S-72.1140 Transmission Methods in Telecommunication Systems (5 cr)

Digital Baseband Transmission

Digital Baseband Transmission

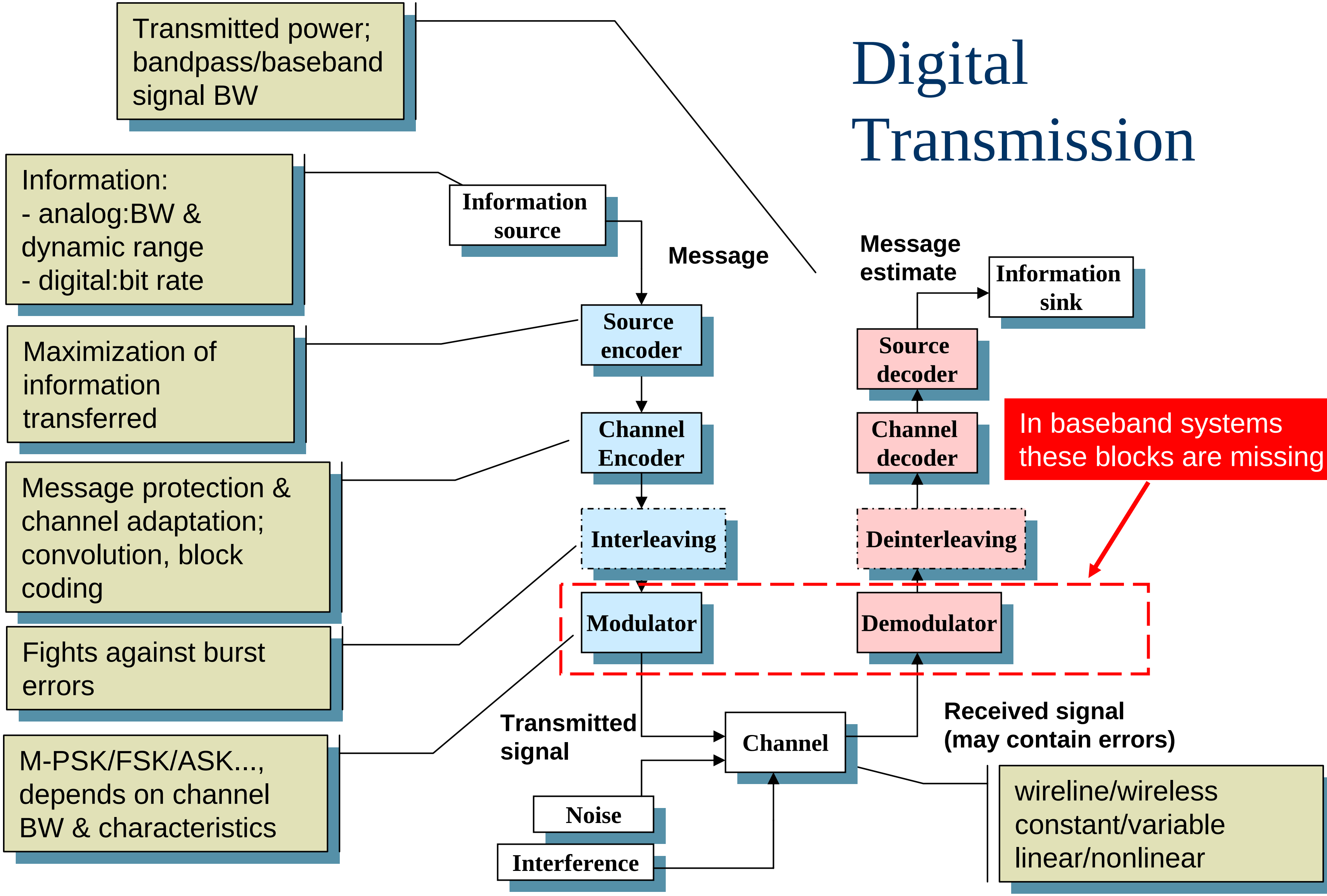
- Why to apply digital transmission?
- Symbols and bits
- Binary PAM Formats
- Baseband transmission
  - Binary error probabilities in baseband transmission
- Pulse shaping
  - minimizing ISI and making bandwidth adaptation - cos roll-off signaling
  - maximizing SNR at the instant of sampling - matched filtering
  - optimal terminal filters
- Determination of transmission bandwidth as a function of pulse shape
  - Spectral density of Pulse Amplitude Modulation (PAM)
- Equalization - removing residual ISI - eye diagram

Why to Apply Digital Transmission?

- Digital communication withstands channel <u>noise, interference and distortion</u> better than analog system. For instance in PSTN inter-exchange STP*-links NEXT (Near-End Cross-Talk) produces several interference. For analog systems interference must be below 50 dB whereas in digital system 20 dB is enough. With this respect digital systems can utilize lower quality cabling than analog systems
- <u>Regenerative repeaters</u> are efficient. Note that cleaning of analog-signals by repeaters does not work as well
- Digital <u>HW/SW implementation</u> is straightforward
- Circuits can be easily <u>configured and programmed</u> by DSP techniques
- Digital signals can be <u>coded</u> to yield very low error rates
- Digital communication enables efficient <u>exchange of SNR to BW-> easy adaptation into different channels</u>
- The <u>cost</u> of digital HW continues to halve every two or three years

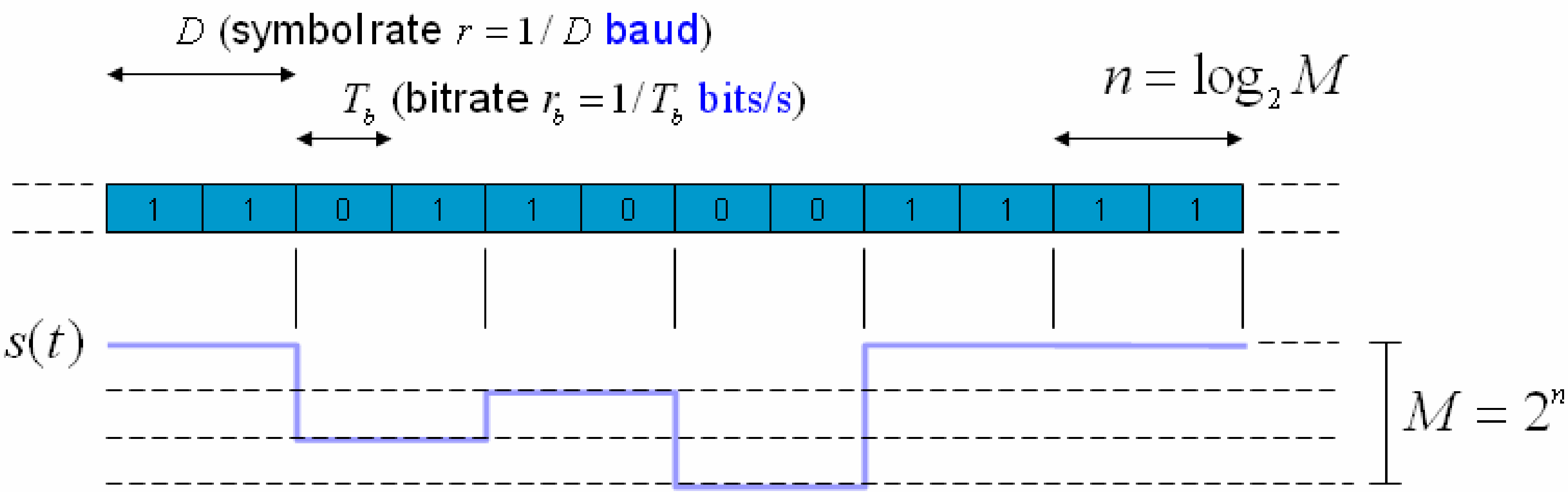
STP: Shielded twisted pair

Digital Transmission



- 'Baseband' means that no carrier wave modulation is used for transmission

Symbols and Bits – M-ary PAM



Generally: $s(t) = \sum_k a_k p(t - kD)$ (a PAM* signal)

For M=2 (binary signalling): $s(t) = \sum_k a_k p(t - kT_b)$

For non-Inter-Symbolic Interference (ISI), *p(t)* must satisfy:

$$p(t) = \begin{cases} 1, t = 0 \\ 0, t = \pm D, \pm 2D \ldots \end{cases}$$

unipolar, 2-level pulses

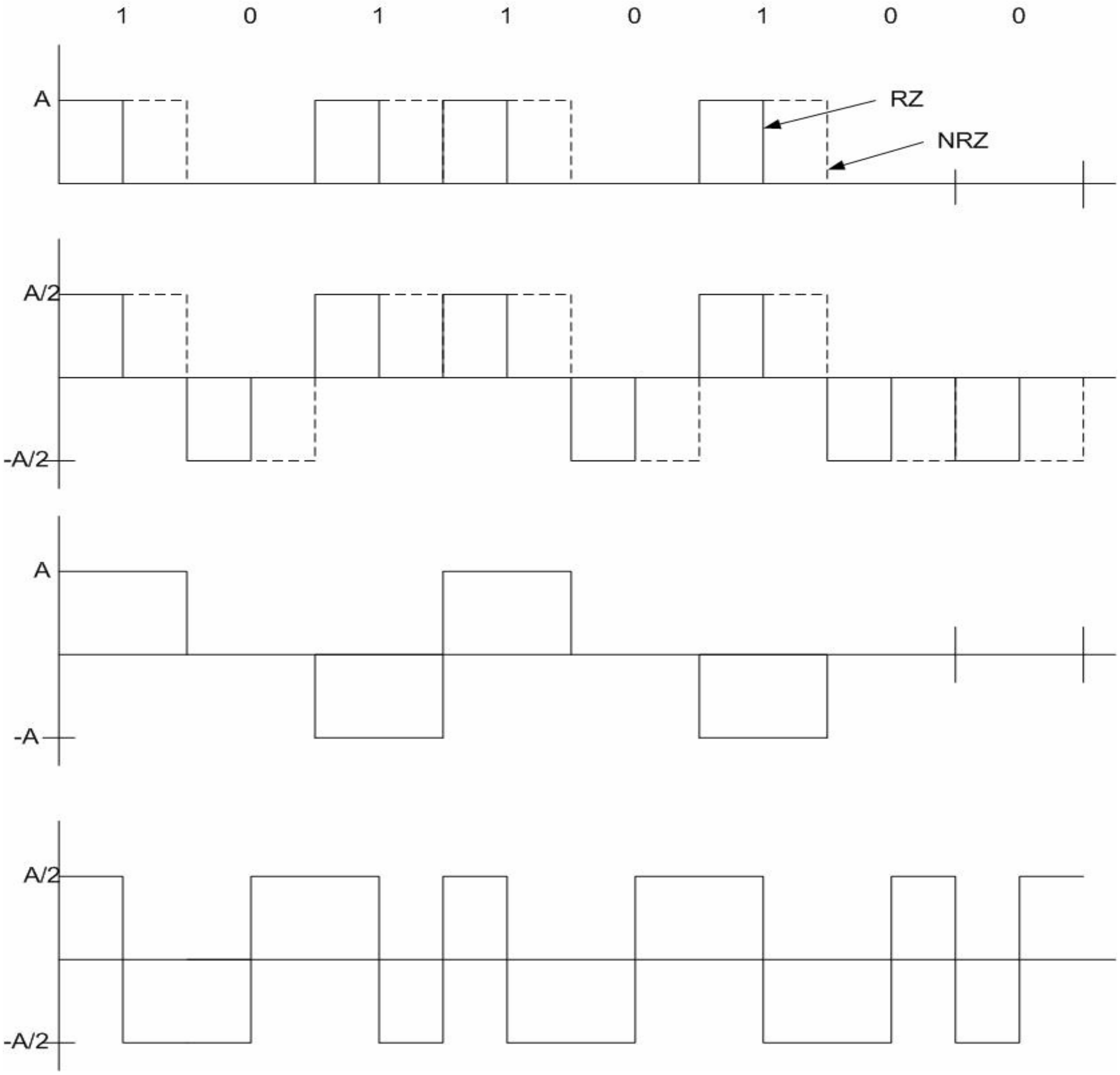
$$\begin{cases} n: & \text{number of bits} \\ M: & \text{number of levels} \\ D: & \text{Symbol duration} \\ T_b: & \text{Bit duaration} \end{cases}$$

This means that at the instant of decision, received signal component is

$$s(t) = \sum_k a_k p(t - kD) = a_k$$

*Pulse Amplitude Modulation

Binary PAM Formats (1)



Bit stream

Unipolar RZ and NRZ

Polar RZ and NRZ

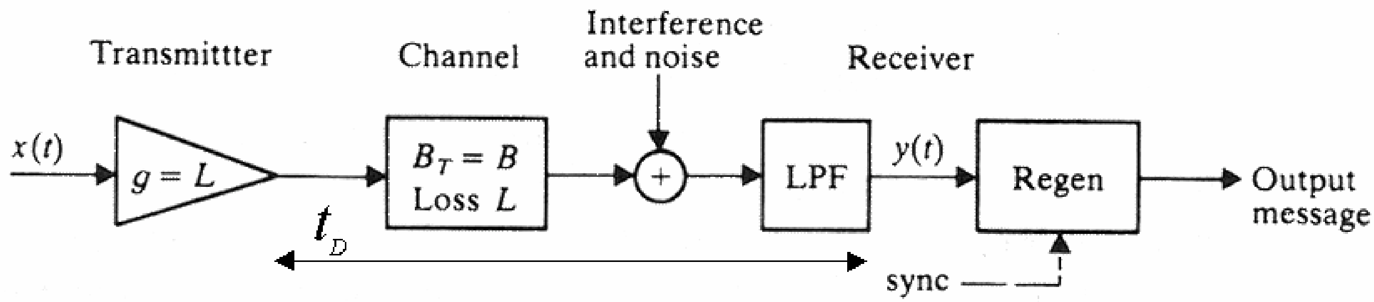
Bipolar NRZ or alternate mark inversion (AMI)

Split-phase Manchester

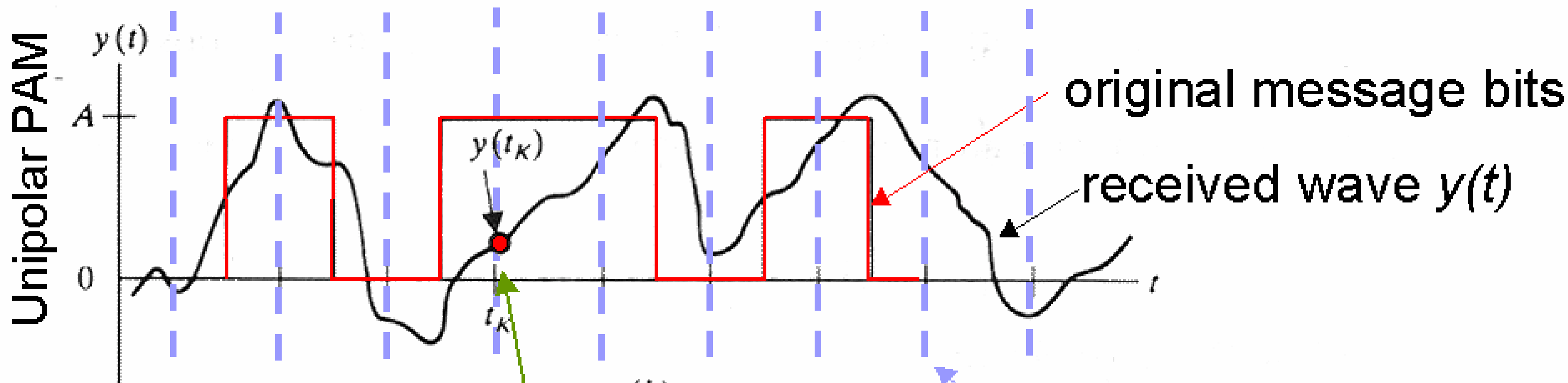
Binary PAM Formats (2)

- Unipolar RZ, NRZ:
  - DC component
    - No information, wastes power
    - Transformers and capacitors in route block DC
  - NRZ, more energy per bit, synchronization more difficult
- Polar RZ, NRZ:
  - No DC term if ´0´and ´1´ are equally likely
- Bipolar NRZ
  - No DC term
- Split-phase Manchester
  - Zero DC term regardless of message sequence
  - Synchronization simpler
  - Requires larger bandwidth

Baseband Digital Transmission Link



(a)



(b)

decision instances

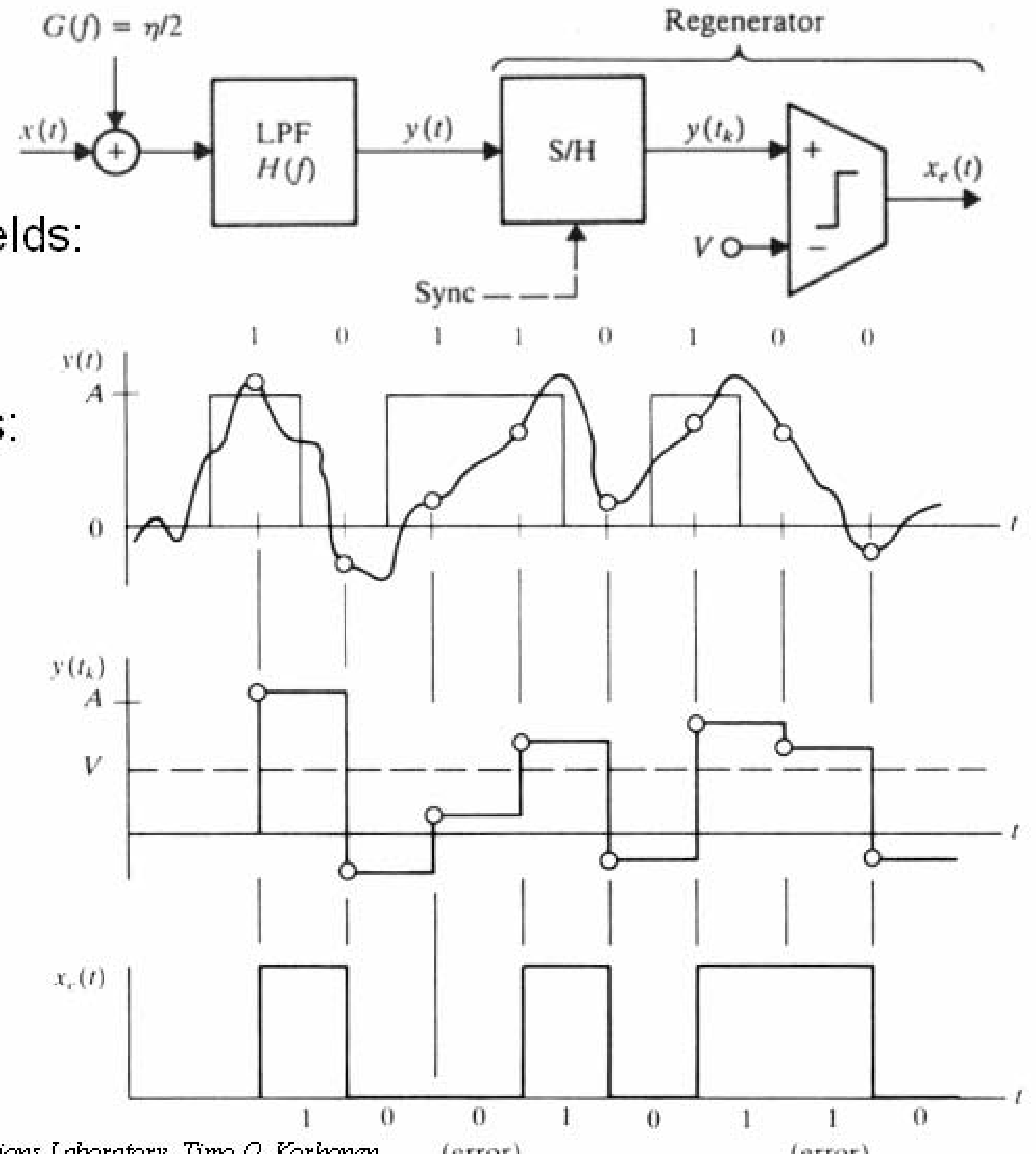
$$y(t)=\sum_k a_k p(t-t_d-kD)+n(t)$$

message reconstruction at $t_K = KD + t_d$ yields

$$y(t_K) = a_K + \sum_{k\neq K} a_k \tilde{p}(KD-kD) + n(t)$$

message ISI Gaussian bandpass noise

Baseband Unipolar Binary Error Probability



Assume binary & unipolar $x(t)$

The sample-and-hold circuit yields:

$$\text{r.v. } Y : y(t_k) = a_k + n(t_k)$$

Establish H_0 and H_1 hypothesis:

$$H_0 : a_k = 0, Y = n$$

$$p_Y(y \mid H_0) = p_N(y)$$

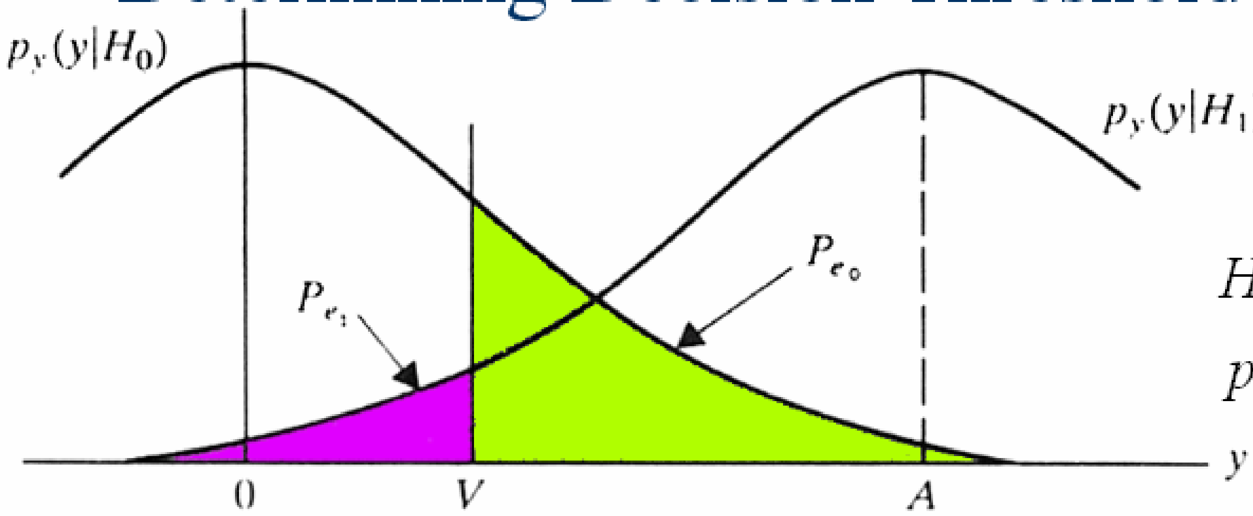
and

$$H_1 : a_k = 1, Y = A + n$$

$$p_Y(y \mid H_1) = p_N(y - A)$$

$p_N(y)$: Noise probability density function (PDF)

Determining Decision Threshold



$H_0 : a_k = 0, Y = n$

$p_Y(y\,|\,H_0) = p_N(y)$

$H_1 : a_k = 1, Y = A + n$

$p_Y(y\,|\,H_1) = p_N(y - A)$

The comparator implements decision rule:

Choose Ho *(a_k=0)* if *Y<V*
Choose H1 *(a_k=1)* if *Y>V*

$$p_{e1} \equiv P(Y < V\,|\,H_1) = \int_{-\infty}^{V} p_Y(y\,|\,H_1)dy$$

$$p_{e0} \equiv P(Y > V\,|\,H_0) = \int_{V}^{\infty} p_Y(y\,|\,H_0)dy$$

Transmitted ‘0’ but detected as ‘1’

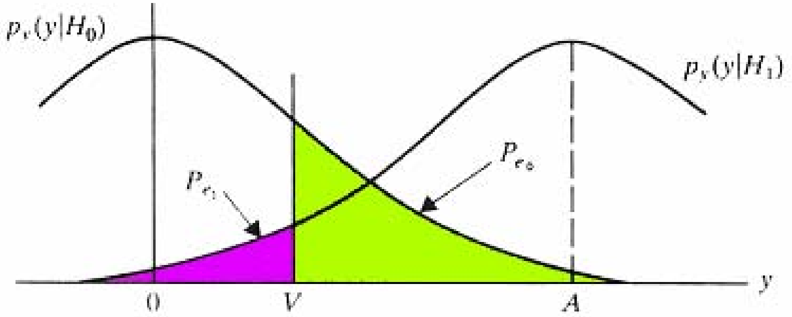
Average error error probability: $P_e = P_0 P_{e0} + P_1 P_{e1}$

$$P_0 = P_1 = 1/2 \Rightarrow P_e = ½(P_{e0} + P_{e1})$$

Channel noise is Gaussian with the pfd:

$$p_N(x) = \frac{1}{\sigma\sqrt{2\pi}} \exp\left(-\frac{x^2}{2\sigma^2}\right)$$

Error rate and Q-function



$$p_{e0} = \int_V^{\infty} p_N(y)dy$$

$$p_{e0} = \frac{1}{\sigma\sqrt{2\pi}} \int_V^{\infty} \exp\left(-\frac{x^2}{2\sigma^2}\right)dx$$

This can be expressed by using the Q-function

$$Q(k) \triangleq \frac{1}{\sqrt{2\pi}} \int_k^{\infty} \exp\left(-\frac{\lambda^2}{2}\right)d\lambda$$

by

$$p_{e0} = \int_V^{\infty} p_N(y)dy = Q\left(\frac{V}{\sigma}\right)$$

and also

$$P_{e1} = \int_{-\infty}^{V} p_N(y-A)dy = Q\left(\frac{A-V}{\sigma}\right)$$

Baseband Binary Error Rate in Terms of Pulse Shape and γ

setting $V=A/2$ yields then

$$p_e = ½(p_{e0}+p_{e1}) = p_{e0} = p_{e1} \Rightarrow \boxed{p_e = Q\left(\frac{A}{2\sigma}\right)}$$

for unipolar, **rectangular** NRZ [0,A] bits

$$S_R = \frac{1}{2}(A)^2 + \frac{1}{2}(0)^2 = A^2/2$$

Probability of occurrence

for polar, **rectangular** NRZ [-A/2,A/2] bits

$$S_R = \frac{1}{2}(A/2)^2 + \frac{1}{2}(-A/2)^2 = A^2/4$$

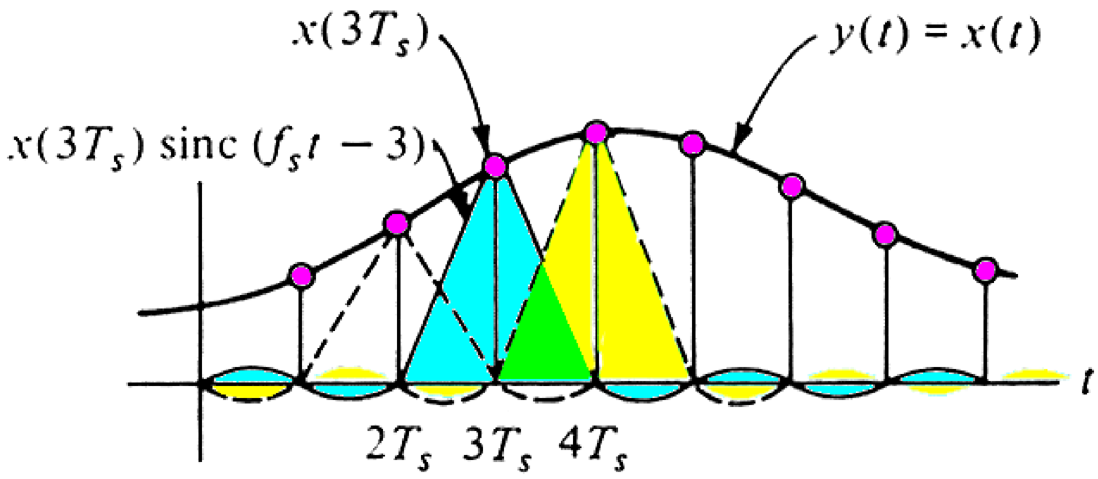
and hence

$$\left(\frac{A}{2\sigma}\right)^2 = \frac{A^2}{4N_R} = \begin{cases} S_R/(2N_R), \text{unipolar} \\ S_R/N_R, \text{polar} \end{cases}$$

$$\begin{cases} \gamma_b = E_b/N_0 = S_R/N_0 r_b \\ N_R = N_0 r_b/2 \end{cases} \Rightarrow \left(\frac{A}{2\sigma}\right)^2 = \begin{cases} 2\gamma_b N_0 r_b/(2N_0 r_b) = \gamma_b, \text{unipolar} \\ 2\gamma_b N_0 r_b/N_0 r_b = 2\gamma_b, \text{polar} \end{cases}$$

Note that $N_R = N_0 B_N \geq N_0 r_b/2$ (lower limit with sinc-pulses)

Pulse Shaping and Band-limited Transmission



- In digital transmission signaling pulse shape is chosen to satisfy the following requirements:
  - yields **maximum SNR at the time instance of decision** (matched filtering)
  - accommodates signal **to channel bandwidth:**
    - rapid **decrease of pulse energy** outside the main lobe in frequency domain alleviates filter design
    - **lowers cross-talk** in multiplexed systems

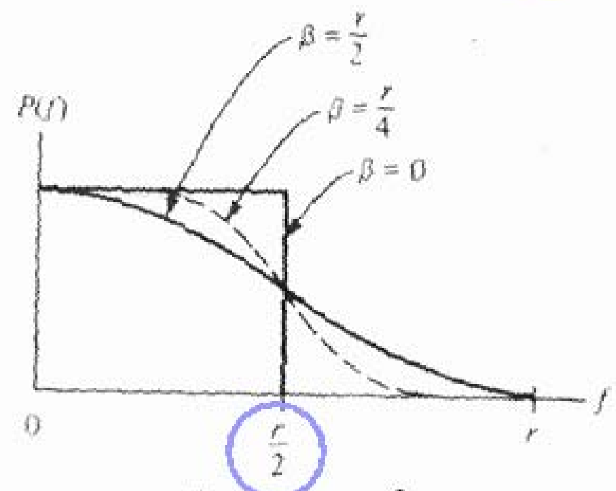
Signaling With Cosine Roll-off Signals

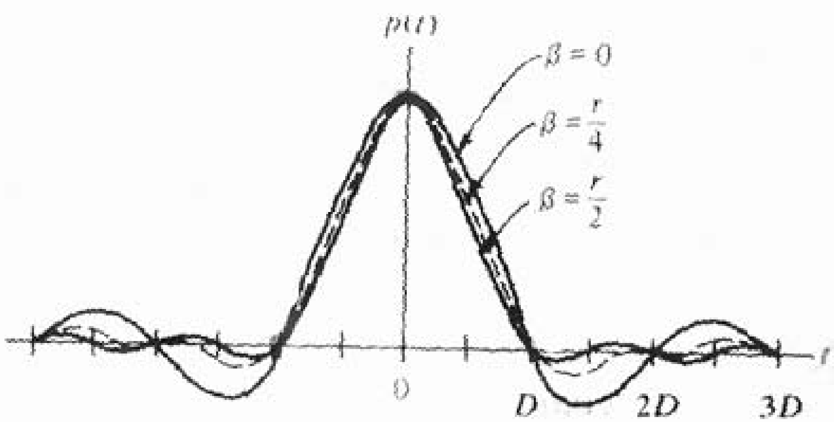
- **Maximum transmission rate** can be obtained with sinc-pulses

$$\begin{cases} p(t) = \operatorname{sinc}(rt) = \operatorname{sinc}(t/D) \\ P(f) = F[p(t)] = \dfrac{1}{r}\Pi\left(\dfrac{f}{r}\right) \end{cases}$$

- However, they are not time-limited. A more practical choice is the **cosine roll-off signaling**:

for **raised cos-pulses** $\beta = r/2$



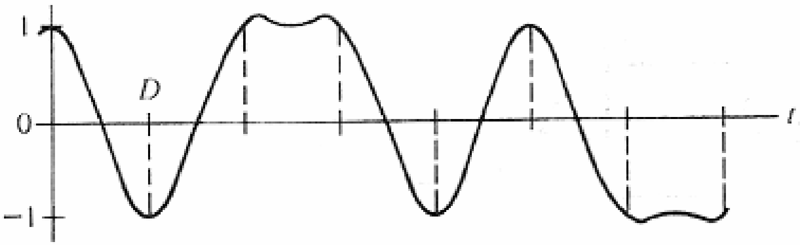


$$P(f)\Big|_{\beta=r/2} = \frac{1}{r}\cos^2\frac{\pi f}{2r}\Pi(f/2r)$$

$$p(t) = \frac{\cos 2\pi\beta t}{1-(4\beta t)^2}\operatorname{sinc} rt$$

Example

- By using $\beta = r/2$ and polar signaling, the following waveform is obtained:



- Note that the zero crossing are spaced by *D* at

$$t = \pm 0.5D, \pm 1.5D, \pm 2.5D, \ldots$$

(this could be seen easily also in eye-diagram)

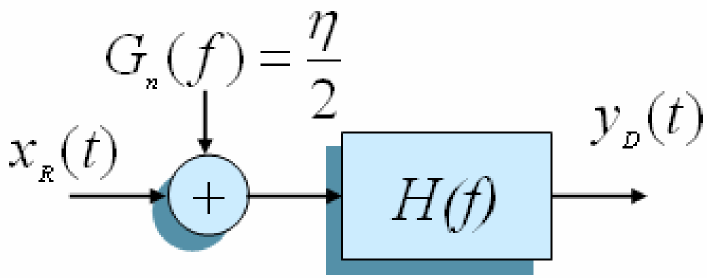
- The zero crossing are easy to detect for clock recovery. Note that unipolar baseband signaling involves performance penalty of 3 dB compared to polar signaling:

$$p_e = \begin{cases} Q(\sqrt{\gamma_b}), \text{ unipolar } [0/1] \\ Q(\sqrt{2\gamma_b}), \text{ polar}[\pm 1] \end{cases}$$

$$\gamma_b = E_b / N_0 = S_R / N_0 r_b$$
$$N_R = N_0 r_b / 2$$

Matched Filtering

$$\begin{cases} x_R(t) = A_R p(t - t_0) \\ X_R(f) = A_R P(f)\exp(-j\omega t_0) \end{cases}$$



$$A = F^{-1}[H(f)X_R(f)]\Big|_{t=t_0+t_d}$$ Peak amplitude to be maximized

$$= A_R \int_{-\infty}^{\infty} H(f)P(f)\exp(j\omega t_d)df$$

$$\sigma^2 = \int_{-\infty}^{\infty} |H(f)|^2 G_n(f)df = \frac{\eta}{2}\int_{-\infty}^{\infty} |H(f)|^2 df$$ Post filter noise

$$\left(\frac{A}{\sigma}\right)^2 = A_R^2 \frac{\left|\int_{-\infty}^{\infty} H(f)P(f)\exp(j\omega t_d)df\right|^2}{\frac{\eta}{2}\int_{-\infty}^{\infty} |H(f)|^2 df}$$ Should be maximized

Using Schwartz's inequality $\longrightarrow$ $\left|\int_{-\infty}^{\infty} V(f)W^*(f)df\right|^2 \leq \int_{-\infty}^{\infty} |W(f)|^2 df \int_{-\infty}^{\infty} |V(f)|^2 df$

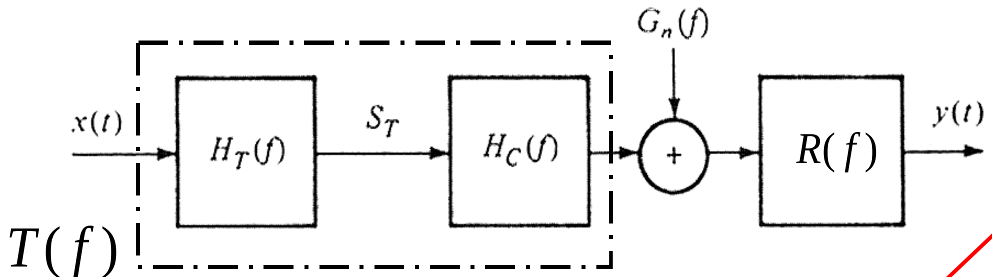
$$\Rightarrow H(f) = KP(f)\exp(j\omega t_d) \Rightarrow h(t) = Kp(t_d - t)$$

Optimum terminal filters

- Assume
  - arbitrary TX pulse shape $x(t)$
  - arbitrary channel response $h_c(t)$
  - multilevel PAM transmission

$$\begin{cases} P_x : \text{transmitting waveform} \\ H_T : \text{transmitter shaping filter} \\ H_C : \text{channel transfer function} \\ R : \text{receiver filter} \end{cases}$$

- What kind of filters are required for TX and RX to obtain matched filtered, non-ISI transmission?



Nyqvist shaped pulse in $y(t)$

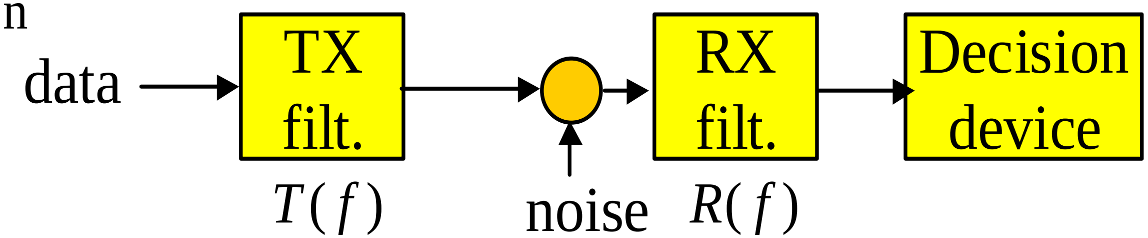
- The following condition must be fulfilled:

$$P_x(f)H_T(f)H_C(f)R(f) = P(f)\exp(-j\omega t_d)$$

that means that undistorted transmission is obtained

Avoiding ISI and enabling band-limiting in radio systems

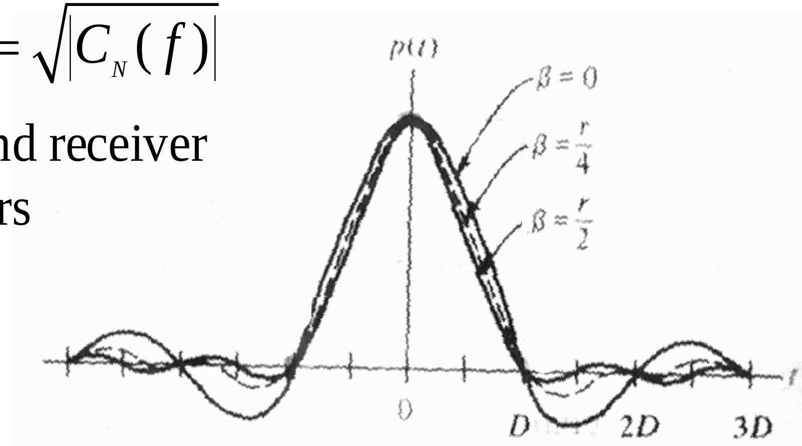
- Two goals to achieve: band limited transmission & matched filter reception



$$\begin{cases} T(f)R(f) \equiv C_N(f), & \text{raised-cos shaping} \\ T(f) = R^*(f), & \text{matched filtering} \end{cases}$$

$$\Rightarrow |R(f)| = |T(f)| = \sqrt{|C_N(f)|}$$

- Hence at the transmitter and receiver alike **root-raised** cos-filters must be applied



raised cos-spectra $C_N(f)$

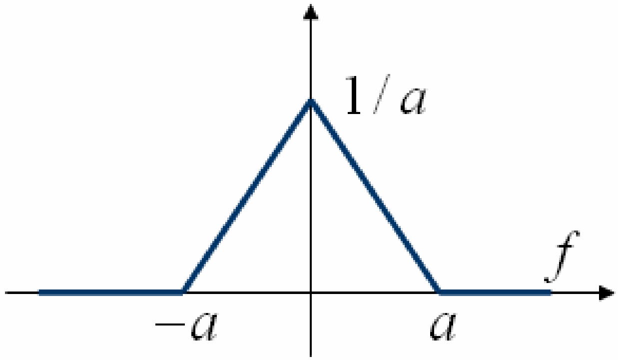
Determining Transmission Bandwidth for an Arbitrary Baseband Signaling Waveform

- Determine the relation between *r* and *B* when *p(t)=sinc*^{2} *at*
- First note from time domain that

$$\mathrm{sinc}^2\, at = \begin{cases} 1, t = 0 \\ 0, t = \pm 1/a, \pm 2/a \ldots \end{cases} \Rightarrow r = a$$

  hence this waveform is suitable for signaling
- There exists a Fourier transform pair

$$\mathrm{sinc}^2\, at \leftrightarrow \frac{1}{a}\Lambda\left(\frac{f}{a}\right)$$



- From the spectra we note that $B_T \geq a$ and hence it must be that for **baseband**

$$\Rightarrow B_T \geq r$$

PAM Power Spectral Density (PSD)

- PSD for PAM can be determined by using a general expression

Amplitude autocorrelation

$$G_x(f)=\frac{1}{D}\left|P(f)\right|^2\sum_{n=-\infty}^{\infty}R_a(n)\exp(-j2\pi nfD)$$

- For uncorrelated message bits

$$R_a(n)=\begin{cases}\sigma_a^2+m_a^2, n=0 & \text{Total power}\\ m_a^2, n\neq 0 & \text{DC power}\end{cases}$$

and therefore

$$\sum_{n=-\infty}^{\infty}R_a(n)\exp(-j2\pi nfD)=\sigma_n^2+m_a^2\sum_{n=-\infty}^{\infty}\exp(-j2\pi nfD)$$

on the other hand $\boxed{\sum_{n=-\infty}^{\infty}\exp(-j2\pi nfD)=\frac{1}{D}\sum_{n=-\infty}^{\infty}\delta\left(f-\frac{n}{D}\right)}$ and $r=1/D$

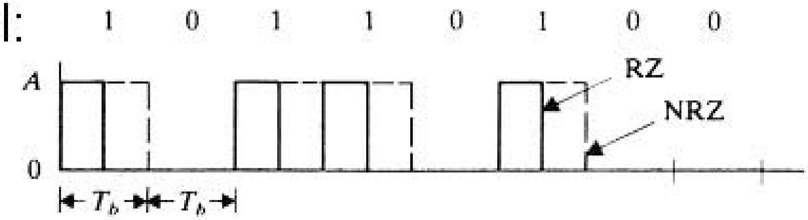
$$G_x(f)=\sigma_a^2 r\left|P(f)\right|^2+m_a^2r^2\sum_{n=-\infty}^{\infty}\left|P(nr)\right|^2\delta(f-nr)$$

Example

- For unipolar binary RZ signal:

$$P(f)=\frac{1}{2r_b}\operatorname{sinc}\frac{f}{2r_b}$$

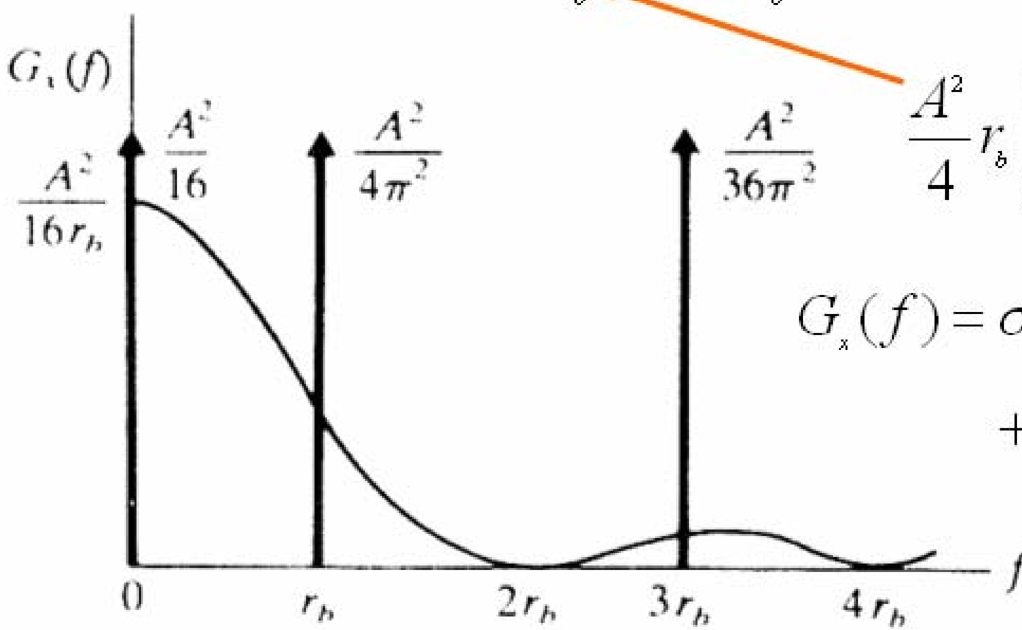
(a)



- Assume source bits are equally likely and independent, thus

$$m_a=\overline{a_k}=A/2,\ \overline{a_k^2}=A^2/2,\ \sigma_a^2=A^2/4,$$

$$\Rightarrow G_x(f)=\frac{A^2}{16r_b}\operatorname{sinc}^2\frac{f}{2r_b}+\frac{A^2}{16}\sum_{n=-\infty}^{\infty}\delta(f-nr_b)\operatorname{sinc}^2\frac{n}{2}$$

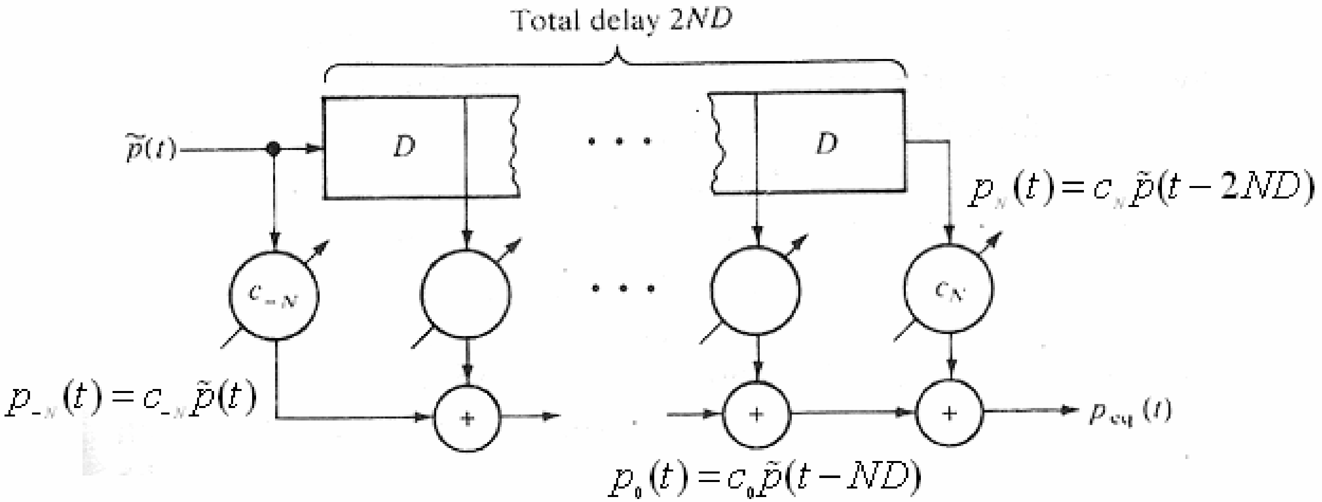


$$\frac{A^2}{4}r_b\left(\frac{1}{2r_b}\right)^2$$

$$G_x(f)=\sigma_a^2 r\left|P(f)\right|^2+m_a^2r^2\sum_{n=-\infty}^{\infty}\left|P(nr)\right|^2\delta(f-nr)$$

Equalization: Removing Residual ISI

- Consider a tapped delay line equalizer with



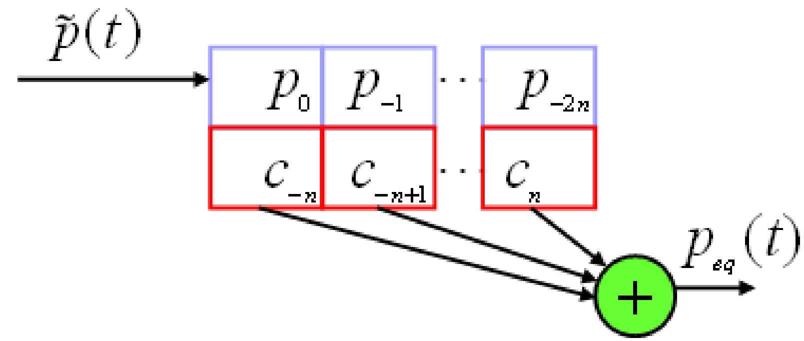
- Search for the tap gains c_N such that the output equals zero at sample intervals D except at the decision instant when it should be unity. The output is (think for instance paths c_{-N}, c_N or c_0)

$$p_{eq}(t) = \sum_{n=-N}^{N} c_n \tilde{p}(t - nD - ND)$$

that is sampled at $t_k = kD + ND$ yielding

$$p_{eq}(t_k) = p_{eq}(kD + ND) = \sum_{n=-N}^{N} c_n \tilde{p}(kD - nD) = \sum_{n=-N}^{N} c_n \tilde{p}\left[D(k-n)\right]$$

Tapped Delay Line: Matrix Representation



- At the instant of decision:

$$p_{eq}(t_k) = \sum_{n=-N}^{N} c_n \tilde{p}\left[D(k-n)\right] = \sum_{n=-N}^{N} c_n \tilde{p}_{k-n} = \begin{cases} 1, k=0 \\ 0, k=\pm 1, \pm 2, \ldots, \pm N \end{cases}$$

- That leads into *(2N+1)x(2N+1)* matrix where *(2N+1)* tap coefficients can be solved:

taps $\longrightarrow$

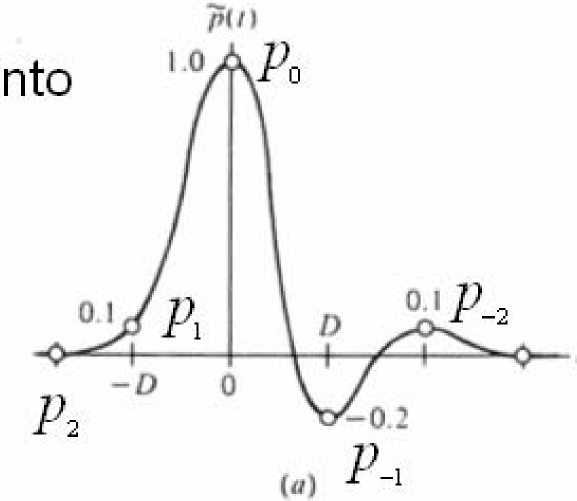
time $\downarrow$

$$\tilde{p}_0 c_{-n} + \tilde{p}_{-1} c_{-n+1} + \ldots + \tilde{p}_{-2n} c_n = 0$$
$$\tilde{p}_1 c_{-n} + \tilde{p}_0 c_{-n+1} + \ldots + \tilde{p}_{-2n+1} c_n = 0$$
$$\ldots$$
$$\tilde{p}_n c_{-n} + \tilde{p}_{n-1} c_{-n+1} + \ldots + \tilde{p}_{-n} c_n = 1$$
$$\ldots$$
$$\tilde{p}_{2n} c_{-n} + \tilde{p}_{2n-1} c_{-n+1} + \ldots + \tilde{p}_0 c_n = 0$$

$$\begin{bmatrix} \tilde{p}_0 & \cdots & \tilde{p}_{-2N} \\ \vdots & \cdots & \vdots \\ \tilde{p}_{N-1} & \cdots & \tilde{p}_{-N-1} \\ \tilde{p}_N & \cdots & \tilde{p}_{-N} \\ \tilde{p}_{N+1} & \cdots & \tilde{p}_{-N+1} \\ \vdots & & \vdots \\ \tilde{p}_{2N} & \cdots & \tilde{p}_0 \end{bmatrix} \begin{bmatrix} c_{-N} \\ \vdots \\ c_{-1} \\ c_0 \\ c_1 \\ \vdots \\ c_N \end{bmatrix} = \begin{bmatrix} 0 \\ \vdots \\ 0 \\ 1 \\ 0 \\ \vdots \\ 0 \end{bmatrix}$$

Example of Equalization

- Read the distorted pulse values into matrix from fig. (a)

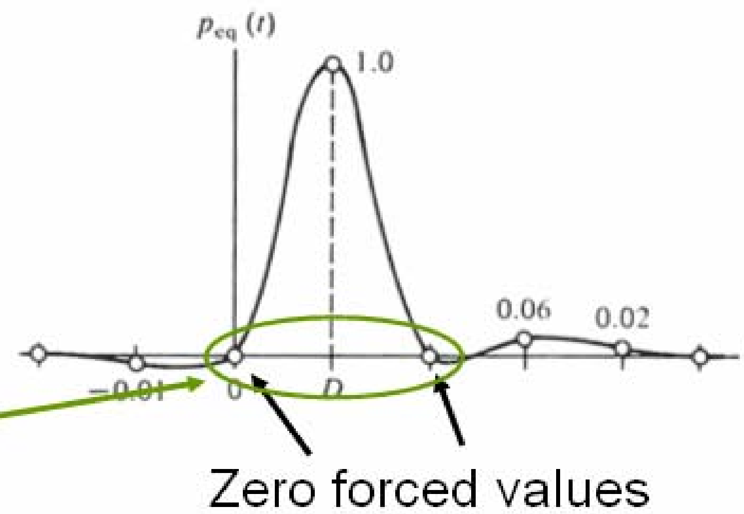


$$\begin{bmatrix} 1.0 & 0.1 & 0.0 \\ -0.2 & 1.0 & 0.1 \\ 0.1 & -0.2 & 1.0 \end{bmatrix} \begin{bmatrix} c_{-1} \\ c_0 \\ c_1 \end{bmatrix} = \begin{bmatrix} 0 \\ 1 \\ 0 \end{bmatrix}$$

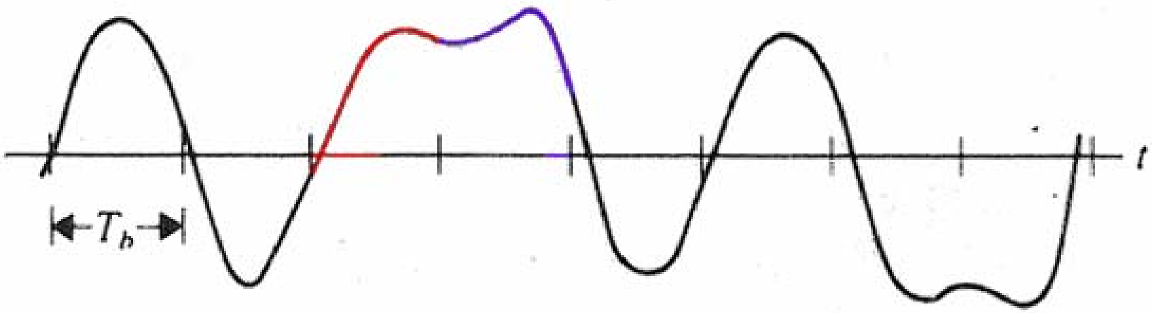
and the solution is

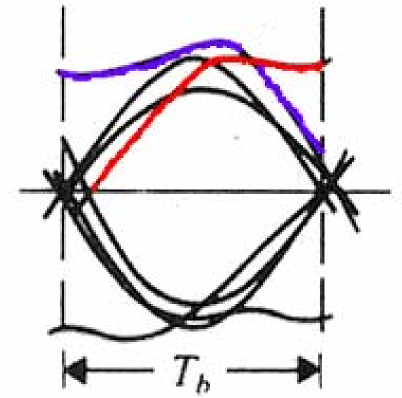
$$\begin{bmatrix} c_{-1} \\ c_0 \\ c_1 \end{bmatrix} = \begin{bmatrix} -0.096 \\ 0.96 \\ 0.2 \end{bmatrix}$$

Question: what does these zeros help because they don't exist at the sampling instant?



Monitoring Transmission Quality by Eye Diagram





Required minimum bandwidth is

$$B_T \geq r/2$$

Nyqvist's sampling theorem:

> Given an ideal LPF with the bandwidth B it is possible to transmit independent symbols at the rate:
>
> $$B_T \geq r/2 = 1/(2T_b)$$