72.630 Capacity enhancement methods for radio interface
To be returned before 5.02.2005

Home assignment 1

1.
The transmitter generates two hypotheses H_1 and H_0 with the equal probability 0.5. When hypothesis H_1 is true the source generates either amplitude 3 or -1. The signal amplitude 3 is selected with probability $p_{1,3} = 0.1$ and the amplitude -1 with probability $p_{1,-1} = 0.9$.
When hypothesis H_0 is true over the channel is transmitted either amplitude -3 or 1. The signal amplitude -3 is selected with probability $p_{0,3} = 0.1$ and to amplitude 1 with probability $p_{0,-1} = 0.9$.
If we do correct decision there is no penalty $C_{i,j} = 0,\ \ i = j$. In case of wrong decision the cost is $C_{i,j} = 1,\ \ i \neq j$.
The noise in the channel is Gaussian.

1) Derive the optimal test.
2) Calculate the error probability by using exact function, $\frac{1}{2} erfc\left(\sqrt{SNR}\right)$, and by using approximation with $\frac{1}{2}\exp\left(-SNR\right)$. Evaluate the bound for the error probability when the noise variance is -3 and $-10\ \ dB$: $Noise\_Var\_dB = [-3 \quad -10]\ dB$.

2.
In case of hypothesis H_1 the source generates the sequence [-1 1 1 -1 0]. To hypothesis H_0 corresponds the sequence [0.7 -0.4 0.4 -0.4 -0.3]. The noise in the channel is Gaussian. If we do correct decision there is no penalty $C_{i,j} = 0,\ \ i = j$. In case of wrong decision the cost is $C_{i,j} = 1,\ \ i \neq j$.
Derive the optimal test for deciding between the hypothesis H_1 and H_0.