S-72.610 Mobile Communications Services and Systems

Tutorial 2, November 19, 2004

1. Once a physicist Robert Wood did not stop his car behind the red traffic light. He excuses himself by using Doppler effect. Because of Doppler shift the red light had turned to the green one. How quickly he had to move in order his claim to be true.

1. Solution
The theory has shown the Doppler shift between the transmitted and received wavelengths as:

$$\frac{l}{l'} \quad 1 \quad \frac{v}{c},$$

where
l is wavelength emitted from the source,
l' is wavelength recognized by a moving observer,
v is speed of the user,
c is speed of light 300000 *km*/*s*.
The shortest wave corresponding to the red light has wavelength 630 *nm*, the longest wavelength corresponding to the green light has wavelength 560 *nm*.
Inserting these values to the equation we get:

$$\frac{630}{560} \quad 1 \quad \frac{v}{300000}.$$

From there we found the speed of the car

$$v = \frac{300000}{8} = 37500\frac{km}{s} \Rightarrow 1.35 \cdot 10^{8}\frac{km}{h}.$$

2. Calculate the Doppler bandwidth in the GSM900 and in the UMTS-system operating at 960 and 2150 MHz when the mobile station speed is 3, 50, 120, and 500 *km*/*h* and:
a) Only reflections from stationary structures are considered,
b) Also reflections from cars moving with the same speed are considered.

2. Solution

a) The Doppler bandwidth is $B_D = \frac{2\nu}{c} f_c$

When $\nu = 3\frac{km}{h}$, f_c=960 *MHz*, then

$$B_d = 2 \times \frac{3\frac{km}{h}}{3\times 10^5 \frac{km}{s} \times 3600\frac{s}{h}} \times 9.6\times 10^8\, Hz = 5.33\ Hz$$

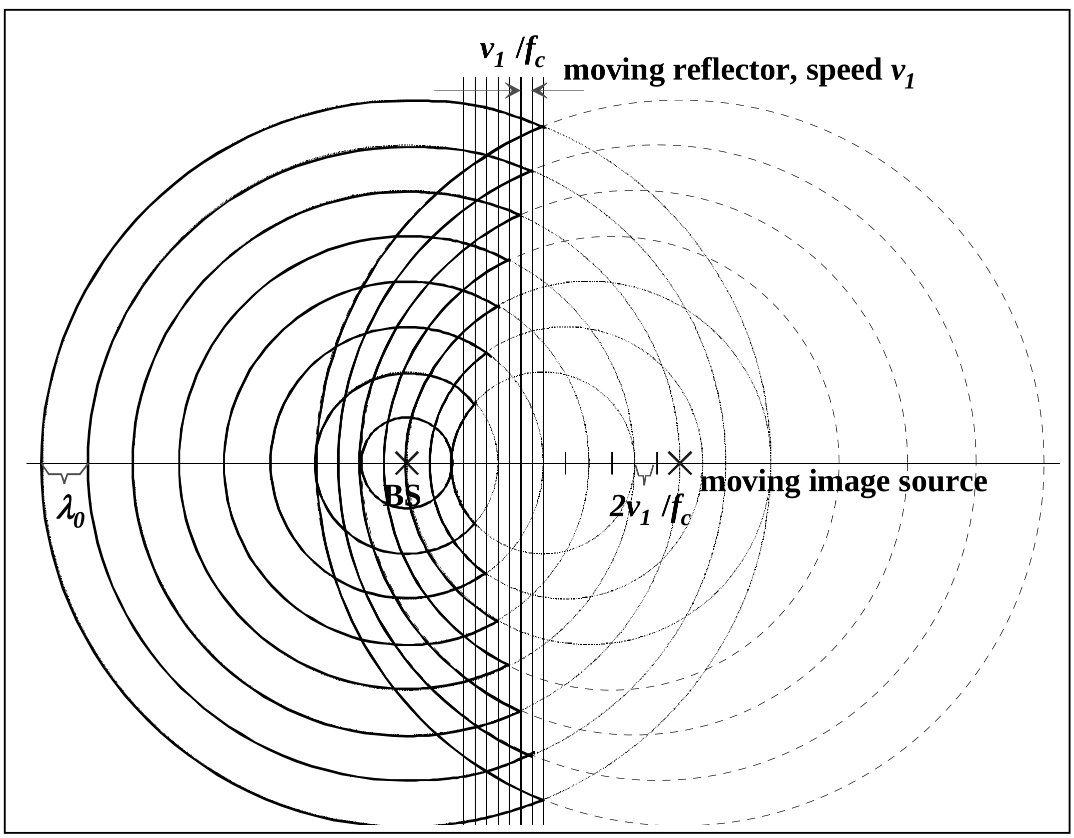
The Doppler bandwidth for the other parameter combinations are obtained in the same way (proportional to the speed and frequency)

	3 *km/h*	50 *km/h*	120 *km/h*	500 *km/h*
f_c=960 *MHz*	5.33 *Hz*	88.89 *Hz*	213.33 *Hz*	888.89 *Hz*
f_c=2150 *MHz*	11.94 *Hz*	199.07 *Hz*	477.78 *Hz*	1990.74 *Hz*

b) The figure shows the direct and reflected field obtained by the image principle From the figure can be seen that

$$\lambda_1 = \lambda_0 - \frac{2v_1}{f_c} \rightarrow \frac{c}{f_1} = \frac{c}{f_c} - \frac{2v_1}{f_c} \rightarrow f_1 = \frac{f_c}{1-\frac{2v_1}{c}}$$

which gives



$$f_D = f_c - f_1 = f_c\left(\frac{1}{1-\frac{2v_1}{c}} - 1\right) = f_c\left(\frac{1-1+\frac{2v_1}{c}}{1-\frac{2v_1}{c}}\right) = \frac{\frac{2v_1}{c}f_c}{1-\frac{2v_1}{c}} \cong \frac{2v_1}{c}f_c$$

Adding the Doppler shift from MS movement the maximum Doppler-shift is $f_{D_{\max}} \cong \frac{2v_1+v}{c}f_c$, which in this case will triple the Doppler-bandwidth obtained with fixed reflectors.

3. Investigate the UMTS system where the user is moving with speed $50\frac{km}{h}$ WCDMA

a) Determine the number of positive – going zero crossing about the *rms* value that occur over 10 s interval.

b) Determine the average duration of the fade below the *rms* level.

c) Determine the average duration of the fade below the -10 dB level.

3.

Reference. Rappaport T.S. Wireless Communications. Prentice Hall 1996.

a) The *level crossing rate* is defined as the expected rate at which the Rayleigh fading envelop, normalized to the local *rms* signal level, crosses a specified level in a positive -going direction.

$$N_R = \int_0^\infty r'p\left(R,r'\right)dr' = \sqrt{2\pi}f_m\rho e^{-\rho^2}$$

where f_m is maximum Doppler frequency and

$\rho = \frac{R}{R_{rms}}$ is the value of the specified level *R*, normalized to the local *rms* amplitude of the fading envelop.

For our case the $f_m = 199.07\ Hz$

The number of level crossings per second is given by

$$10\frac{N_R}{2} = 5\cdot\sqrt{2\pi}\cdot 199.07\cdot 1\cdot e^{-1^2} = 917.8$$

b) The average fade duration is defined as the average period of time for which the received signal is below a specified level *R*. For the Rayleigh fading signal this is

$$\overline{\tau} = \frac{1}{N_R}\Pr\left[r \le R\right]$$

where $\Pr\left[r \le R\right]$ is probability that *r* is less than *R*.

The probability that the received signal r is less than the threshold R is found from the Rayleigh distribution

$$\Pr\left[r \le R\right] = \int_0^R p(r)\,dr = 1 - e^{-\rho^2}$$

where $p(r)$ is *pdf* of a Rayleigh distribution.

The average fade duration as function of f_m can be expressed as

$$\overline{\tau} = \frac{e^{\rho^2} - 1}{\rho f_m \sqrt{2\pi}}$$

For the *rms* level $\rho = 1$

$$- \quad \frac{e^{1^2} \quad 1}{1 f_m \sqrt{2}} \quad 0.0034\ s$$

For the $-10\,dB$ level $\rho = 0.1$

$$- \quad \frac{e^{0.1^2} \quad 1}{0.1 f_m \sqrt{2}} \quad 0.00021\ s$$

4. Channel characterization

Power $[dB]$	-3	0	-2	-6	-8	-10
Delay $[\mu s]$	0.0	0.2	0.5	1.6	2.3	5.0

Determine the *rms* delay and mean excess time for the channel
Determine the maximum excess delay –10 dB.
Estimate the coherence bandwidth of the channel.
(Coherence time). Assume that mobile using UMTS system traveling with $50\,\frac{km}{h}$ receives the signal through this channel determine the time over which the channel appears stationary (or at least highly correlated).

4. Reference. Rappaport T.S. Wireless Communications. Prentice Hall 1996.
a) Mean excess delay is the first moment of the power delay profile and is defined as,

$$\overline{\tau} = \frac{\sum_k P(\tau_k)\tau_k}{\sum_k P(\tau_k)} = 0.67\,[\mu s]$$

For our channel it becomes
The *rms* delay is the square root of the second central moment of the power delay profile and is defined to be

$$\sigma_\tau^2 = \overline{(\tau^2)} + (\overline{\tau})^2$$

Where $\overline{(\tau^2)}$ is defined as $\overline{(\tau^2)} = \frac{\sum_k P(\tau_k)\tau_k^2}{\sum_k P(\tau_k)} = 1.5816\,[\mu s]$

$$\sigma_\tau^2 = \overline{(\tau^2)} + (\overline{\tau})^2 = 1.0616\,.$$

b) *Maximum excess delay* is defined to be delay during which multipath energy falls X dB below the maximum. In our case it would be $-10\,dB$. For the given channel that is delay till the last multipath component component. $5\,\mu s$.

c) The *coherence bandwidth* can be assumed to be the frequency separation at which the channel is highly correlated. For example it can be estimated to be at the level where the normalized value of the correlation coefficient attains a value 0.7.
The *coherence bandwidth* can be calculated as $\Delta f = \frac{1}{2\pi T}$ where T is maximum excess delay. For the given channel it is 31.8 kHz.

d) The Doppler speed and *coherence time* are inversely proportional to each other $T_{C,1} \approx \frac{1}{f_m}$ where f_m is the maximum Doppler shift.

If the coherence time is defined as the time over which the time correlation function is above 0.5, then the coherence time is approximately $T_{C,2} \approx \frac{9}{16\pi f_m}$.

The coherence time can be calculated as geometric mean of equations of these two definitions of T_C

$$T_C \approx \sqrt{\frac{9}{16\pi f_m}} = \frac{0.423}{f_m} = 0.0021\,[s]$$

5. Do we need an equalizer for the GSM signal in the channel given in previous exercise.

5. If the transmitted signal bandwidth is narrow compared to the channel's coherence bandwidth, all transmitted frequency components encounter early identical propagation delay. The channel can be characterized as a narrowband channel where all frequency components have the same fading.
We compare GSM bandwidth to the coherence bandwidth of the channel. For the GSM signal is allocated 200 *kHz* bandwidth this is much higher than the correlation bandwidth and the equalizer is useful.