CDMA as Signal Space Waveform I

- SF chips (T_c) combined into symbol (duration $T = SF \cdot T_c$)
- signal-space waveform $s(t) = \sum_{n=1}^{SF} c_n f(t - nT_c)$
  - $f(t)$ is the pulse shape, most of power concenterated inside chip duration T_c
  - $\mathbf{c} = \begin{bmatrix} c_1 & c_2 & \cdots & c_{SF} \end{bmatrix}^{\mathrm{T}}$ is the spreading code
  - most power concentrated inside symbol duration $[0, T]$
  - $s(t)$ modulated with different complex symbol each period T
- if same entity transmits multiple symbols in $[0, T]$, *multicode* transmission used
  - multiple orthogonal waveforms in $[0, T]$
  - orthogonal spreading codes

Multicode CDMA

- signal space model of transmitted chips:

$$\mathbf{s} = \begin{bmatrix} c_{11} & c_{12} & \cdots & c_{1M} \\ c_{21} & c_{22} & \cdots & c_{2M} \\ \vdots & \vdots & \ddots & \vdots \\ c_{SF,1} & c_{SF,2} & \cdots & c_{SF,M} \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ \vdots \\ x_M \end{bmatrix} \equiv \mathbf{Cx}$$

  - M symbols x_m transmitted simulataneously, using M spreading codes
  - each column in the $SF \times M$ spreading matrix $\mathbf{C}$ is a spreading code
  - with orthogonal codes, $\mathbf{C}^{\mathrm{H}}\mathbf{C} = \mathbf{I_M}$

CDMA as Signal Space Waveform II

- Bandwidth B occupied by CDMA waveform is $\sim 1/T_c$
  - for sinc pulses, $B = 1/T_c$ exactly
  - for raised cosine pulses with roll-off factor α, $B = (1 + \alpha)/T_c$
- The frequency domain corresponding to a finite time (or periodic) signal $s(t)$ is discrete, consists of Fourier series coefficients at frequencies k/T

$$f_k = \int_0^T dt\ s(t)\ \mathrm{e}^{2\pi \mathrm{j}\, \mathrm{kt/T}}$$

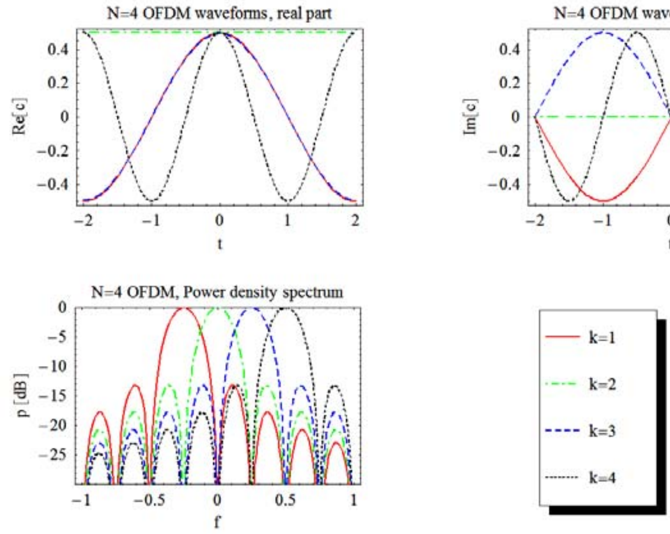
  - frequencies arising from transitions at symbol borders ignored
  - CDMA signal $s(t)$ has non-vanishing frequencies $|k| \geq SF/2$
  - full power density spectrum: F-transform of $s(t)\mathrm{rect}(0, T)$

CDMA as Signal Space Waveform III

- the SF orthogonal spreading codes constitute a basis in the space of complex periodic functions with period T and bandwidth B
  - any function within $[0, T]$ and Fourier series components vanishing outside $[-B/2, B/2]$ can be expressed with orthogonal spreading codes and complex coefficents (the symbols)
  - elements c_i in Walsh-Hadamard spreading codes are real, but generic spreading code may have complex elements
- in time interval $[0, T]$ and bandwidth $B = 1/T_c$, there are $TB = T/T_c = SF$ orthogonal resources

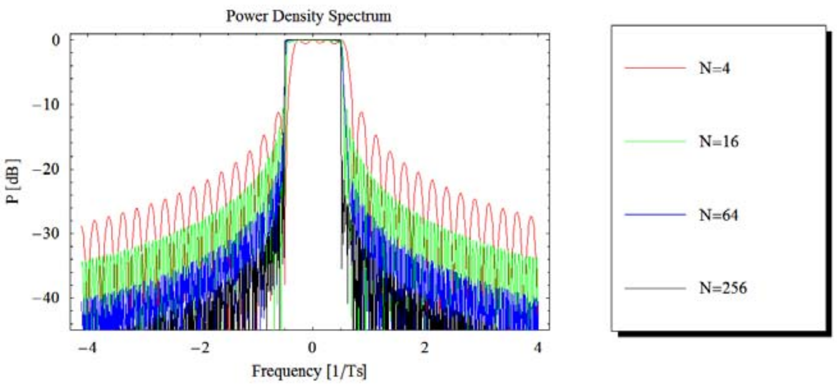
Fourier Basis

- Another natural basis: $\{f_k(t) = e^{2\pi j (N/2-k)t/NT_s}\}_{k=1}^{N}$
  - Periodic in $[0, T]$ (or $[-T/2, T/2]$), symbol duration $T = NT_s$
  - the Fourier sampling time T_s analogous to T_c
  - Fourier transform size N analogous to SF
  - Fourier bandwidth $B = 1/T_s$
  - $f_k(t)$ is *subcarrier*, bandwidth $B_s = 1/NT_s$
    - constant amplitude, rotating phase—complex valued waveform
    - localized in frequency $[k - 1/2, k + 1/2]/NT_s - 1/2T_s$
- any function within $[0, T]$ and Fourier series components vanishing outside $[-B/2, B/2]$ can be expressed with subcarriers waveforms and complex coefficents (the symbols)



OFDM power density spectrum

- Out-of band emissions due to rectangular symbol window
- decrease as N increases



OFDM symbol

- subccarriers modulated with a complex symbol
  - changes the phase (frequency) during transmission
  - changes amplitude, if MQAM with $M > 4$
  - narrowband transmission
  - the information is packed in the frequencies used
  - frequencies remain constant in $[0, T]$
- the sum of $N' \leq N$ modulated subcarriere signals in time $T = NT_s$ is an OFDM symbol
  - Orthogonal Frequency Domain Multiplexing

- multipath components of an OFDM symbol arrive with different timing, but same frequencies
- assume delay spread τ_s, transmission of a single OFDM symbol,
  - received in time $[0, T + \tau_s]$
  - multipath components do not change the frequency content of the signal
  - reception can be based on any period of length T within $[0, T + \tau_s]$
- when transmitting multiple OFDM symbols, Cyclic Prefix (CP) of length $\geq \tau_s$ added in front of OFDM symbol
  - at receiver, CP dropped $\Rightarrow$ all Inter-Symbol Interference vanishes
- movement (Doppler shift) changes the frequency content of the signal $\Rightarrow$ Inter-Carrier Interference

DFT and FFT

- sampling the subcarrier waveforms at intervals of T_s one gets the sample representation of the subcarriers, the Hermitian conjugate of a DFT matrix M, e.g. $N = 4$ sampled at $t = -2, -1, 0, 1$:

$$\mathbf{M} = \frac{1}{2}\begin{bmatrix} 1 & \mathrm{j} & -1 & -\mathrm{j} \\ 1 & 1 & 1 & 1 \\ 1 & -\mathrm{j} & 1 & \mathrm{j} \\ 1 & -1 & 1 & -1 \end{bmatrix}$$

  - by convention, OFDM transmission is defined as *inverse* Fourier transform $\Rightarrow$ Hermitian conjugate
  - the sampled subcarrier waveforms are columns in $\mathbf{M}^{\mathrm{H}}$, conjugated rows in $\mathbf{M}$.

- often $N \times N$ DFT matrix expressed with permuted columns (subcarriers) in the form with matrix elements $m_{ik} = \mathrm{e}^{-2\pi \mathrm{j}\,(\mathrm{i}-1)(\mathrm{k}-1)/\mathrm{N}}$, e.g.:

$$\mathbf{M} = \frac{1}{2}\begin{bmatrix} 1 & 1 & 1 & 1 \\ 1 & \mathrm{j} & -1 & -\mathrm{j} \\ 1 & -1 & 1 & -1 \\ 1 & -\mathrm{j} & 1 & \mathrm{j} \end{bmatrix}$$

- time domain samples of an OFDM symbol are Inverse Discrete Fourier Transforms of subcarrier symbols:

$$\mathbf{s} = M^{\mathrm{H}}\begin{bmatrix} x_1 \\ x_2 \\ \vdots \\ x_N \end{bmatrix}$$

- some of the highest & lowerst frequency subcarriers may be left unused
- with $N = 2^m$, the DFT can be performed with Fast Fourier Transform (FFT)
  - $N \log_2 N$ multiplications required to construct the DFT
- at receiver, DFT or FFT is used to recover the symbols transmitted on the subcarriers

OFDM symmary

- using a discrete (or fast) Fourier transform, modulation is performed in frequency domain
- using a cyclic prefix removes all inter-symbol interference
- added overhead from transmitting CP
- perfect removal of multipath-generated interference
- in ADSL, OFDM (a.k.a. discrete multitone) is used
- OFDM is the modulation used in most recent mobile standards
  - WiMAX
  - 3G LTE