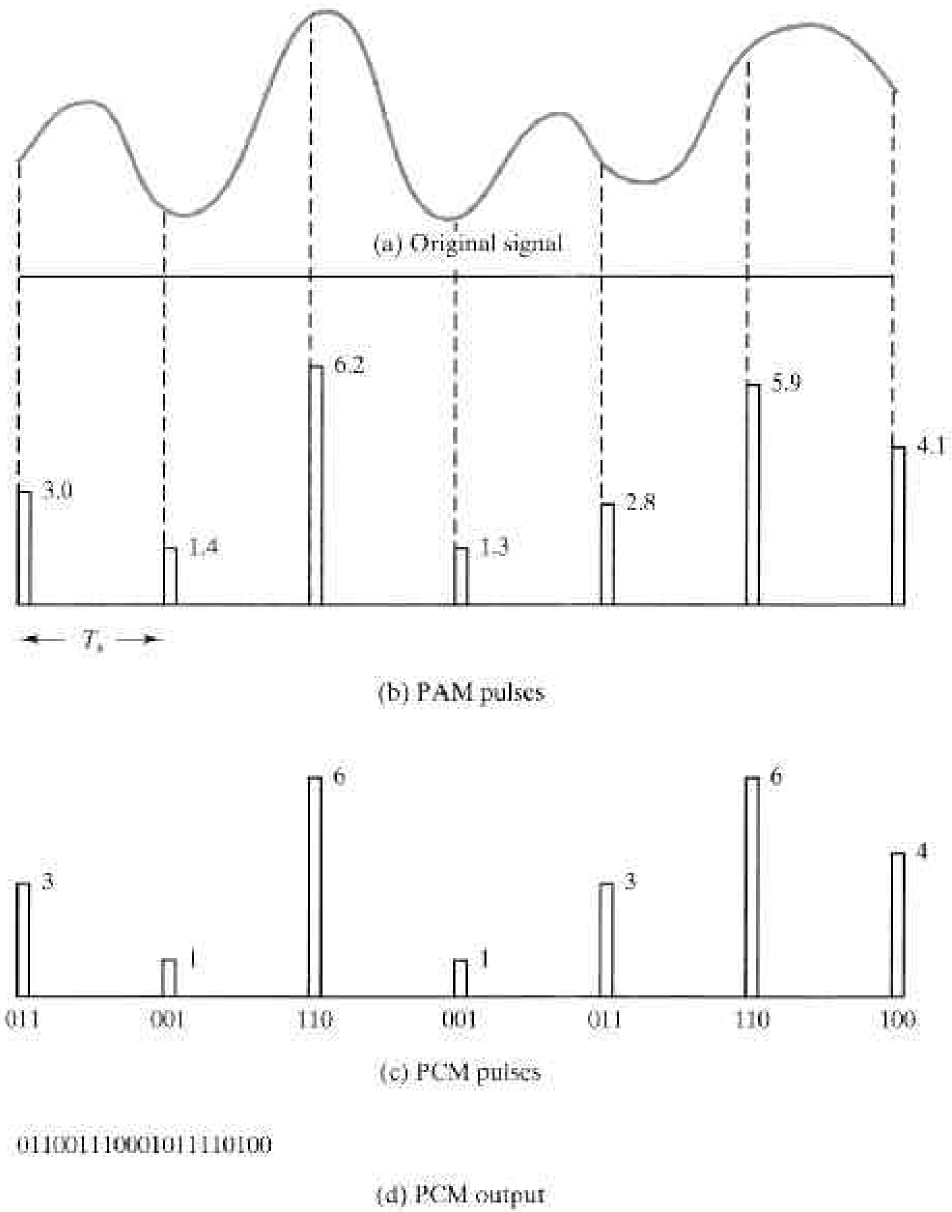
Nonreturn-to-Zero-Level (NRZ-L)
0 = high level
1 = low level
Nonreturn to Zero Inverted (NRZI)
0 = no transition at beginning of interval (one bit time)
1 = transition at beginning of interval
Bipolar-AMI
0 = no line signal
1 = positive or negative level, alternating for successive ones
Pseudoternary
0 = positive or negative level, alternating for successive zeros
1 = no line signal
Manchester
0 = transition from high to low in middle of interval
1 = transition from low to high in middle of interval
Differential Manchester
Always a transition in middle of interval
0 = transition at beginning of interval
1 = no transition at beginning of interval
B8ZS
Same as bipolar AMI, except that any string of eight zeros is replaced by a string with two code violations
HDB3
Same as bipolar AMI, except that any string of four zeros is replaced by a string with one code violation

Quantization

Analog signal to be transmitted

Continuous PAM-pulse train

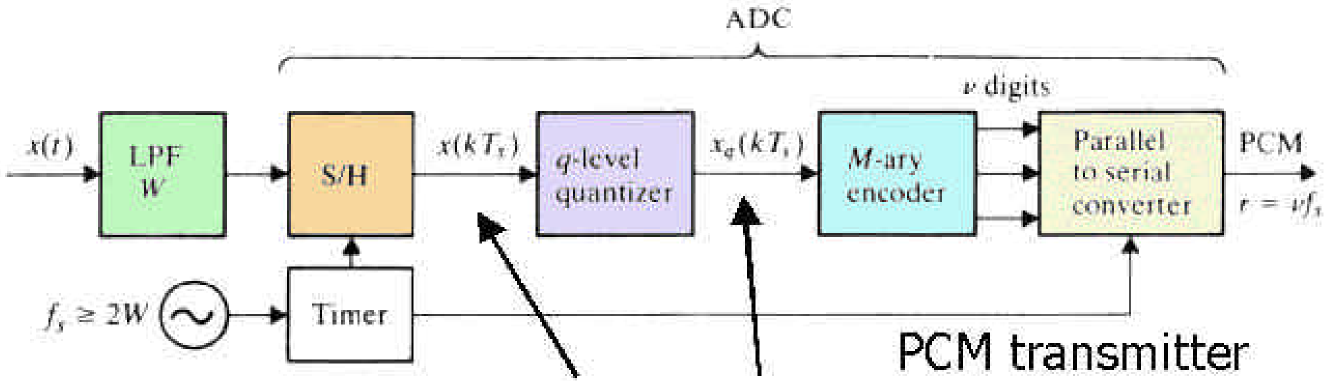
Quantized PAM-pulse train

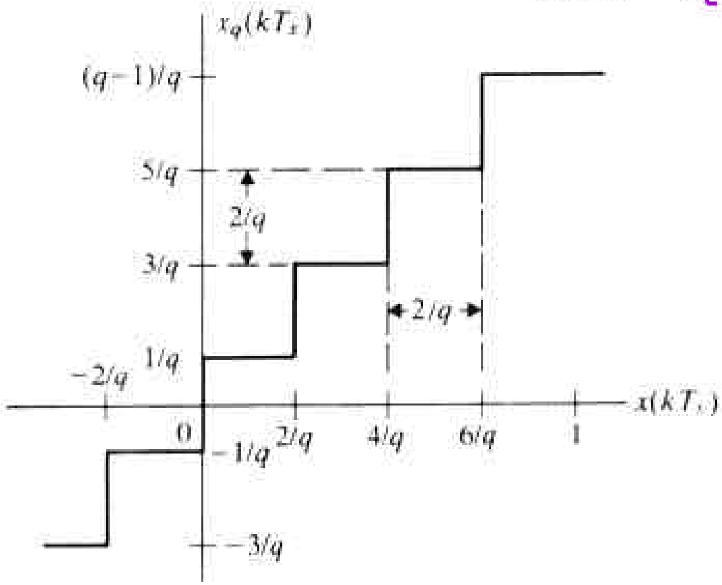


- Original signal has values continuously in its dynamic range
- PAM - signal is a discrete constant frequency, pulse train having continuous amplitude values
- Quantized PAM signal has only the values that can be quantized by the words available (here by 3 bit words)

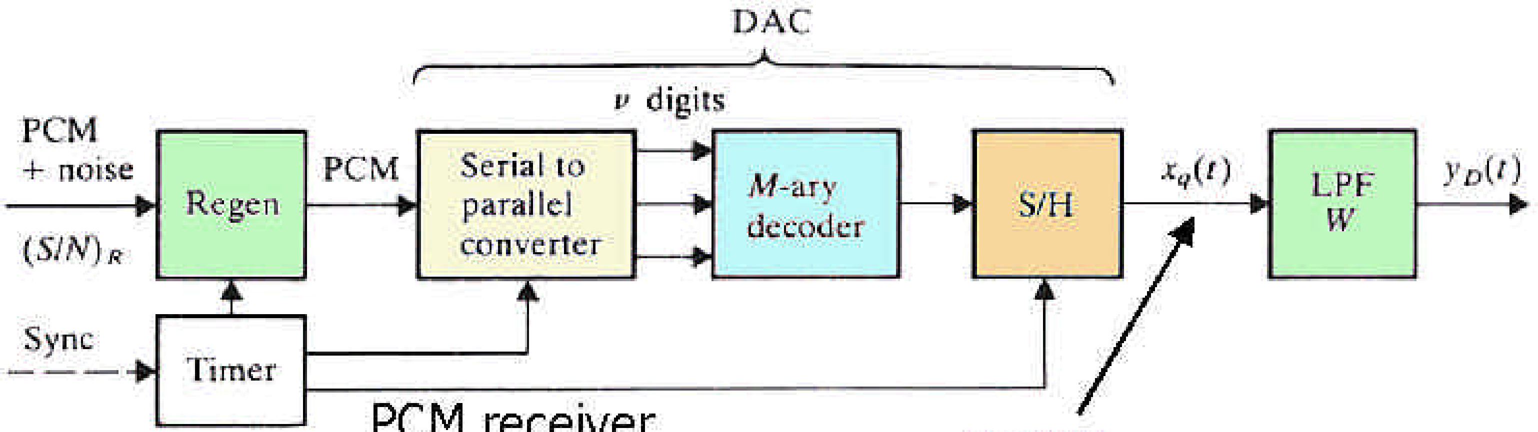
Uniform quantization

- Transforming the continuos samples into discrete level samples is called quantization
- In uniform quantization quantization step size is constant



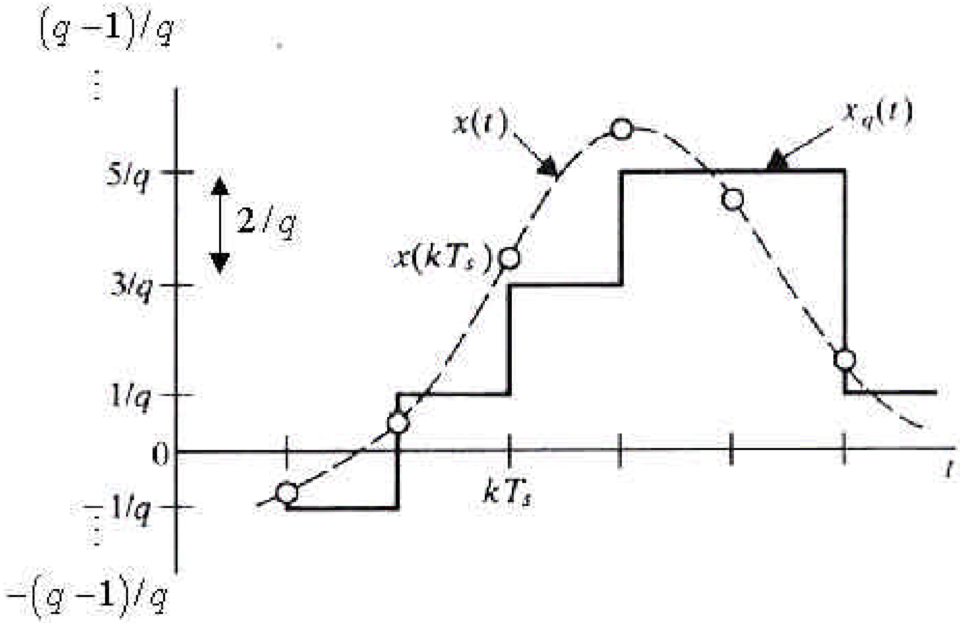


Reconstruction from the quantized signal



PCM receiver

Q-PAM



- Note that quantization noise is limited to

$$|\varepsilon_k| \le 1/q$$

$$q = M^{\nu}$$

$$\nu = \log_M q$$

q: number of quantization levels

ν: number of quantization bits

Quantization noise: uniform quantization

- Model the quantized signal by assuming ideal PAM sampling using the quantization error ε_k:

$$y(t)=\sum_k\left[x(kT_s)+\varepsilon_k\right]\delta(t-kT_s)$$

- Quantization error is the difference of the reconstructed and the quantized signal

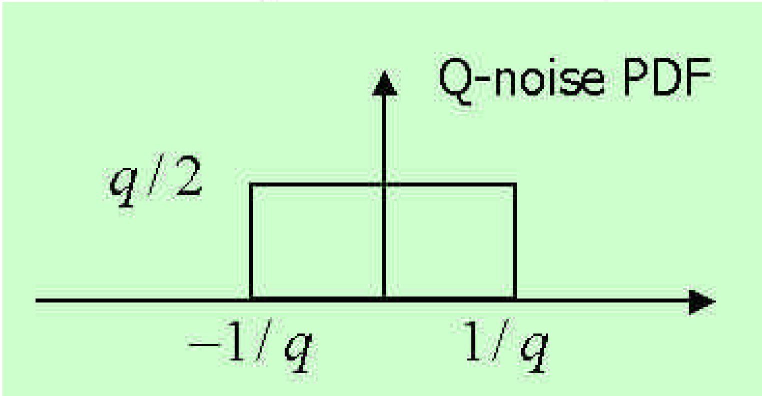
$$\varepsilon_k=x_s(kT_s)-x(kT_s)$$

- The final output is obtained by using the ideal LPF:

$$y_D(t)=x(t)+\sum_k\varepsilon_k\,\mathrm{sinc}(f_st-k)$$

- Assuming signal equal probable at all amplitude levels yields for quantization noise average power

$$\overline{\sigma_k^2}=\frac{q}{2}\int_{-1/q}^{1/q}\varepsilon^2 d\varepsilon=\frac{1}{3q^2}$$



note: $\frac{2}{q}\frac{q}{2}=1$

Uniform quantization: Destination SNR

- Define the destination SNR by

$$\left(\frac{S}{N}\right)_D = \frac{S_X}{\sigma_q^2} = 3q^2 S_X$$

that is by using $q=2^v$ and [dB]s

$$\left(\frac{S}{N}\right)_D = 10\log_{10}(3\cdot 2^{2v} S_X) = 10\log_{10} 3 + 10\log_{10}(2^{2v} S_X)\Big|_{S_X=1}$$

$$\leq 4.8 + 6.0v \ \text{dB}$$

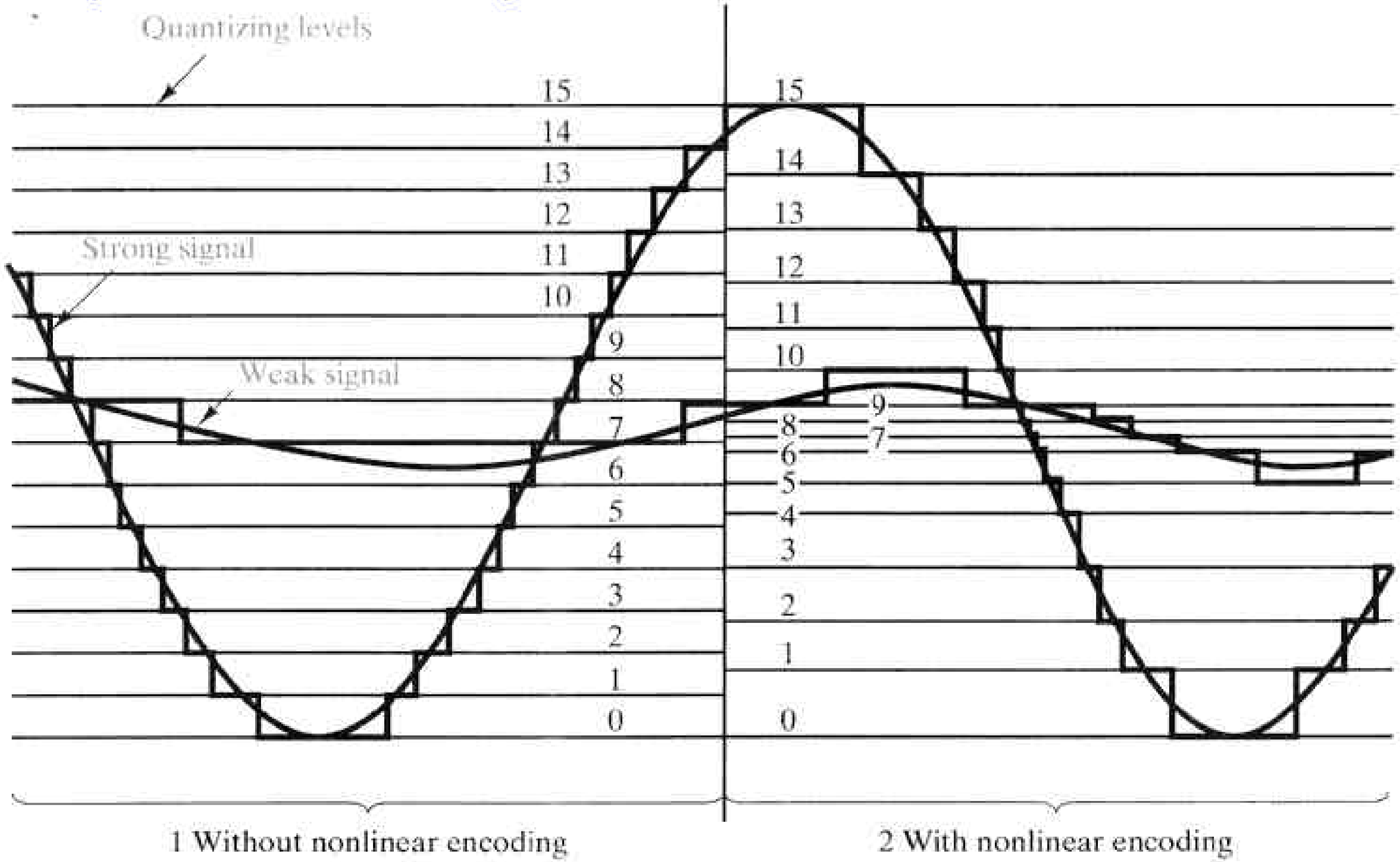
- Note that for 8 bits this yields $\left(\frac{S}{N}\right)_D \approx 4.8 + 6\cdot 8 = 52.8 \text{ dB}$

- However this is an upper bound and in practice $S_x << 1$ and typically signals follow LP-type PDF as for speech the Laplace-pdf:

$$p_X(x) = \frac{\alpha}{2}\exp(-\alpha|x|), \alpha = \sqrt{\frac{2}{S_X}}$$

- Therefore non-uniform sampling is frequently applied

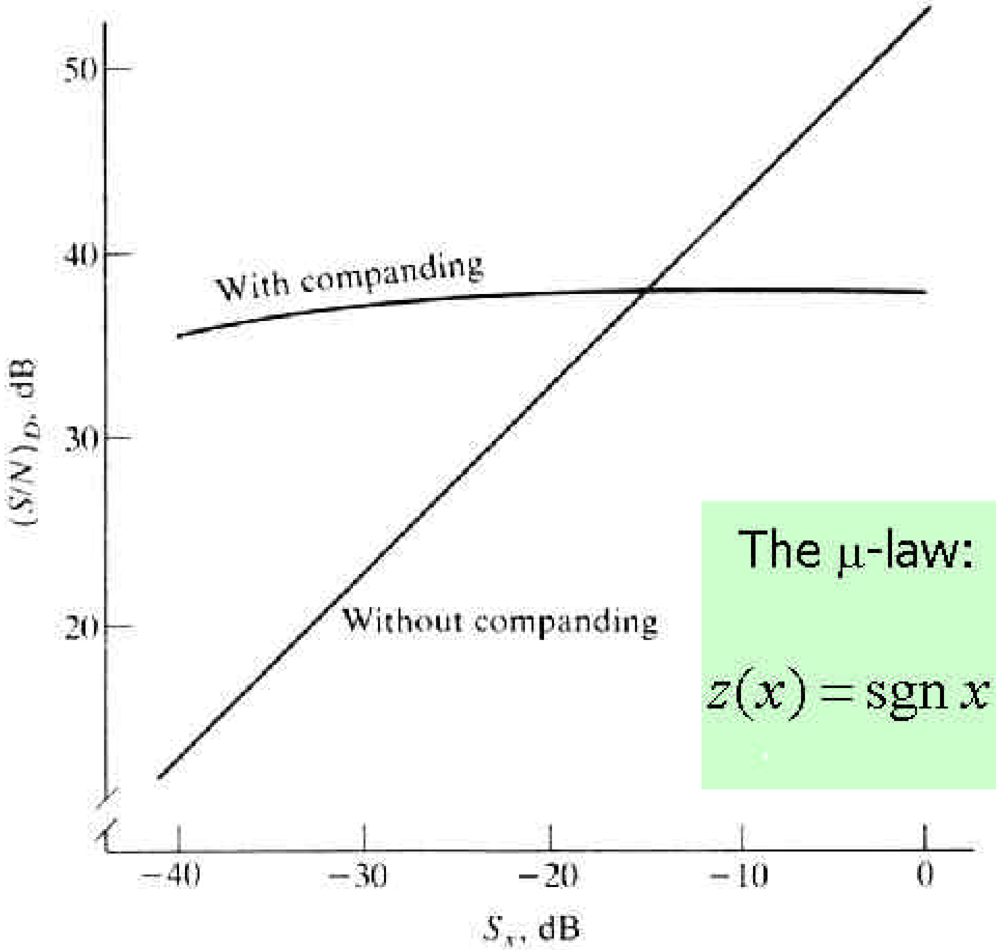
Non-uniform and uniform sampling: A qualitative comparison



- Note that for nonlinear quantization lower signal levels get more accurately quantized. That is how it should be because in practical voice and video applications their probability is much larger

Companding

- In PSTN-PCM two compounding laws are frequently used. The A-law (G.711) and the μ-law for Europe and USA respectively.
- Below is a figure showing how μ-law effects PCM-quality:



The μ-law:

$$z(x) = \operatorname{sgn} x \frac{\ln(1+\mu|x|)}{\ln(1+\mu)}, \mu = 255, |x| \leq 1$$

PCM with channel noise

- Random noise added into code words causes some code words to change their values
- Effect on signal error depends where it falls in the code word: Errors in the most significant bits (MSB) are a bigger problem than errors in the LSB
- The *m*:th bit distinguishes between quantum levels spaced by 2^m times the step height 2/q. Therefore the error on the *m*:th bit shifts the decoded level by

$$\varepsilon_m = (2/q)2^m$$

- The average channel noise power for a single bit at the decoded signal is therefore

$$\overline{\varepsilon_m^2} = \frac{1}{v}\sum_{m=0}^{v-1}\left(\frac{2}{q}2^m\right)^2 = \frac{4}{3v}\frac{q^2-1}{q^2} \approx \frac{4}{3v}$$

and for the whole code word bit-error probability P_e

$$\sigma_D^2 = vP_e\overline{\varepsilon_m^2} = 4P_e/3$$

PCM noise characteristics for uniform quantization and Gaussian channel noise

- Total noise of the PCM system consists of channel noise and quantization noise or

$$N_D = \sigma_D^2 + \sigma_Q^2 = \frac{1}{3q^2} + \frac{4P_e}{3}$$

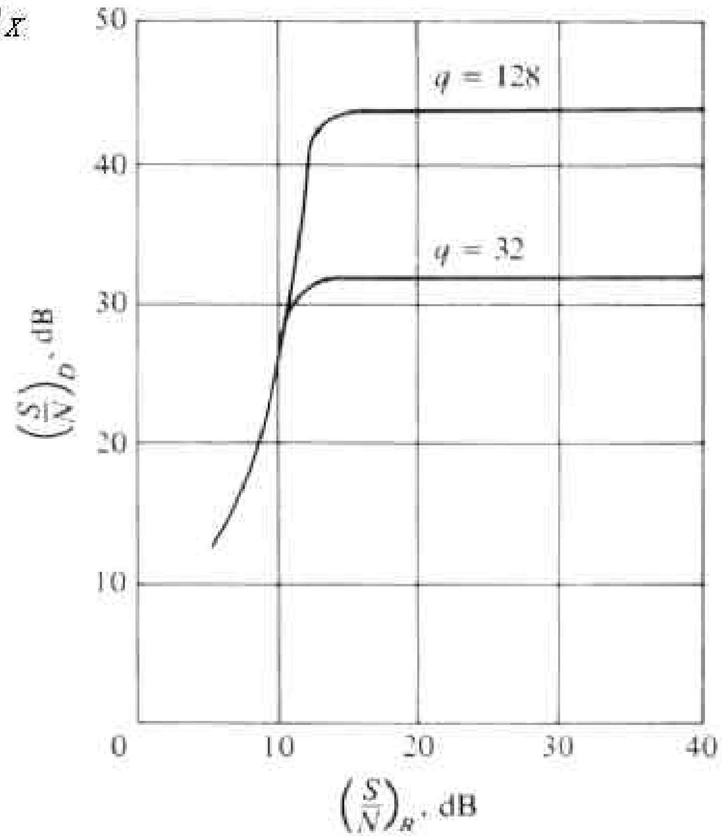
and the SNR is $\left(\frac{S}{N}\right)_D = \frac{3q^2}{1+4q^2P_e} S_X$

- Assume now polar signaling with

$$P_e = Q\left[\sqrt{(SNR)_R}\right]$$

and S_x=0.5 yields then the following figure:

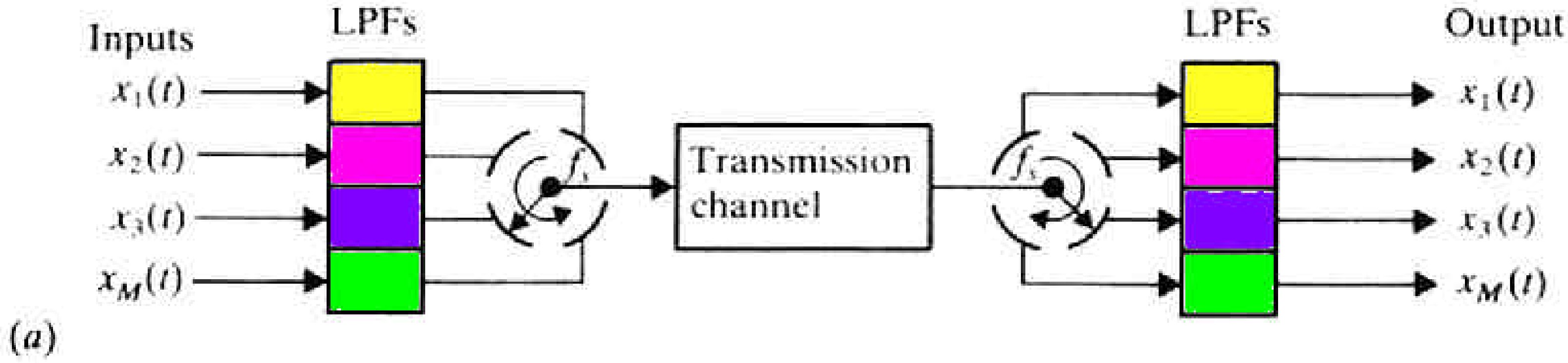
- Note that PCM system maintains solid quality until performance drops dramatically

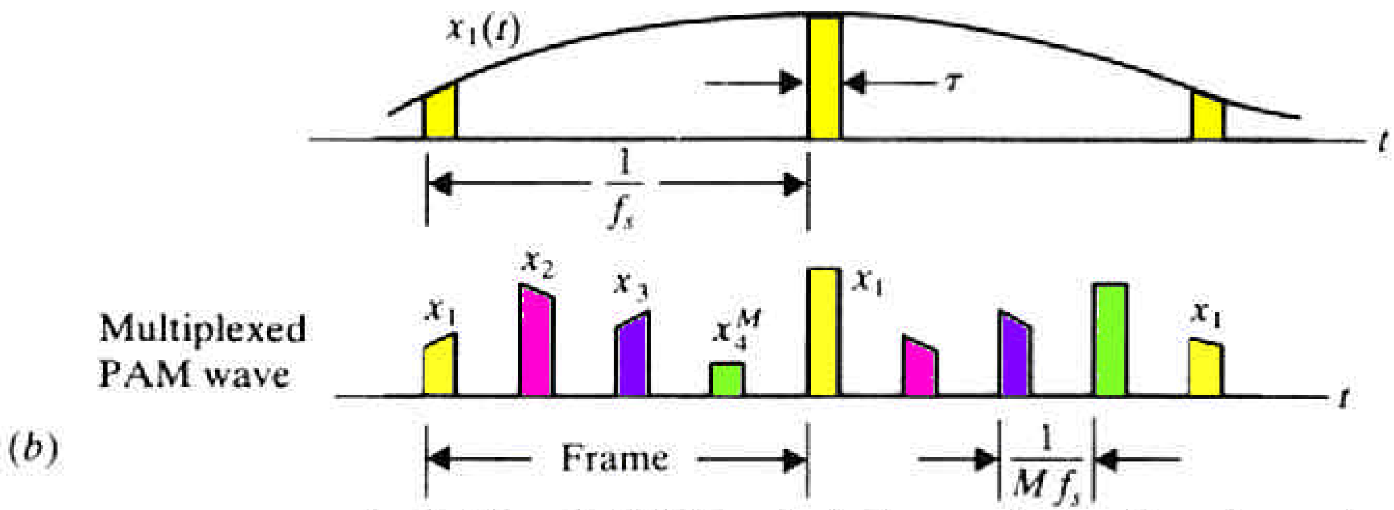


PCM-method summarized

- Analog speech signal is applied into a LP-filter restricting its bandwidth into 3.4 kHz
- Sampling circuit forms a PAM pulse train having rate of 8 kHz
- Samples are quantized into 256 levels that requires a 8 bit-word for each sample ($2^8=256$).
- Thus a telephone signal requires 8x8 kHz = 64 kHz bandwidth
- The samples are line coded by using the HDB-3 scheme to alleviate synchronization problems at the receiver
- Usually one transmits several channels simultaneously following SDH hierarchy (as 30 pcs)
- Transmission link can be an optical fiber, radio link or an electrical cable
- At the receiver the PAM signal is first reconstructed where after it is lowpass filtered to yield the original-kind, analog signal

Time-division multiplexing (TDM) can be used to combine PAM or PCM signals

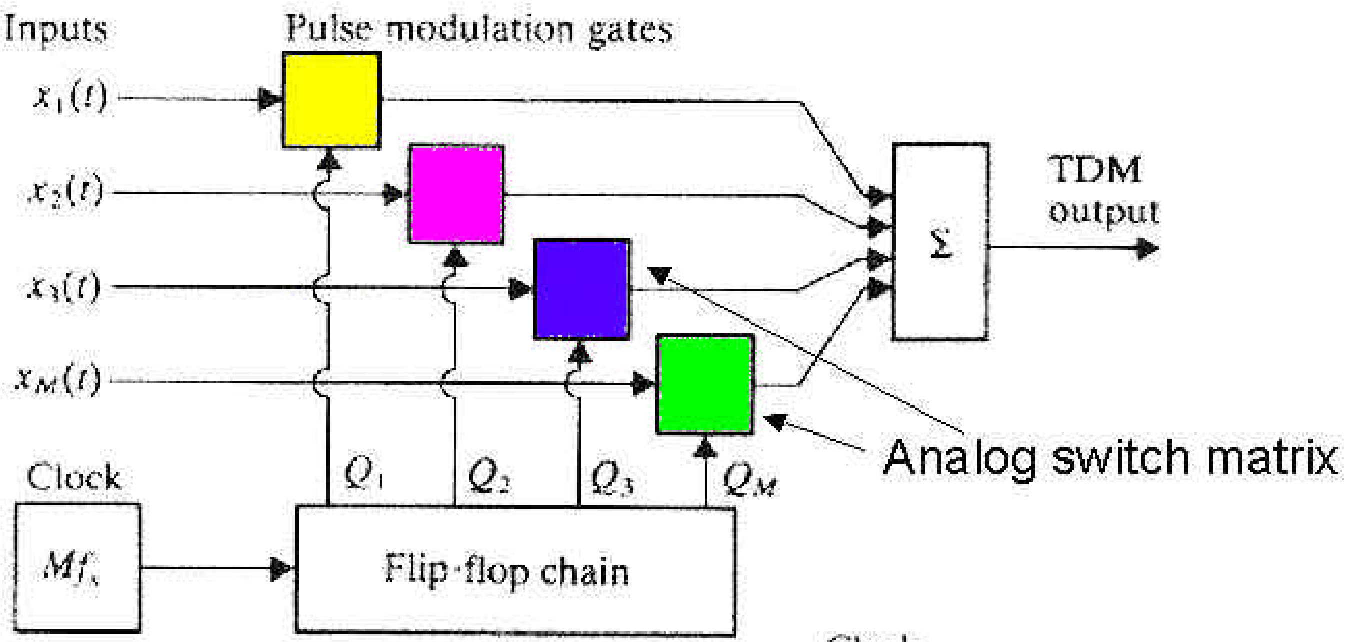




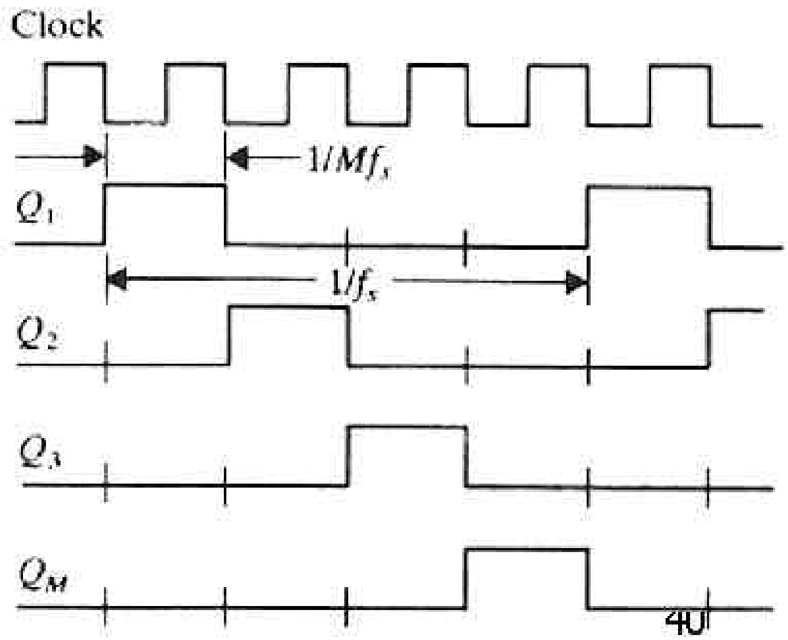
$$r \geq Mf_s = M\,2W = \text{total number of pulses / second}$$

therefore with raised cos-pulses TDM baseband bandwidth is limited to $B_T = r/2 \ldots r$

A TDM realization



- TDM systems are critical in timing
- Timing can be arranged by
  - marker pulses
  - pilot tones
  - statistical properties of the TDM signals



Comparing TDM and FDM

- TDM and FDM (see the last lecture) accomplish the same transfer efficiency (dual methods)
- TDM: simpler instrumentation; only commutator switches + LPF (FDM: subcarrier modulator, bandpass filter and demodulator for every message channel)
- TDM requires good synchronization
- TDM can be accommodated to different signals and BWs by using different modulation formats
- With respect of fading wireless channel both methods have advantages and disadvantages
- TDM is discussed more while discussing SDH later