S-72.610 Mobile Communications Services and Systems

Tutorial 3, November 26, 2004

1. The modulation used in a mobile communication system is DPSK, for which the bit error probability as function of the signal to noise ratio $\gamma = P_{rx} / N_o R_s$ with an ideal receiver in the AWGN-channel is $P_b(\gamma) = 0.5\exp(-\gamma)$.

a) Determine the receiver sensitivity in the AWGN-channel, where the sensitivity is defined as the received power level (dBm) giving the bit error probability 10^{-2}. The transmission rate is 16 kbit/s and $N_o = 10^{-20}\, W/Hz$

b) Derive the average bit error probability expression in a slowly and flat fading Rayleigh-channel, where the probability density function of the signal to noise ratio is $p(\gamma) = \exp\left(-\gamma/\gamma_m\right)/\gamma_m \cdot U(\gamma)$, where γ_m is the average signal to noise ratio.

c) Determine the receiver sensitivity in the Rayleigh-channel (bit error probability 10^{-2}). How many dB the higher SNR is needed in Rayleigh fading channel compared to AWGN channel.

1. **Solution.**

a) $P_b(\gamma) = 0.5\exp(-\gamma) = 0.5\exp\left(-P_{rx}/N_o R_s\right) = 0.01$

$$\rightarrow P_{rx} = N_o R_s \ln\left(1/2P_b\right) = 10^{-20} \cdot 16000 \ln(50) = 6.26 \cdot 10^{-16}\ \text{W}$$

$$= 6.26 \cdot 10^{-13}\ \text{mW} \leftrightarrow -122.0\ \text{dBm} = \text{Sensitivity}$$

The SNR at what this error level is achieved is $\gamma = \dfrac{P_{rx}}{N_0 R_s} = 3.91$.

b)

$$P_b = \int_0^\infty p(\gamma) P_b(\gamma) d\gamma = \int_0^\infty \frac{\exp\left(-\gamma/\gamma_m\right)}{\gamma_m} \cdot \frac{\exp(-\gamma)}{2} d\gamma$$

$$= \int_0^\infty \frac{\exp\left(-\gamma\left(1+1/\gamma_m\right)\right)}{2\gamma_m} d\gamma = \Big|_0^\infty - \frac{\exp\left(-\gamma\left(1+1/\gamma_m\right)\right)}{2\gamma_m\left(1+1/\gamma_m\right)}$$

$$= -0 + \frac{1}{2\gamma_m\left(1+1/\gamma_m\right)} = \frac{1}{2\left(\gamma_m+1\right)}$$

c)

$$P_b = \frac{1}{2\left(\gamma_m+1\right)} \to \gamma_m = \frac{P_{rx}}{N_o R_s} = \frac{1}{2P_b} - 1$$

$$\to P_{rx} = 10^{-20} \cdot 16000 \cdot 49 = 7.84 \cdot 10^{-15} \text{ W}$$

$$= 7.84 \cdot 10^{-12} \text{ mW} \leftrightarrow -111.0 \text{ dBm}$$

The target SNR is $\gamma = \dfrac{P_{rx}}{N_0 R_s} = 49$.

For the same performance the required SNR increase in Rayleigh fading channel is $10\log_{10}\left(\dfrac{SNR_{Rayl}}{SNR_{AWGN}}\right) = 10.98 dB$.

2. Determine the power distribution for two branch selection combining. In both branches the power distribution is exponentially distributed with the same parameter γ_m. $f(x) = \dfrac{1}{\gamma_m}\exp\left(-\dfrac{x}{\gamma_m}\right)U(x), f(y) = \dfrac{1}{\gamma_m}\exp\left(-\dfrac{y}{\gamma_m}\right)U(y)$.

2. **Solution**

The selection combining implements the function $\max(x,y) \le z$. The probability of z is equal to the probability that x is equal to z and y is less than x or y is equal to z and x is less than y.

$f(z) = f_x(z)F_y(z) + f_y(z)F_x(z)$ Where $F_x(z), F_y(z)$ are respective CDF functions. The PDF functions were given to be exponential distribution. The corresponding CDF's are $F(x) = \left(1 - \exp\left(-\dfrac{x}{\gamma_m}\right)U(x)\right)$

By inserting this into the probability calculation function we get.

$$f(z) = \frac{1}{\gamma_{mx}} \exp\left(-\frac{z}{\gamma_{mx}}\right)U(z)\left(1 - \exp\left(-\frac{z}{\gamma_{my}}\right)U(z)\right)$$
$$+\frac{1}{\gamma_{my}} \exp\left(-\frac{z}{\gamma_{my}}\right)U(z)\left(1 - \exp\left(-\frac{z}{\gamma_{mx}}\right)U(z)\right)$$

By considering that both initial distributions had the same parameters we can use $\gamma_m = \gamma_{mx} = \gamma_{my}$.

$$f(z) = \frac{2}{\gamma_m} \exp\left(-\frac{z}{\gamma_m}\right)U(z)\left(1 - \exp\left(-\frac{z}{\gamma_m}\right)U(z)\right)$$

3. Let assume a WCDMA receiver operating in a (static) multipath environment. The relative strength a_i and delays τ_i of the propagation paths (with respect to the line of sight (LOS) path) are as follows

a_i	1	0.3	0.6	0.2	0.5	0.2
τ_i $[\mu s]$	0.0	0.2	0.3	0.7	2.4	3.1

We assume that for the perfectly synchronized code the signal processing branch in the receiver produces the relative power 1 (unit) for relative path strength 1 (unit) at the output of the matched filter. When the codes are not in synchronisation the correlator produces $\frac{1}{N}$ of this power. There N is code gain of the WCDMA system.

Calculate the SIR ratio (signal power to multipath interference ratio) for the following cases (other interfering signals and noise is not taken into account):

a) No RAKE reception; the receiver synchronises only to the LOS signal component. The channel bit rate is 32 *kbps* (Note user bit rate undefined), N=128.
b) 4 finger RAKE reception with Equal Gain Combining, N=128.
c) 4 finger RAKE reception with Maximum Ratio Combining, N=128.
d) 4 finger RAKE reception with Maximum Ratio combining. In this case, however, the channel bit rate is 512 *kbps*, N=8.
e) 6 finger RAKE reception with Maximum Ratio Combining, N=128.

Calculate the percentage of the arrived signal energy that the 4 finger RAKE receiver utilise for decoding.

3. **Solution**

The RAKE receiver consist a set of fingers, receivers, and each of them is synchronised to one of the channel taps. The outputs of fingers are combined accordingly to applied policy most common of which are Equal Gain and Maximum Ratio Combining.

Assume that the matched filter of a signal processing branch consist of a correlator and integrator. Correlator is between the signal code with strength a_i (received via propagation path i and the reference code with strength b_i where amplitude of b depends on combining policy.

The received signal is described by signal to noise ratio SNR what is power of the mean of a received signal divided by the second moment of the signal. In code synchronisation the relative average matched filter output power is $\left(a_i \cdot b_i\right)^2$ and otherwise $\frac{\left(a_i \cdot b_i\right)^2}{N}$.

a)

The relative signal power is $\left(a_i \cdot b_i\right)^2 = 1$ (power units) $\rightarrow b_i = 1$. The multipath interference is variance of the sum of the amplitudes from the paths that are not synchronised to the reference signal. The variance of the sum of the independent signals is sum of variances. Because the other multipath are not synchronous to the first path they are scaled down by N.

$$i = \frac{1}{N}\sum_{i=1}^{5}\left(a_i \cdot b_0\right)^2 = \frac{0.78}{N}$$

The SIR ratio is thus $\frac{N}{0.78} = 164.1 \Rightarrow 22.15\ dB$.

b)

We first investigate if there is inter symbol interference (ISI) in the receiver. ISI is generated whenever the channel delay spread D is larger than the bit duration T. Since $T = \left(\text{bit rate}\right)^{-1} = 31\ \mu s \gg D\ \mu s$ there is no ISI.

The signal power after Equal Gain Combining (EGC) is

$$S = \left(a_0 \cdot 1 + a_1 \cdot 1 + a_2 \cdot 1 + a_4 \cdot 1\right)^2 = 5.76$$

Since in EECG the reference code strength b_i are assumed equal to be $b_0 = 1$.

When we calculate the interference power we have to recall that it is the variance of the signal at the finger. When we consider the interference from different RAKE fingers to be independent the variance of the signal from each finger is sum of the variances at each finger. The interference power is correspondingly

$$I = \frac{1}{N}\begin{pmatrix} \sum_{\substack{i=0\\ i\neq 0}}^{5}\left(a_i \cdot 1\right)^2 + \sum_{\substack{i=0\\ i\neq 1}}^{5}\left(a_i \cdot 1\right)^2 \\ +\sum_{\substack{i=0\\ i\neq 2}}^{5}\left(a_i \cdot 1\right)^2 + \sum_{\substack{i=0\\ i\neq 4}}^{5}\left(a_i \cdot 1\right)^2 \end{pmatrix} = \frac{5,47}{N},$$

and the SIR ratio is $5.76 \cdot \dfrac{N}{5.46} = 134.82 \Rightarrow 21.29$

c)

Again, there is no ISI. In Maximum Ratio Combining (MRC) we assume $b_i = a_i$. The signal power is

$$S = \left(a_0 \cdot a_0 + a_1 \cdot a_1 + a_2 \cdot a_2 + a_4 \cdot a_4\right)^2 = 2.89,$$

the interference power is

$$I = \frac{1}{N}\begin{pmatrix} \sum_{\substack{i=0\\ i\neq 0}}^{5}\left(a_i \cdot a_0\right)^2 + \sum_{\substack{i=0\\ i\neq 1}}^{5}\left(a_i \cdot a_1\right)^2 \\ +\sum_{\substack{i=0\\ i\neq 2}}^{5}\left(a_i \cdot a_2\right)^2 + \sum_{\substack{i=0\\ i\neq 4}}^{5}\left(a_i \cdot a_4\right)^2 \end{pmatrix} = \frac{1.83}{N}$$

and SNR is $2.89 \cdot \frac{N}{1.83} = 202.6 \Rightarrow 23.07$

d)

In this case there is some ISI since $T = \left(\text{bit rate}\right)^{-1} = 1.95\ \mu s < d = 3.1\ \mu s$.

The receiver either recognises that one multipath component is spread to the following information bit or it does not recognise that. In case the receiver does not recognise that it attempts to synchronise one finger to the multipath component that is in the interval of the current bit and carries information of a previous bit. In our case this means that three branches of the RAKE circuit, with relative powers

$S^2 = (1\cdot 1 + 0.3\cdot 0.3 + 0.6\cdot 0.6)^2 = 2.10$

will synchronise to the right bit and the fourth branch with relative power S_2 to the previous bit This fourth branch is thus producing serious interference. A kind of worst case estimate of the SNR ratio is

$$I = \left(\left(a_4 \cdot a_4\right)^2 + \frac{1}{N} \left(\begin{array}{l} \sum_{\substack{i=0 \\ i\neq 0}}^{5} \left(a_i \cdot a_0\right)^2 + \sum_{\substack{i=0 \\ i\neq 1}}^{5} \left(a_i \cdot a_1\right)^2 \\ + \sum_{\substack{i=0 \\ i\neq 2}}^{5} \left(a_i \cdot a_2\right)^2 + \sum_{\substack{i=0 \\ i\neq 4}}^{5} \left(a_i \cdot a_4\right)^2 \end{array} \right) \right) = 0.29$$

and SNR is $2.10 \cdot \dfrac{1}{0.29} = 7.23 \Rightarrow 8.59$.

In contrast when the receiver recognises that the channel spread exceeds the symbol length it can synchronise to the path that arrives in next symbol interval in what case the signal power will be.

$S^2 = (1\cdot 1 + 0.3\cdot 0.3 + 0.6\cdot 0.6 + 0.4 + 0.4)^2 = 2.89$

and interference is

$$I = \frac{1}{N} \left(\begin{array}{l} \sum_{\substack{i=0 \\ i\neq 0}}^{5} \left(a_i \cdot a_0\right)^2 + \sum_{\substack{i=0 \\ i\neq 1}}^{5} \left(a_i \cdot a_1\right)^2 \\ + \sum_{\substack{i=0 \\ i\neq 2}}^{5} \left(a_i \cdot a_2\right)^2 + \sum_{\substack{i=0 \\ i\neq 4}}^{5} \left(a_i \cdot a_4\right)^2 \end{array} \right) = \frac{1.83}{N}$$

and SNR is $2.89 \cdot \frac{N}{1.83} = 12.63 \Rightarrow 11.07$.

e)

Again, there is no ISI. In Maximum Ratio Combining (MRC) we assume $b_i = a_i$. The signal power is

$$S = \left(a_0 \cdot a_0 + a_1 \cdot a_1 + a_2 \cdot a_2 + a_3 \cdot a_3 + a_4 \cdot a_4 + a_5 \cdot a_5\right)^2 = 3.17,$$

the interference power is

$$I = \frac{1}{N}\begin{pmatrix} \sum_{\substack{i=0 \\ i\neq 0}}^{5}\left(a_i \cdot a_0\right)^2 + \sum_{\substack{i=0 \\ i\neq 1}}^{5}\left(a_i \cdot a_1\right)^2 \\ +\sum_{\substack{i=0 \\ i\neq 2}}^{5}\left(a_i \cdot a_2\right)^2 + \sum_{\substack{i=0 \\ i\neq 3}}^{5}\left(a_i \cdot a_3\right)^2 \\ +\sum_{\substack{i=0 \\ i\neq 4}}^{5}\left(a_i \cdot a_4\right)^2 + \sum_{\substack{i=0 \\ i\neq 5}}^{5}\left(a_i \cdot a_5\right)^2 \end{pmatrix} = \frac{1.96}{N}$$

and SNR is $3.17 \cdot \frac{N}{1.96} = 206.39 \Rightarrow 23.14$.

4. Drow the phase trellis and the information signal phase for the MSK signal (CPFSK with coding index ½) if the bit sequence is [1 0 0 1 1 1 0 0]. How does the carrier modulated signal look like.

4. **Solution**

MSK is a special form of binary CPFSK in which the modulation index is $h = \frac{1}{2}$.

The carrier-modulated CPFSK signal can be expressed as

$s(t) = A\cos\left[2\pi f_c t + \phi(t, I) + \phi_0\right]$ and the equivalent lowpass waveform is:

$x'(t) = A\exp\left\{ j\left[4\pi T f_d \int_{-\infty}^{t} x(\tau)\, d\tau + \phi_0\right]\right\}$.

where $x(t) = \sum_n a_n u(t - nT)$ describes the pulses of the information sequence with signal shape corresponding to $u(t)$.

$\phi(t, I)$ is the time varying phase of the carrier, which defined as

$$\begin{aligned}
\phi(t, I) &= 4\pi T f_d \int_{-\infty}^{t} x(\tau)\, d\tau \\
&= 4\pi T f_d \int_{-\infty}^{t} \sum_n a_n u(\tau - nT) \\
&= 2\pi T f_d \sum_{k=-\infty}^{n-1} a_k + 2\pi f_d (t - nT) a_n \\
&= \pi h \sum_{k=-\infty}^{n-1} a_k + 2\pi h a_n q(t - nT)
\end{aligned}$$

Where

$h = 2f_d T$

$$q(t) = \begin{cases} 0 & t < 0 \\ t/2T & 0 \le t \le T \\ 1/2 & t > T \end{cases}$$

From this we see that at the end of the symbol the phase has constant value and between this levels it changes linearly.

For MSK $h = \frac{1}{2}$ and $\phi(t, I)$ can be simplified.

$$\phi(t, I) = \frac{\pi}{2} \sum_{k=-\infty}^{n-1} a_k + \pi a_n q(t - nT).$$

The corresponding carrier modulated signal is

$$\begin{aligned} s(t) &= A\cos\left[2\pi f_c t + \phi(t, I) + \phi_0\right] \\ &= A\cos\left[2\pi\left(f_c + \frac{1}{4T}a_n\right)t - \frac{n\pi}{2}a_n + \frac{\pi}{2}\sum_{k=-\infty}^{n-1} a_k\right] \end{aligned}.$$

This expression can be expressed as a sinusoid having one of the possible frequencies in the interval $nT \le t \le (n+1)T$

If these frequencies are defined as

$$\begin{aligned} f_1 &= f_c - \frac{1}{4T} \\ f_2 &= f_c + \frac{1}{4T} \end{aligned}$$ then the MSK signal is given by:

$$s(t) = A\cos\left[2\pi f_i t + \frac{n\pi}{2}(-1)^{i-1} + \frac{\pi}{2}\sum_{k=-\infty}^{n-1} a_k\right] \quad i = 1, 2.$$

The frequency separation is $\Delta f = f_1 - f_2 = \dfrac{1}{2T}$.

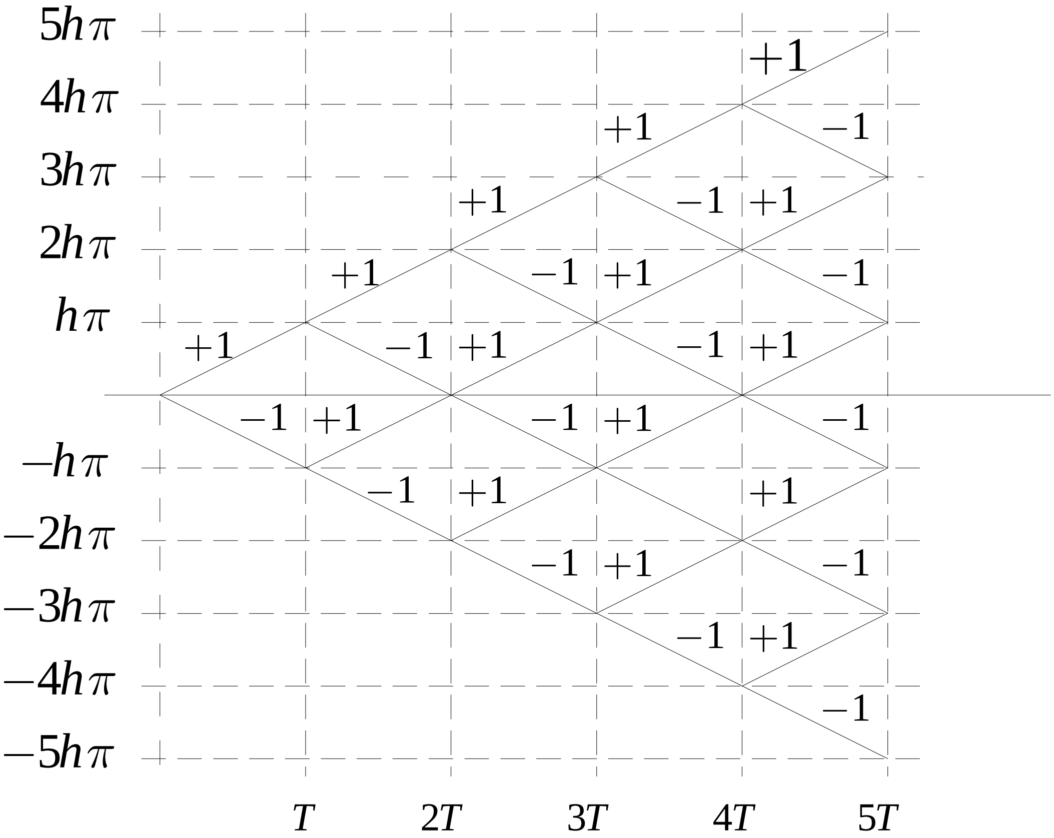
The MSK can also be interpreted as four phase PSK signal in which the pulse shape is one half of cycle of a sinusoid.

$$s(t) = A\left\{\begin{aligned} &\left[\sum_{k=-\infty}^{\infty} a_{2k} u(t - 2nT)\right]\cos 2\pi f_c t \\ &+\left[\sum_{k=-\infty}^{\infty} a_{2k+1} u(t - 2nT - T)\right]\sin 2\pi f_c t \end{aligned}\right\}$$

where $u(t) = \begin{cases} \sin\frac{\pi t}{2T} & 0 \le t \le T \\ 0 & \text{otherwise} \end{cases}.$

The possible trellis of the MSK signal is given below:

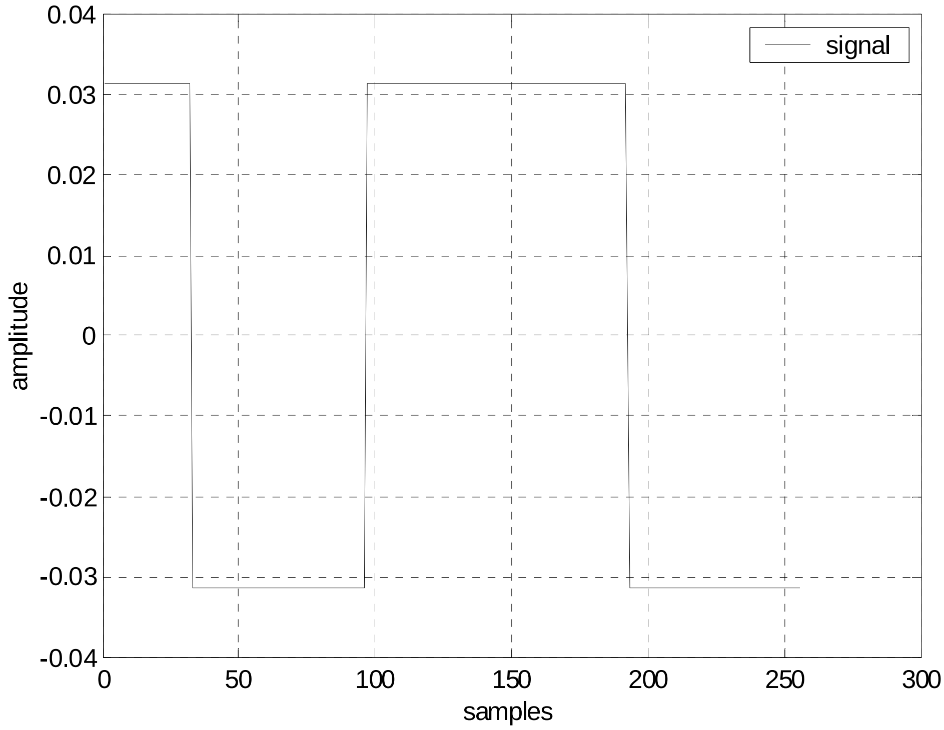
For our signal the sequence (1 0 0 1 1 1 0 0) the sum term at different



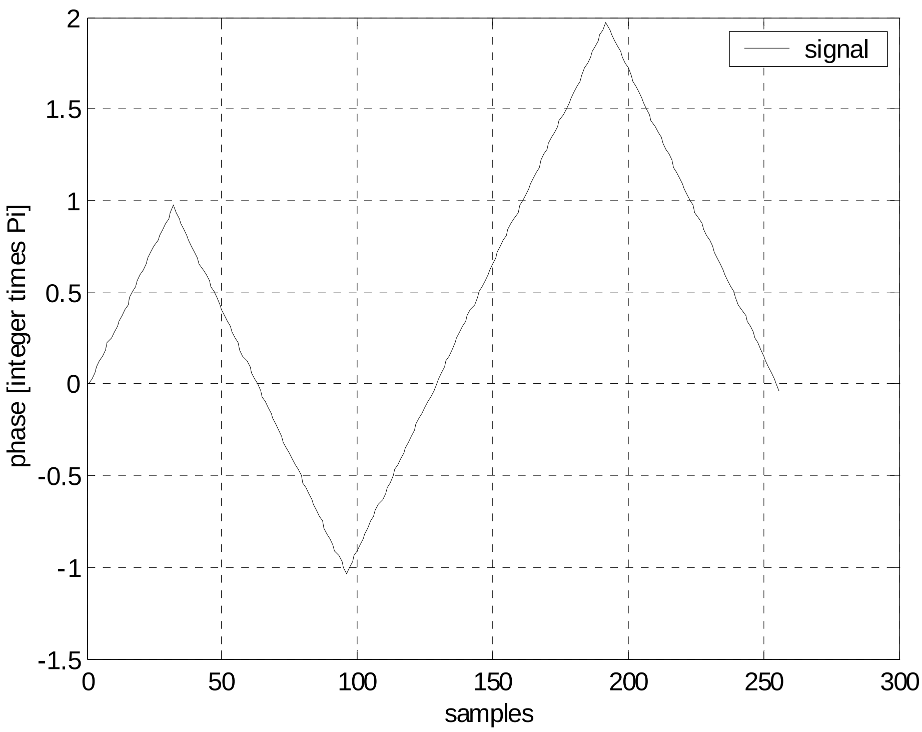
moments gives:

Bits	1	0	0	1	1	1	0	0
Phase	π	0	$-\pi$	0	π	$2\pi = 0$	π	0

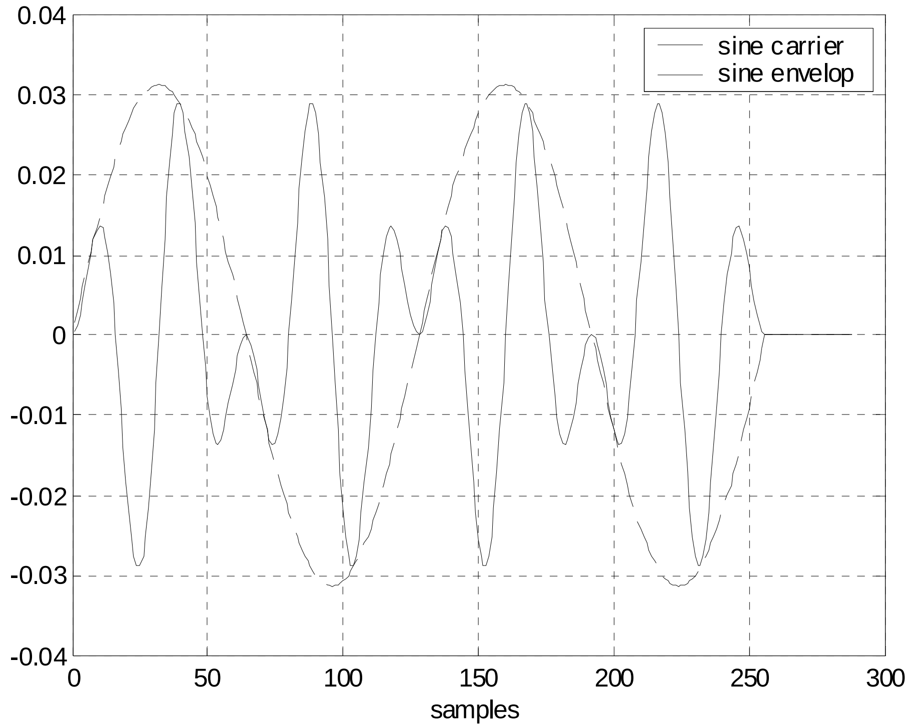
We illustrate the example with the carrier signal frequency $\frac{2\pi}{32}$ with respect to sampling time and oversampled 32 times.

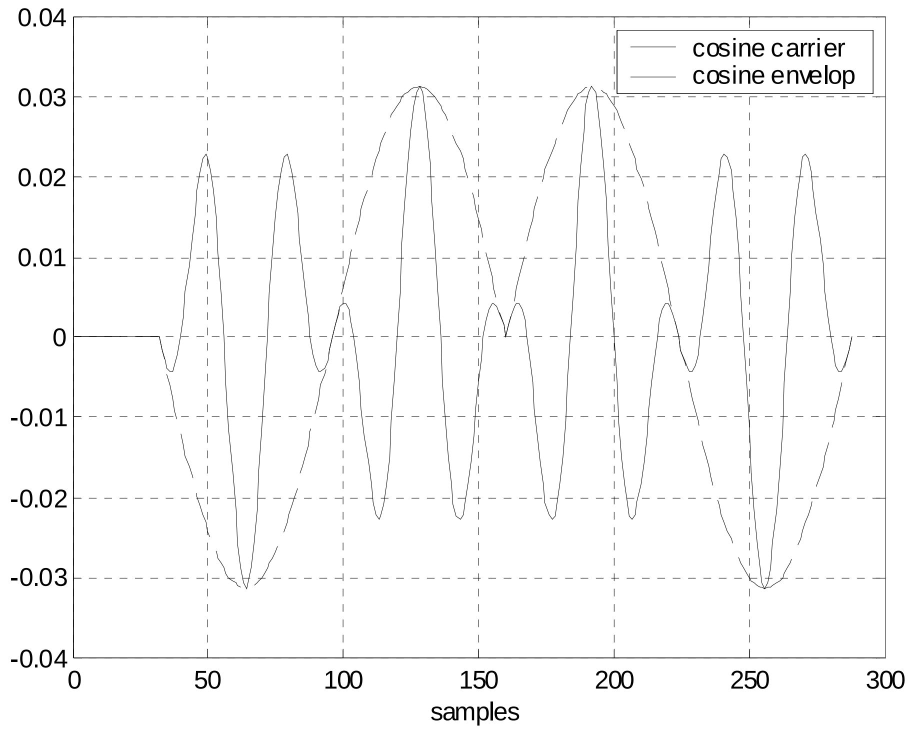


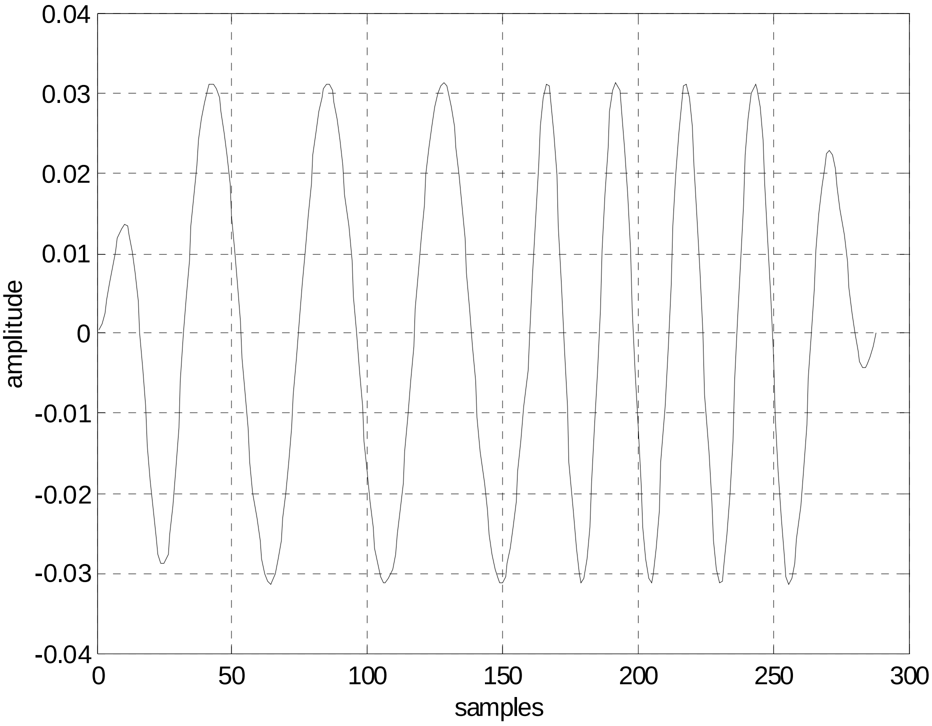
signal
amplitude
samples
0.04
0.03
0.02
0.01
0
-0.01
-0.02
-0.03
-0.04
0
50
100
150
200
250
300



signal
phase [integer times Pi]
samples
2
1.5
1
0.5
0
-0.5
-1
-1.5
0
50
100
150
200
250
300


0.04
0.03
0.02
0.01
0
-0.01
-0.02
-0.03
-0.04
0
50
100
150
200
250
300
samples
sine carrier
sine envelop


0.04
0.03
0.02
0.01
0
-0.01
-0.02
-0.03
-0.04
0
50
100
150
200
250
300
samples
cosine carrier
cosine envelop



Modulated signal

5. How much time can be spend for the delays in core network and for signal processing for the voice in GSM network when the QOS requirement for the delay is 0.15 s?

5. The time that we can spend for signal processing and delays in core network is time left from transmitting the voice over the radio interface. We have to calculate how much time the system spends for transmitting the voice on the radio interface and subtract it from the total QOS requirement.

5. **Solution**

The continuous speech stream is split into 20 ms frames. Those frames are compressed into 260 bits. The channel coding rise the symbol rate to 456 bits.

The resulting 456 bit block is transmitted by using interleaving scheme with depth of eight. Eight frames are used to transmit these bits.

Burst duration (timeslot) is 0.577. One burst is transmitted in every frame with duration 4.615 ms. For transmitting 8 burst we need 8 frames with total duration 4.615*8=36.92 ms. Together with 20 ms frames that makes 56.92.

For the total delay we can assign $0.15 - 0.057 = 0.093\ s$. That will contain the signal processing delay and also delays in the core network.

6. GSM channel rate is 270.33 Kbit/s. The user bit rate is 13 kbit/s. Give the reason why the channel speed is not 8x13 kbit/s = 104 kbit/s.

6. **Solution**

Channel coding rises users transmited symbol rate to 22.8 kbit/s

Duration of the burst is 0.577 ms during which 156.25 bits are transmitted. From those 114 are channel coded information bits.

In every burst is transmitted $\frac{156.25}{114} = 1.37$ bits per one information bit. (input, stop, signalointi, training sequence, flags). The datarate in the all burst where information is transmitted is:

$$\frac{156.25}{114} 22.8 = 31.25 \, kbit \,/\, s \times 8 = 250 \, kbit \,/\, s \,.$$

In the multiframe (26 TDMA frames) two of the frames do not contain users data

$$\frac{26}{24} 250 \, kbit \,/\, s = 270.833 \, kbit \,/\, s$$

In order to transmit traffic with rate $8 \times 13 = 104 \, kbit \,/\, s$ in GSM channels is transmitted 270.833 bits. $\left(\frac{104}{270.833} = 0.384 \right)$.

7.

a. Check the channel coded information rate in the GSM-system by analyzing the length and content of the traffic channel multiframe and burst structure.

b. Check also the net information rate (without channel coding) by analyzing the channel-coding algorithm.

7. **Solution**

Multiframe has duration 120 ms. Every multiframe contains 26 TDMA frames of duration $\frac{120}{26} ms = 4.615\, ms$.

The TDMA frame contains 8 bursts -> burst duration $\frac{120}{26 \times 8} ms = 0.577\, ms$

Traffic channel burst content

Start stop bits	6
Quard period	8.25
Training bits	26
Stealing flag	2
Coded data bits	114
	156.25

Bit duration is $\frac{120}{26 \times 8 \times 156.25} ms = 0.0037\, ms$

Channel bit rate $\frac{26 \times 8 \times 156.25}{120} kbits / s = 270.833\, kbit / s$

Channel coded bit rate before burst formatting

$$\frac{114 \times 26}{120} = 24.7\, kbit / s$$

scaled to speech frame duration

$$\frac{114 \times 24}{120} = 22.8\, kbit / s$$

b) Channel coding

bitrate before channel coding

Class 1 Data	50

Class 2 Data	132
Class 3 Data	78
Bit rate before coding	260
Parity	3
Tail	4

Class 1 and Class 2 as more important data is protected by 1/2 convolutional coding. Convolutional coding gives data1+dat2+parity+tail = 378 bits,

total rate 378 +data3 = 456 bits

Coding rate is $\frac{260}{456} = 0.57$. Data rate before coding is

$\frac{260}{456} 22.8\,kbit\ /\ s = 13.0\,kbit\ /\ s$.