S-72.1140 Transmission Methods in Telecommunication Systems (5 cr)

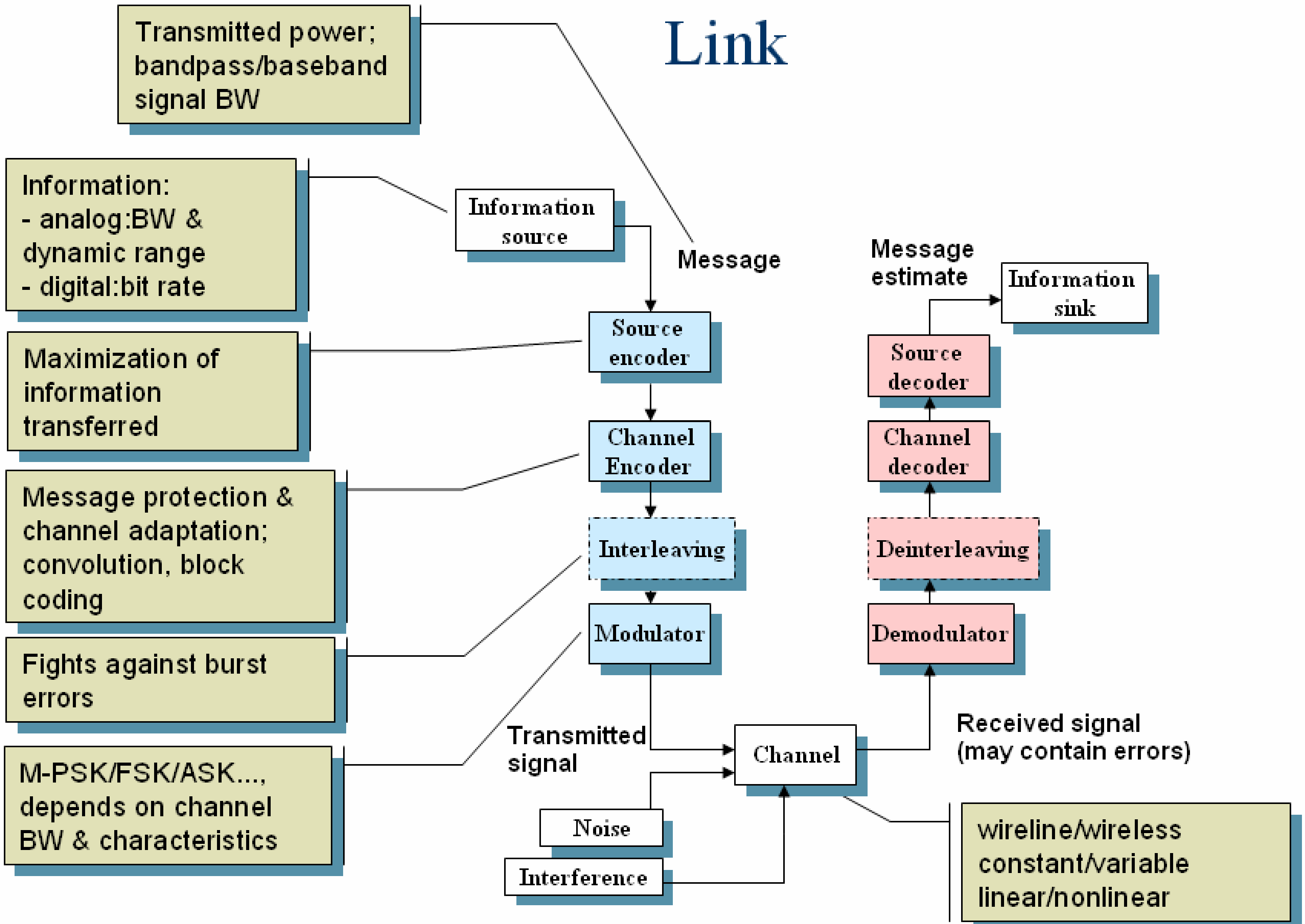
Digital Bandpass Transmission

Digital Bandpass Transmission

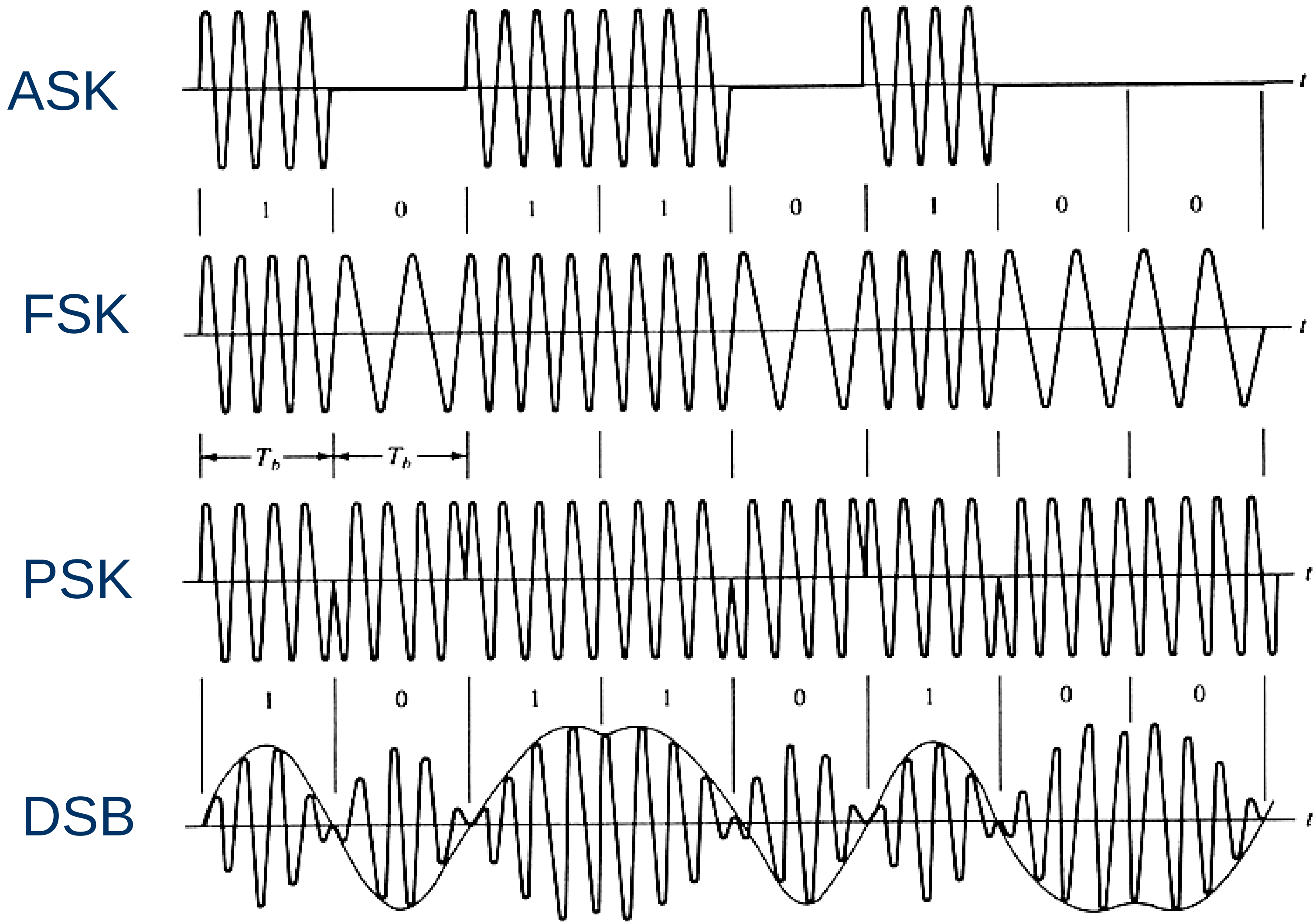
- CW detection techniques
  - Coherent
  - Non-coherent
  - Differentially coherent
- Examples of coherent and non-coherent detection error rate analysis (OOK)
- A method for ‘analyzing’ PSK error rates
- Effect of synchronization and envelope distortion (PSK)
- Comparison: Error rate describing
  - reception sensitivity $P_e = f_1(E_b / N_0)$
  - bandwidth efficiency $P_e = f_2(r_b / B_T)$

Overview

Bandpass Transmission Link



CW Binary Waveforms

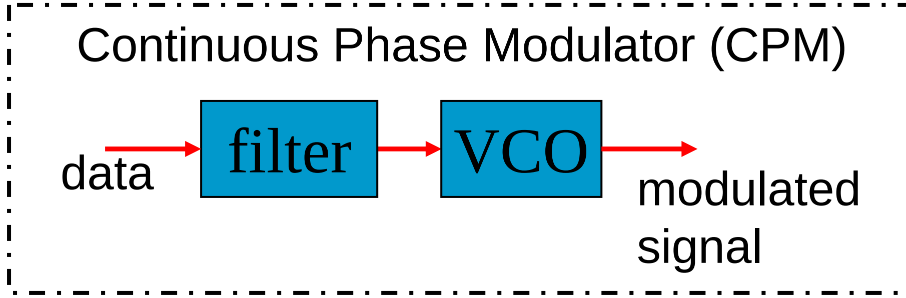


Carrier Wave Communications

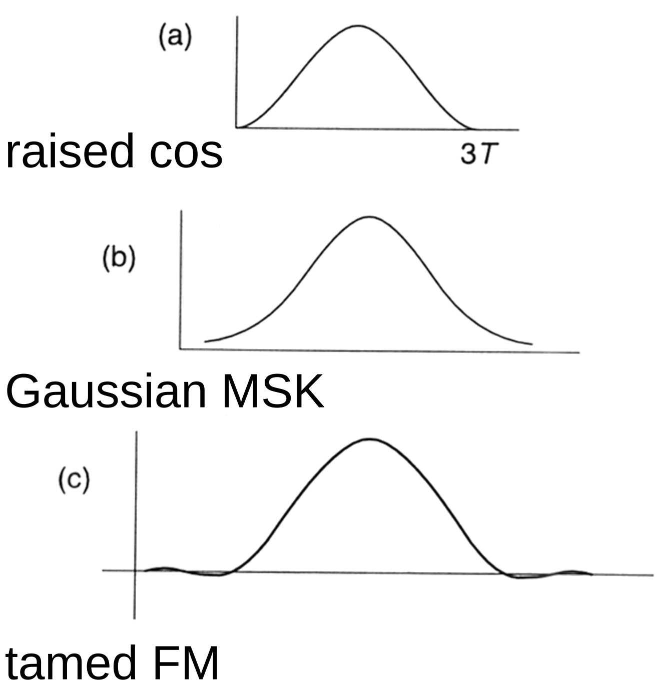
- Carrier wave modulation is used to transmit messages over a distance by radio waves (**air**, copper or coaxial cable), by optical signals (fiber), or by sound waves (air, water, ground)
- CW transmission allocates **bandwidth** around the applied carrier that depends on
  - *message bandwidth and bit rate*
  - *number of encoded levels (word length)*
  - *source and channel encoding methods*
- Examples of transmission bandwidths for certain CW techniques:
  - MPSK, M-ASK $B_T \approx r = r_b / n = r_b / \log_2 M \quad (M = 2^n)$
  - Binary FSK (f_d=r_b/2) $\quad B_T \approx r_b$
  - MSK (CPFSK f_d=r_b/4), QAM: $B_T \approx r_b / 2$

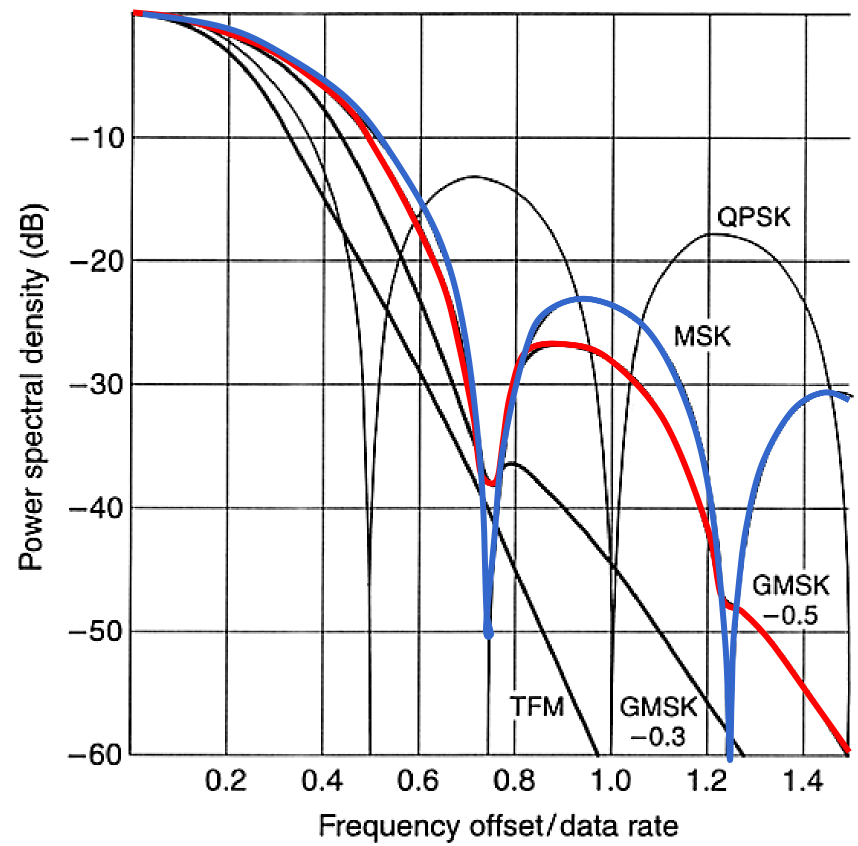
FSK: Frequency shift keying
CPFSK: Continuous phase FSK

Spectral Shaping by Data Pre-filtering



- Making phase changes continuos in time domain by filtering (before modulator) results spectral narrowing & some ISI - often so small that making B_T smaller is more important



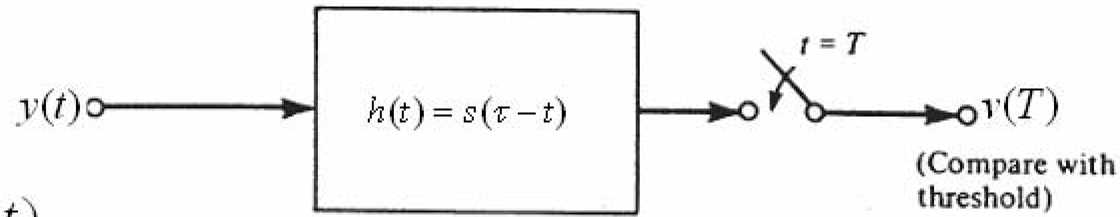


CW Detection Types

- Number of allocated signaling levels determines **constellation diagram** (=*lowpass equivalent of the applied digital modulation format*)
- At the receiver, detection can be
  - **coherent** (carrier phase information used for detection)
  - **non- coherent** (no carrier phase used for detection)
  - **differentially coherent** ('local oscillator' synthesized from received bits)
- CW systems characterized by bit or symbol **error rate** (number of decoded errors(symbols)/total number of bits(symbols))

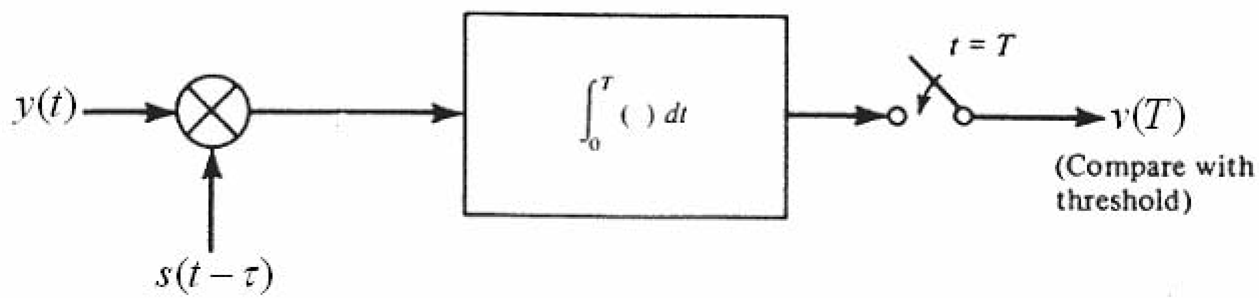
Coherent Detection by Integrate and Dump / Matched Filter Receiver

- Coherent detection utilizes **carrier phase information** and requires in-phase replica of the carrier at the receiver (explicitly or implicitly)
- It is easy to show that these two techniques have the same performance:



(a) Matched filter implementation

$$v(t)=s(\tau-t)\otimes y(t)$$
$$=\int_0^\tau s(t-\tau)y(\tau)d\tau$$

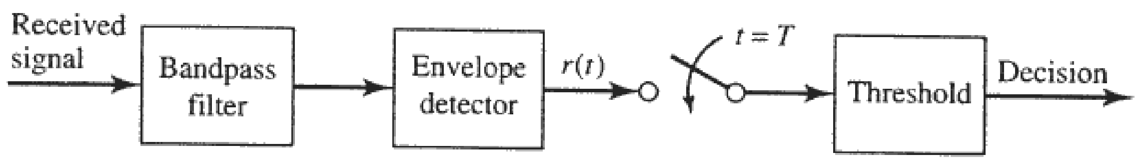


(b) Correlator implementation

$$v(t)=\int_0^\tau s(t-\tau)y(\tau)d\tau$$

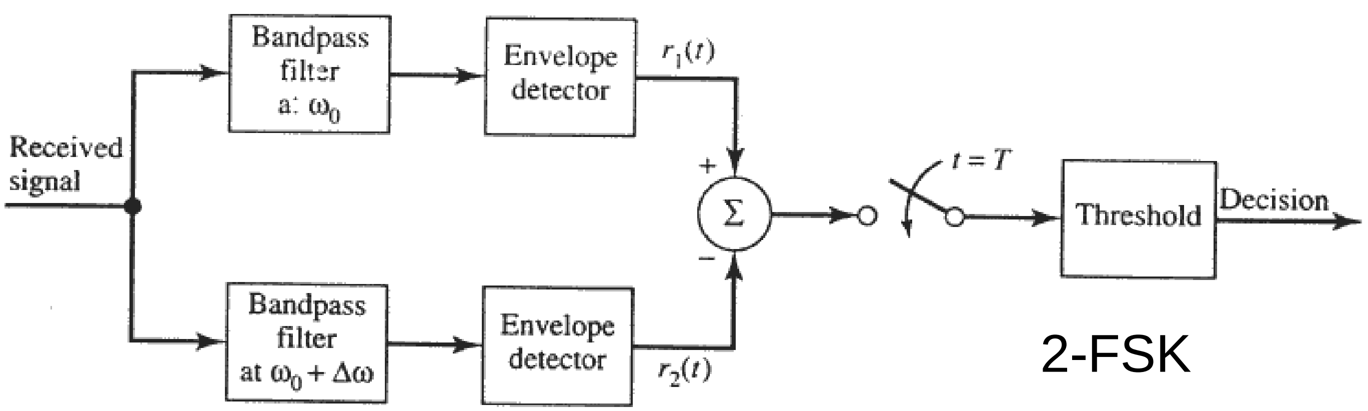
Non-coherent Detection

- Base on filtering signal energy on allocated spectra and using envelope detectors
- Has performance degradation of about 1-3 dB when compared to coherent detection (depending on E_b/N_0)
- Examples:



(a)

2-ASK

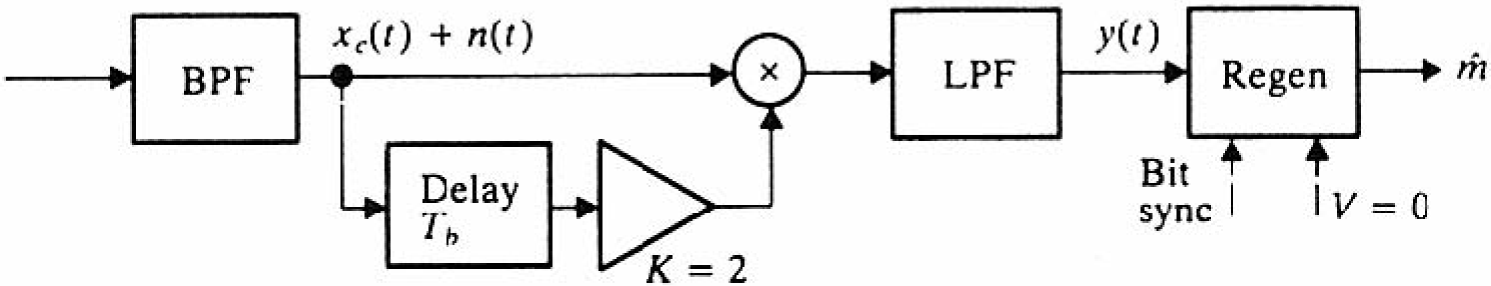


(b)

2-FSK

Differentially coherent PSK (DPSK)

- This methods circumvents usage of coherent local oscillator and can achieve almost the same performance as PSK:



- After the multiplier the signal is

$$
\begin{aligned}
x_C(t)\cdot 2x_C(t-T_b) &= 2A_C^2\cos(\omega_c t+\theta+a_k\pi) \\
&\quad \times\cos\left[\omega_C(t-T_b)+\theta+a_{k-1}\pi\right] \\
&= A_C^2\{\cos(a_k-a_{k-1})\pi \\
&\quad +\cos\left[2\omega_C t+2\theta+(a_k+a_{k-1})\pi\right]\}
\end{aligned}
$$

and the decision variable after the LPF is

$$
z(t_k)=\begin{cases} A_C^2, a_k = a_{k-1} \\ -A_C^2, a_k \neq a_{k-1} \end{cases}
$$

Differential Encoding and Decoding

- Differential encoding and decoding:

Input message	1 0 1 1 0 1 0 0
Encoded message	1 1→0 0 0→1 1→0→1
Transmitted phase	π π 0 0 0 π π 0 π
Phase-comparison sign	+ − + + − + − −
Regenerated message	1 0 1 1 0 1 0 0

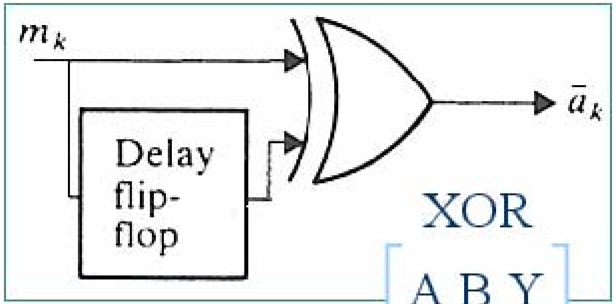
start, say with $a_k = 1$

$$\begin{cases} \text{if } m_k = 1, \text{set } a_k = a_{k-1} \\ \text{if } m_k = 0, \text{set } a_k \neq a_{k-1} \end{cases}$$

- Decoding is obtained by the simple rule:

$$d_k = a_{k-1} \oplus a_k$$

that is realized by the circuit shown right.



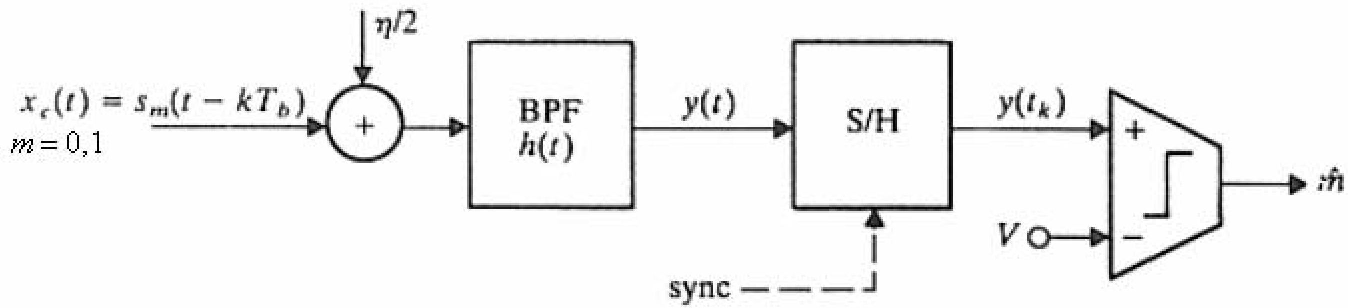
XOR

A	B	Y
0	0	1
0	1	0
1	0	0
1	1	1

- Note that no local oscillator is required
- How would you construct the encoder?

Coherent Detection

Example: Optimum Binary Detection



- Received signal consist of bandpass filtered signal and noise that is sampled at the decision time instants t_K yielding decision variable: $Y = y(t_k) = z_m + n$
- Quadrature presentation of the signaling waveform is

$$s_m(t) = A_C\left\{I_k p_i(t)\cos(\omega_C t) - Q_k p_q(t)\sin(\omega_C t)\right\}$$

- Assuming that the BPF has the impulse response *h(t)*, signal component at the sampling instants is then expressed by

$$z_m = s_m(t - kT_b) \otimes h(t)\big|_{t=t_K} = \int_{kT_b}^{(k+1)T_b} s_m(\lambda - kT_b)h(t_k - \lambda)d\lambda$$

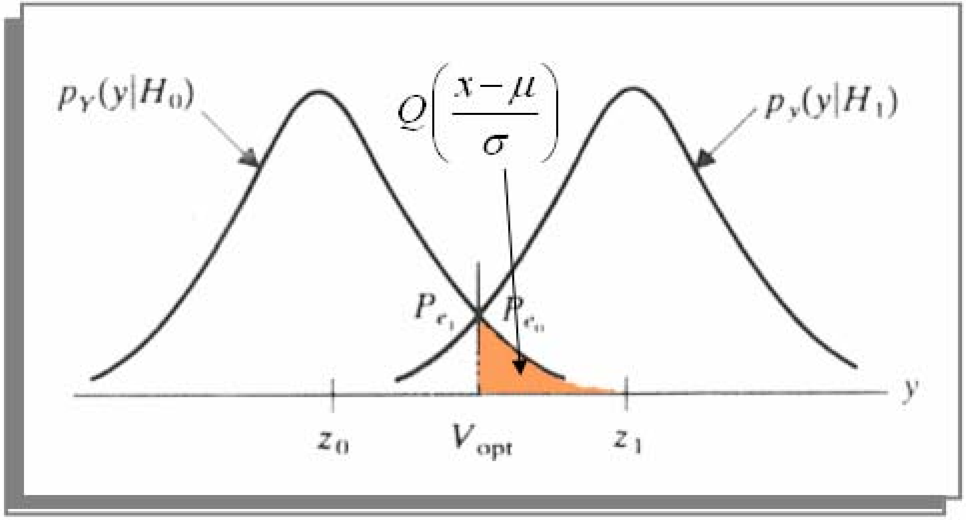
$$= \int_0^{T_b} s_m(\lambda)h(T_b - \lambda)d\lambda$$

$$(x \otimes y(t) = \int_A \mathrm{x}(\lambda)y(t-\lambda)d\lambda)$$

Optimum Binary Detection - Error Rate

- Assuming '0' and '1' reception is equally likely, error happens when H_0 ('0' transmitted) signal hits the dashed region or for H_1 error hits the left-hand side of the decision threshold that is at

$$V_{opt} = (z_1 + z_0)/2$$



For optimum performance we have the maximized **SNR** that is obtained by matched filtering/ integrate and dump receiver

$$\left(|z_1 - z_0|/2\sigma\right)^2$$

Errors for '0' or/and '1' are equal alike, for instance for '0':

$$p_{e0} = \frac{1}{\sigma\sqrt{2\pi}} \int_{V_{opt}}^{\infty} \exp\left[(\lambda + z_0)^2 / 2\sigma^2\right] d\lambda = Q\left(\frac{V_{opt} - z_0}{\sigma}\right)$$

$$p_e = (p_{e0} + p_{e1})/2 = Q\left(|z_1 - z_0|/2\sigma\right)$$

Optimum Binary Detection (cont.)

- Express energy / bit embedded in signaling waveforms by

$$\int_0^{T_b}[s_1(\lambda)-s_0(\lambda)]^2\,d\lambda=\underbrace{\int_0^{T_b}s_1^2(\lambda)d\lambda}_{E_1}$$

$$+\underbrace{\int_0^{T_b}s_0^2(\lambda)d\lambda}_{E_0}-2\underbrace{\int_0^{T_b}s_0(\lambda)s_1(\lambda)d\lambda}_{E_{10}}$$

$$p_e=Q\left(|z_1-z_0|/2\sigma\right)$$

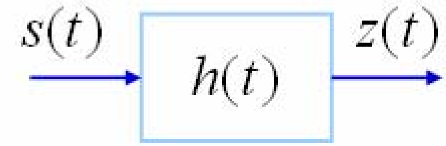
Note that the signaling waveform **correlation** greatly influences the SNR!

- Therefore, for **coherent** CW we have the SNR and error rate

$$\left.\frac{|z_1-z_0|^2}{4\sigma^2}\right|_{\sigma^2=\eta/2}=\frac{E_1+E_0-2E_{10}}{2\eta}\Rightarrow p_e=Q\left(\sqrt{\frac{E_1+E_0-2E_{10}}{2\eta}}\right)=\boxed{Q\left(\sqrt{\frac{E_b-E_{10}}{\eta}}\right)}$$

$$SNR_{\max}=\frac{E_b}{N_o}=\frac{E_b}{\eta/2}\Rightarrow N_o=\sigma^2=\eta/2$$

Timing and Synchronization



- Performance of coherent detection is greatly dependent on how successful local carrier recovery is
- Consider the bandpass signal *s(t)* with a *rectangular pulses* $p_{Tb}(t)$, that is applied to the matched filter *h(t)*:

$$s(t) = A_C p_{Tb}(t)\cos(\omega_C t),$$

$$h(t) = Ks(-T_b - t) = Kp_{Tb}(-T_b - t)\cos(\omega_C t + \theta_\varepsilon)$$

$$\Rightarrow z(\tau) = s(t) \otimes h(t)$$

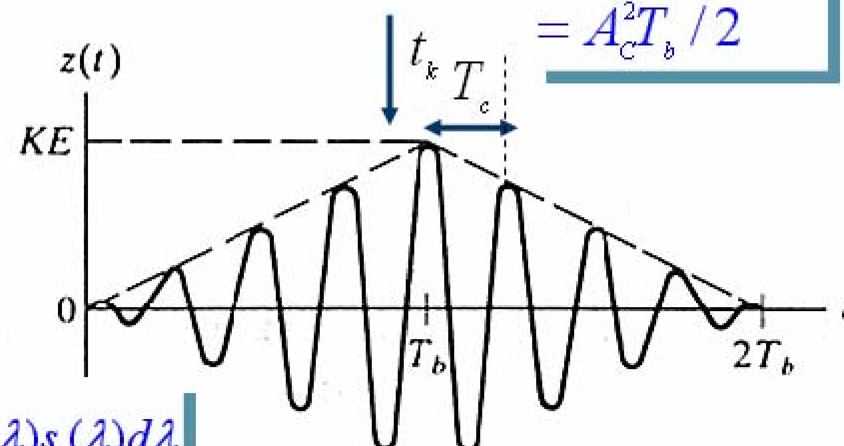
$$= KA_\tau \Lambda\left(\frac{\tau - T_b}{T_b}\right)\left[\cos^2(\pi\tau / T_c) - 1/2\right]$$

$$= KA_\tau \Lambda\left(\frac{\tau - T_b}{T_b}\right)$$

$$\times\left[\cancel{1/2} + \cos(\omega_c \tau) \cancel{-1/2}\right]$$

$$E \approx \int_0^{T_b} s^2(t)dt$$

$$= A_C^2 T_b / 2$$



$$\int_0^{T_b}\left[s_1(\lambda) - s_0(\lambda)\right]^2 d\lambda = \underbrace{\int_0^{T_b} s_1^2(\lambda)d\lambda}_{E_1} + \underbrace{\int_0^{T_b} s_0^2(\lambda)d\lambda}_{E_0} - 2\underbrace{\int_0^{T_b} s_0(\lambda)s_1(\lambda)d\lambda}_{E_{10}}$$

Analyzing phase error by Mathcad

- Therefore, due to phase mismatch at the receiver, the error rate is degraded to

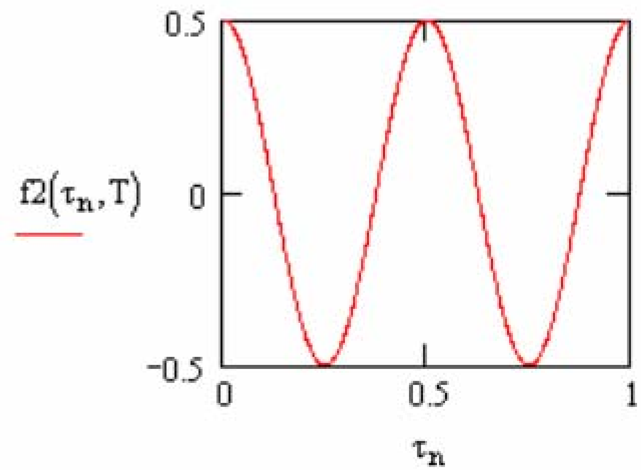
$$p_e \approx Q\left(\sqrt{\frac{E_b - E_{10}}{\eta}\cos^2\theta_\varepsilon}\right)$$

$$\theta_\varepsilon = \omega_c \tau$$

$$\frac{1}{T}\cdot\int_0^T \cos\left(2\cdot\frac{\pi}{T}\cdot t\right)\cdot\cos\left[2\cdot\frac{\pi\cdot(\tau - t)}{T}\right]dt$$

$n := 1 .. 1000 \qquad T := 0.5 \qquad f2(\tau, T) := \cos\left(\pi\cdot\frac{\tau}{T}\right)^2 - \frac{1}{2}$

$\tau_n := n\cdot\frac{1}{1000}$



Example

- Assume data rate is 2 kbaud/s and carrier is 100 kHz for an BPSK system. Hence the symbol duration and carrier period are

$$T_s = 1/2\text{kbaud/s} = 0.5\,\text{ms} \qquad T_c = 1/f_c = 10\ \mu s$$

therefore the symbol duration is in radians

$$\frac{10\mu s}{0.5\text{ms}} \equiv \frac{2\pi}{\text{x}} \Rightarrow x = 314.2\,\text{rad}$$

- Assume carrier phase error is 0.3 % of the symbol duration. Then the resulting carrier phase error is

$$\theta_\varepsilon = 0.003x = 0.94\ \text{rad} = 54^\circ$$

and the error rate for instance for $\gamma = 8 \approx 9\,\text{dB}$ is

$$p_e = Q(\sqrt{16\cos^2 54}) \approx 10^{-2}$$

that should be compared to the error rate without any phase errors or $$p_e = Q(\sqrt{16}) \approx 3 \cdot 10^{-5}$$

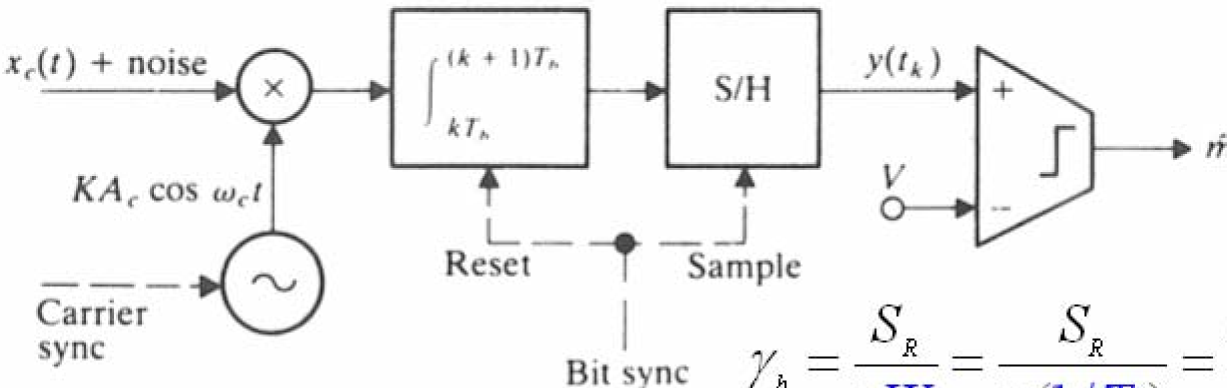
- Hence, phase synchronization is a very important point to remember in coherent detection

Example: Coherent Binary On-off Keying (OOK)

- For on-off keying (OOK) the signaling waveforms are

$$s_1(t) = A_C p_{Tb}(t)\cos\omega_C t,\ \ s_0(t) = 0$$

and the optimum coherent receiver can be sketched by



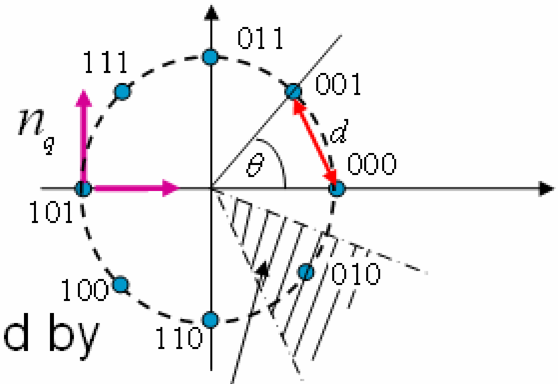
$$\gamma_b = \frac{S_R}{\eta W} = \frac{S_R}{\eta(1/T_b)} = \frac{S_R T_b}{\eta} = \frac{E_b}{\eta}$$

$$E_1 = \int_0^{T_b} s_1^2(\lambda)d\lambda = A_C^2 T_b/2, E_0 = 0, E_{10} = \int_0^{T_b} s_1(\lambda)s_0(\lambda)d\lambda = 0$$

$$E_b = (E_0 + E_1)/2 = A_C^2 T_b/4 \quad p_e = Q\left(\sqrt{\frac{E_b - E_{10}}{\eta}}\right) = Q\left(\sqrt{\frac{E_b}{\eta}}\right) = \underline{Q\left(\sqrt{\gamma_b}\right)}$$

 Helsinki University of Technology, Communications Laboratory, Timo O. Korhonen

Error rate for M-PSK



- In general, PSK error rate can be expressed by

$$p_e = \overline{n}_n Q\left(\frac{d}{2\sigma}\right) = \overline{n}_n Q\left(\frac{a}{\sigma}\right)$$

where *d* is the distance between constellation points (or *a*=*d*/2 is the distance from constellation point to the decision region border) and $\overline{n}_n$ is the average number of constellation points in the immediate neighborhood. Therefore

$$p_e = 2Q\left(\frac{d}{2\sigma}\right) = 2Q\left(\frac{2A\sin(\theta/2)}{2\sigma}\right) = 2Q\left(\frac{A}{\sigma}\sin(\pi/M)\right)$$

Note that for matched filter reception

$$M = 2^n$$

$$\frac{A}{\sigma} = \sqrt{\frac{2E}{\eta}}, E = nE_b = \log_2(M)E_b$$

$$\theta = \frac{2\pi}{M}$$

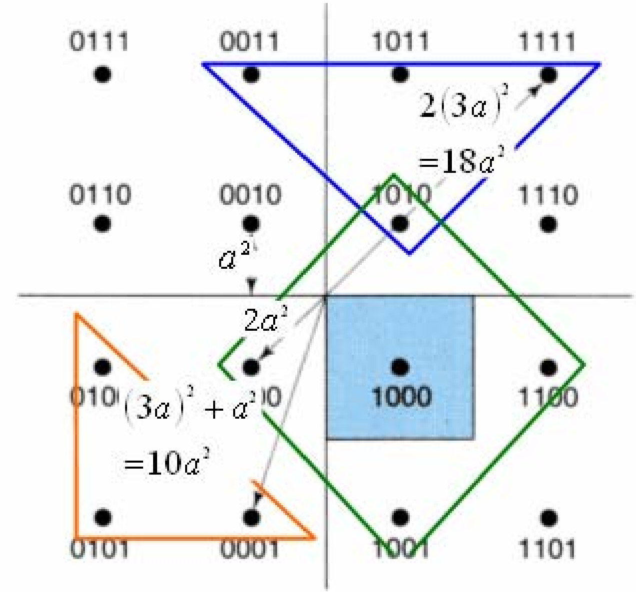
A: RMS signal voltage at the moment of sampling

Error rate for M-QAM, example 16-QAM

$$p_e = \overline{n}_n Q\left(\frac{d}{2\sigma}\right) = \overline{n}_n Q\left(\frac{a}{\sigma}\right)$$

$$\overline{n}_n = \frac{4\cdot 4 + 8\cdot 3 + 4\cdot 2}{4+8+4} = 3$$

$$\overline{A^2} = \frac{4\cdot 2a^2 + 8\cdot 10a^2 + 4\cdot 18a^2}{16} = 10a^2$$



$$p_e = 3Q\left(\frac{a}{\sigma}\right) = 3Q\left(\sqrt{\frac{\overline{A^2}}{10\sigma^2}}\right) = 3Q\left(\sqrt{\frac{2E}{10N_0}}\right)$$ symbol error rate

Constellation follows from 4-bit words and therefore

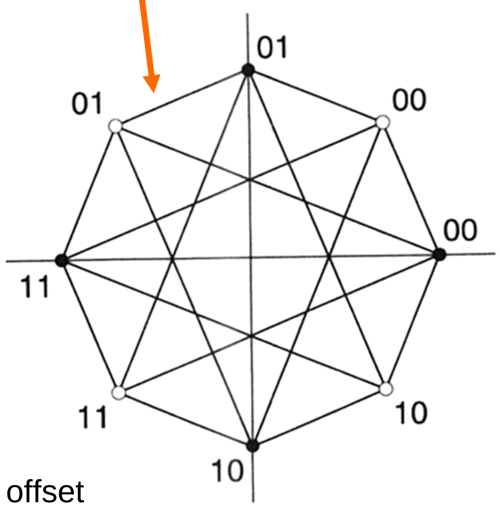
$$p_b = \frac{3}{4}Q\left(\sqrt{\frac{2E}{10N_0}}\right)\Bigg|_{E=4E_b} = \frac{3}{4}Q\left(\sqrt{\frac{4E_b}{5N_0}}\right)$$

$$\begin{cases} n = \log_2 M, E = nE_b \\ p_e = p(E)/n \\ A/\sigma = \sqrt{2E/N_0} \end{cases}$$

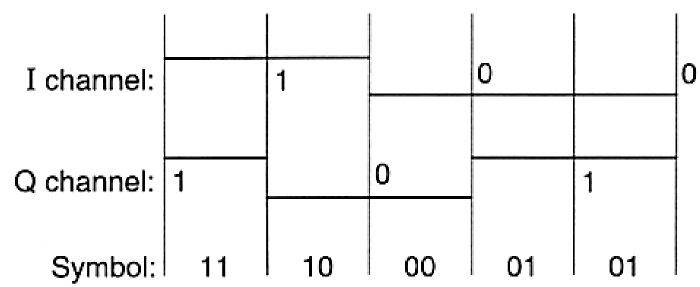
Envelope distortion and QPSK

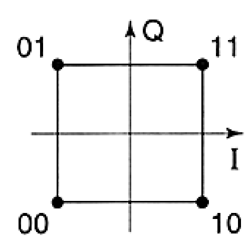
- QPSK is appealing format, however requires constant envelope
- Passing constellation figure via (0,0) gives rise to envelope -> 0
- Prevention:
  - Gray coding
  - Offset - QPSK
  - Pi/4 QPSK

a_k	Natural code	Gray code
$3A/2$	11	10
$A/2$	10	11
$-A/2$	01	01
$-3A/2$	00	00



Apply two $\pi/4$ offset
QPSK constellations by turns

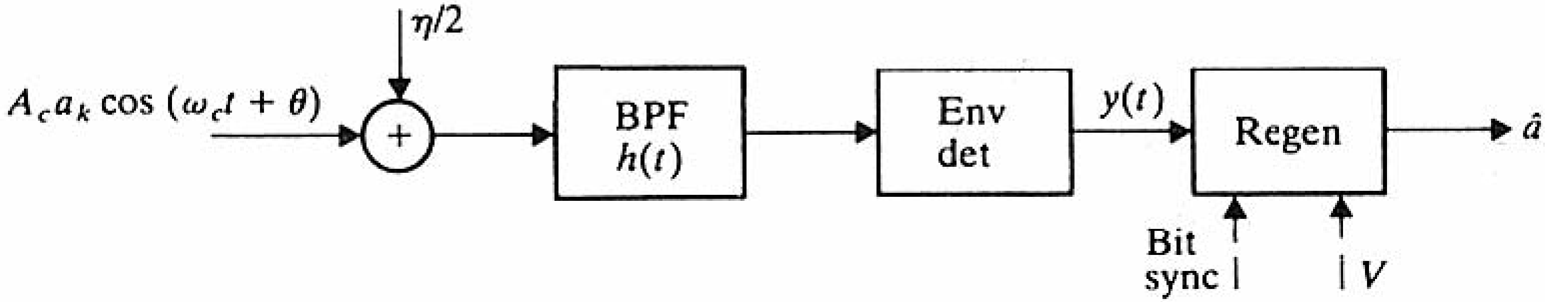




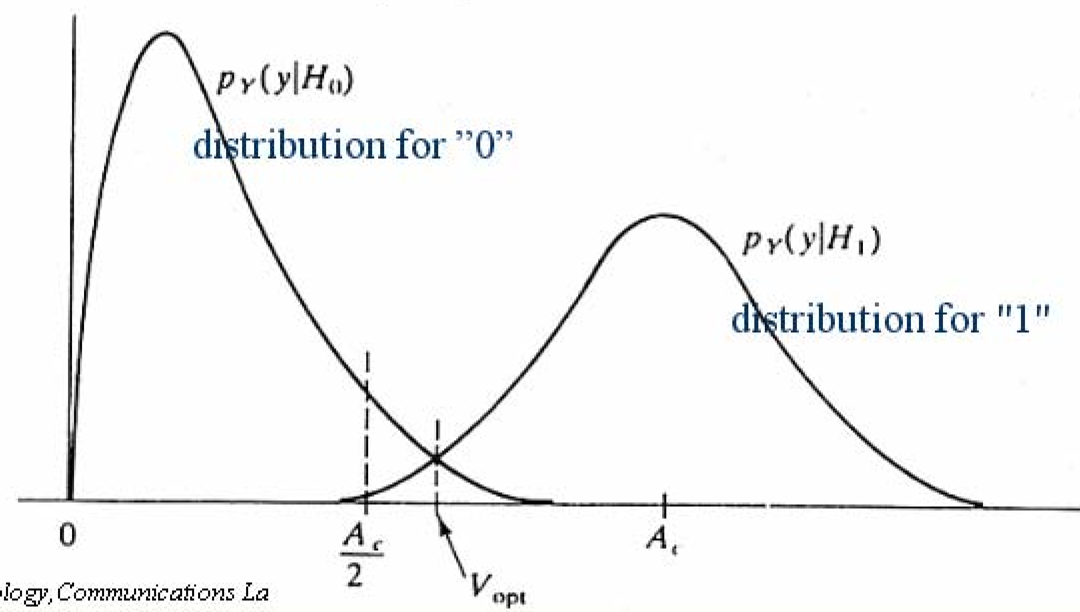
Delay I and Q channels
by symbol time

Non-coherent Detection

Example: Non-coherent On-off Keying (OOK)



- Bandpass filter is matched to the signaling waveform (not to carrier phase), in addition $f_c >> f_m$, and therefore the energy for '1' is simply $E_1 = T_b(A_C^2 / 2)$
- Envelopes follow Rice and Rayleigh distributions for '1' and '0' respectively:



Non-coherent Binary Systems: Noisy Envelopes

- AWGN plus carrier signal have the envelope whose probability distribution function is
  - For nonzero, *constant* carrier component A_C, Rician distributed:

$$p_Y(Y \mid H_1) = \frac{x}{\sigma^2} \exp\left(-\frac{x^2 + A_C^2}{2\sigma^2} \right) I_0\left(\frac{xA_C}{\sigma^2} \right), x \geq 0$$

  - For zero carrier component Rayleigh distributed:

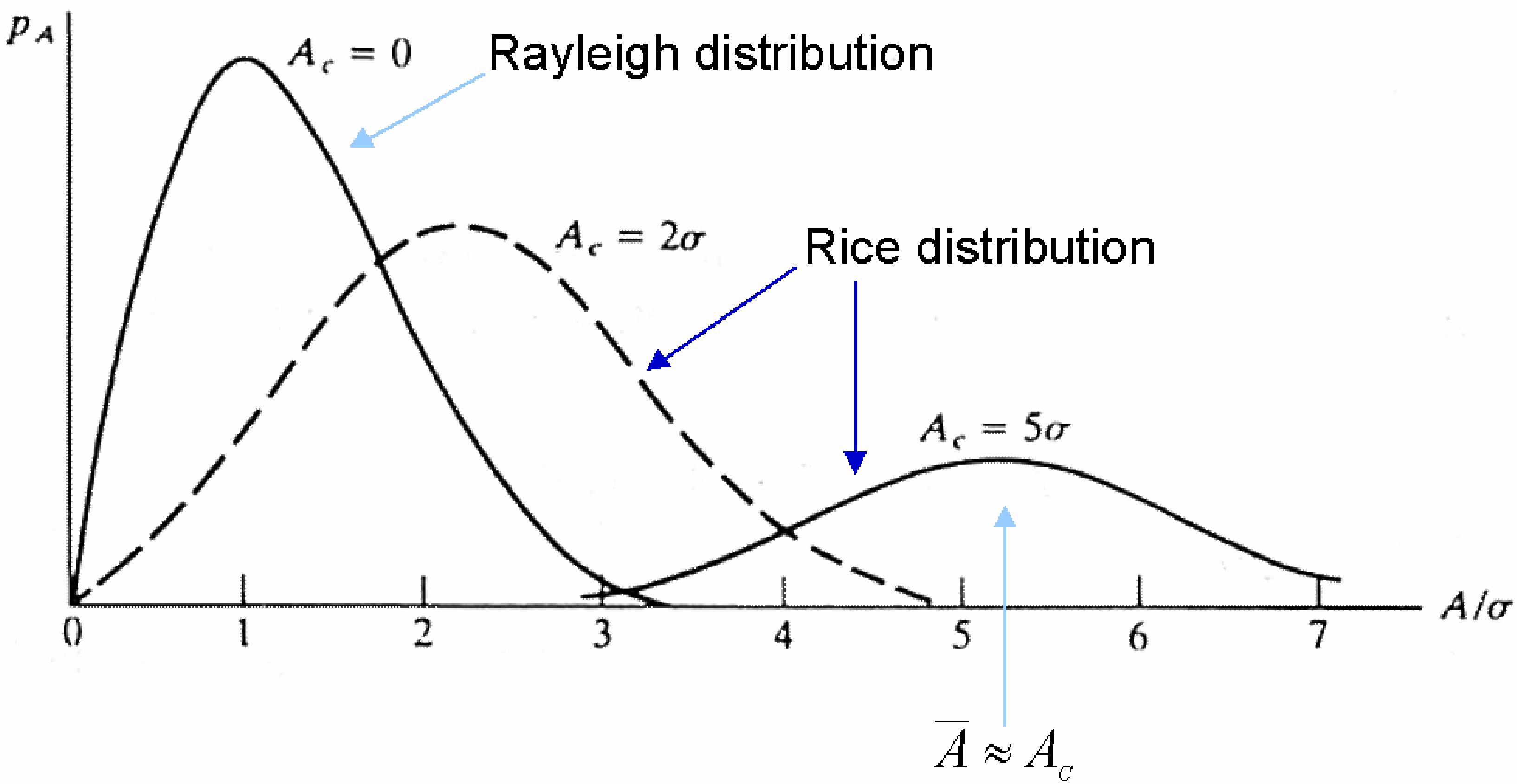
$$p_Y(y \mid H_0) = \frac{x}{\sigma^2} \exp\left(-\frac{x^2}{2\sigma^2} \right), x \geq 0$$

- For large SNR ($A_C >> \sigma$) the Rician envelope simplifies to

$$p_Y(Y \mid H_1) \approx \sqrt{\frac{x}{2\pi A_C \sigma^2}} \exp\left(-\frac{(x - A_C)^2}{2\sigma^2} \right), x \geq 0$$

- Therefore in this case the received envelope is then essentially Gaussian with the variance σ^2 and mean equals $\overline{p_A(x)} \approx A_C$

Envelope Distributions with different Carrier Component Strengths



Noncoherent OOK Error Rate

- The optimum threshold is at the intersection of Rice and Rayleigh distributions (areas are the same on both sides)
- Usually high SNR is assumed and hence the threshold is approximately at the half way and the error rate is the average of '0' and '1' reception probabilities

$$P_e = \tfrac{1}{2}\left(P_{e0} + P_{e1}\right)$$

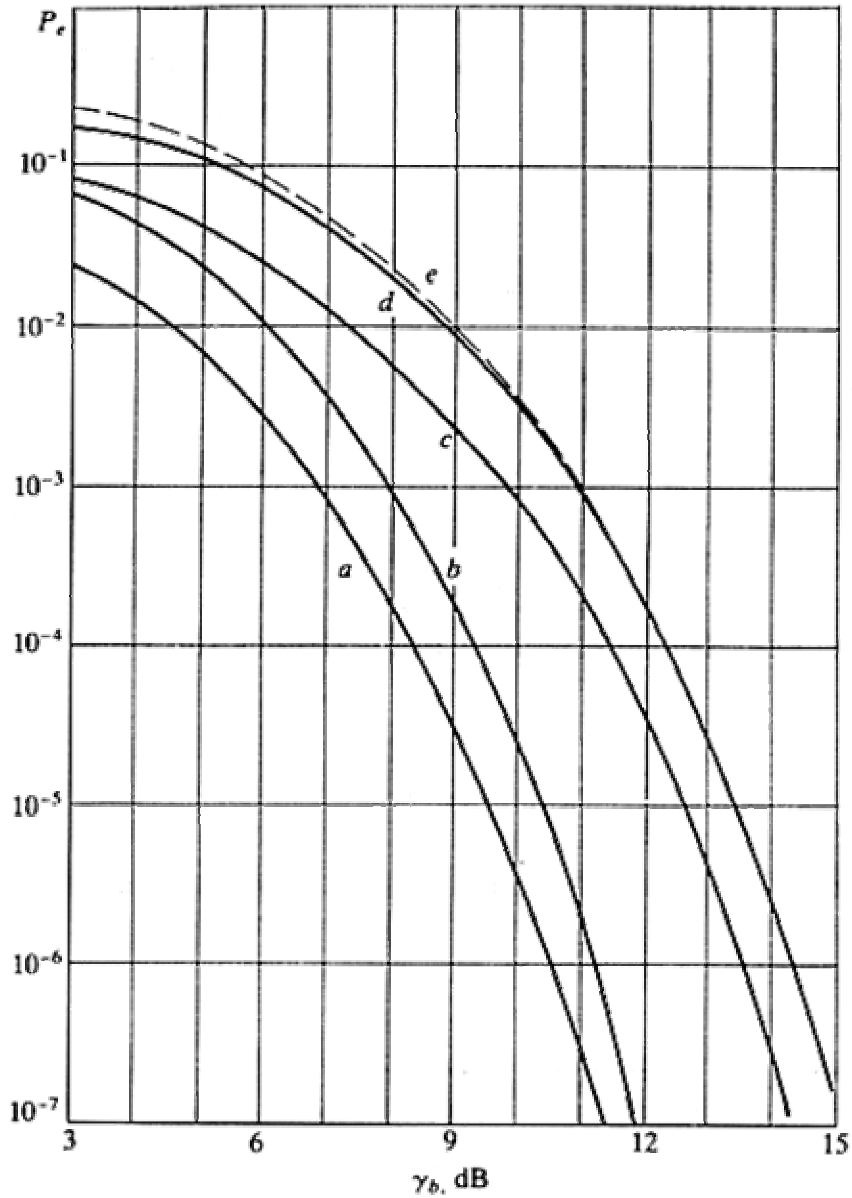
$$\begin{cases} P_{e0} = \displaystyle\int_{A_C/2}^{\infty} p_Y(Y \mid H_0)dy = \exp\left(-A_C^2/8\sigma^2\right) = \exp\left(-\gamma_b/2\right) \\ P_{e1} = \displaystyle\int_{0}^{A_C/2} p_Y(Y \mid H_1)dy \approx Q(A_C/2\sigma) = Q(\sqrt{\gamma_b}) \end{cases}$$

- Therefore, error rate for non-coherent OOK equals

$$P_e \approx \tfrac{1}{2}\left[\exp(-\gamma_b/2) + Q(\sqrt{\gamma_b})\right] \approx \tfrac{1}{2}\exp(-\gamma_b/2), \gamma_b >> 1$$

Comparison

Error Rate Comparison

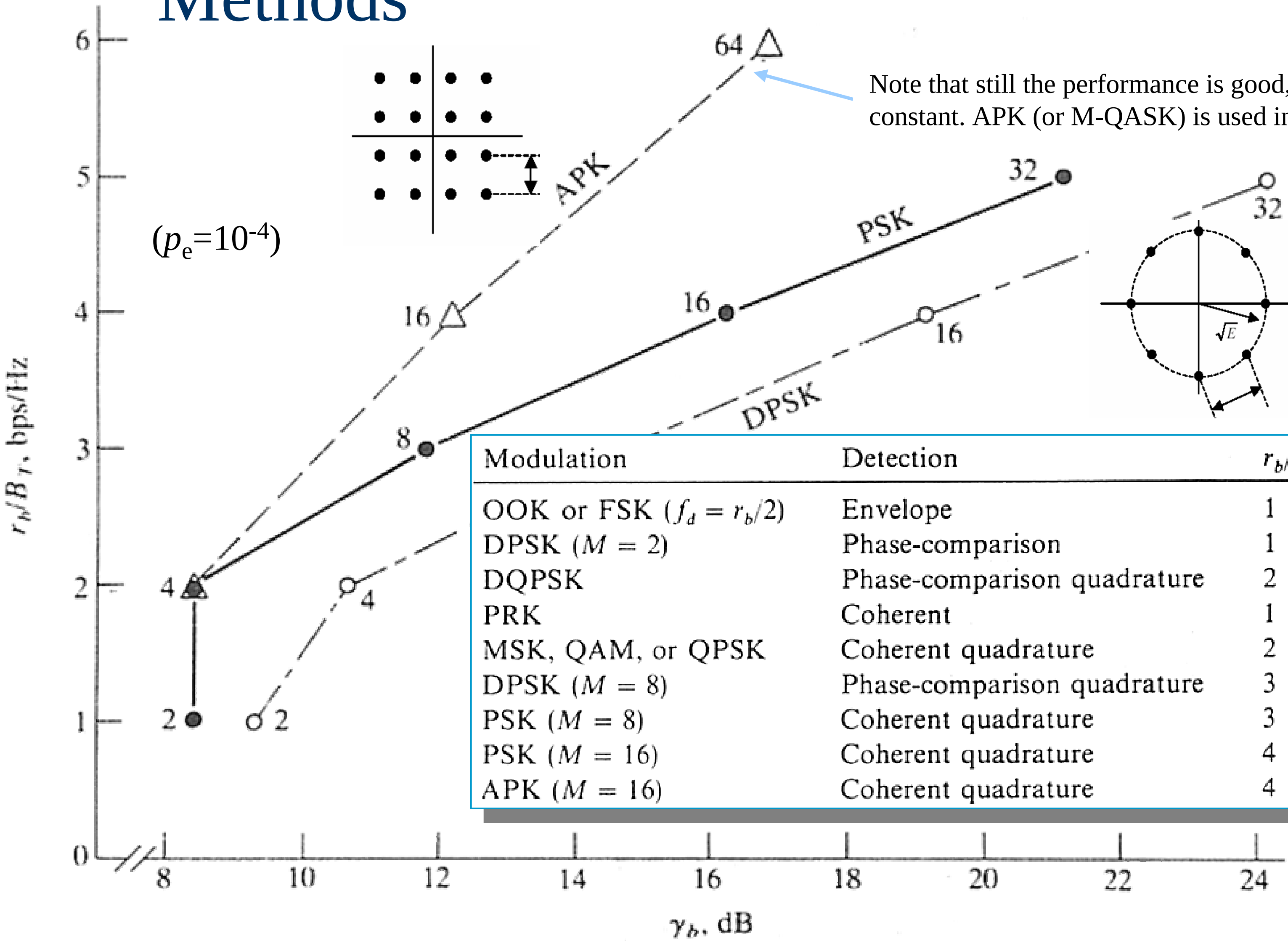


a: Coherent BPSK
b: DPSK
c:Coherent OOK
d: Noncoherent FSK
e: noncoherent OOK

Comparison of Quadrature Modulation Methods

Note that still the performance is good, envelope is not constant. APK (or M-QASK) is used in cable modems

($p_e = 10^{-4}$)



Modulation	Detection	r_b/B_T	γ_b, dB
OOK or FSK ($f_d = r_b/2$)	Envelope	1	12.3
DPSK ($M = 2$)	Phase-comparison	1	9.3
DQPSK	Phase-comparison quadrature	2	10.7
PRK	Coherent	1	8.4
MSK, QAM, or QPSK	Coherent quadrature	2	8.4
DPSK ($M = 8$)	Phase-comparison quadrature	3	14.6
PSK ($M = 8$)	Coherent quadrature	3	11.8
PSK ($M = 16$)	Coherent quadrature	4	16.2
APK ($M = 16$)	Coherent quadrature	4	12.2

M-APK: Amplitude Phase Shift Keying