

S-72.2205 Digital Transmission Methods

Exercise session 5

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1 CDMA

In many systems several users must share the same resources and multiple access (MA) schemes is needed to do so. For example, in GSM a combination of FDMA and TDMA is used. The third way of sharing resources is code division multiple access (CDMA). In CDMA the users transmit simultaneously at same frequency but the users are separated by spreading codes. The spreading code spreads the information bandwidth R_b to occupy a much larger transmission bandwidth R_c . It is the interference attenuation property of spread spectrum that allows multiple users to occupy the same bandwidth at the same time.

1. Define chip rate, bit rate and processing gain.

Solution

Chip rate is proportional to the transmission bandwidth and bit rate to the information bandwidth. The processing gain is the ratio of the spread (transmission) bandwidth to the bit rate and it means that the interference is reduced by the processing gain relative to its effect in a nonspread system.

2. What is actually this interference attenuation property? Where this results from?

Solution

With CDMA the users are approximately code orthogonal. To achieve orthogonality different symbol-shaping function

$$g_k(t) = \sum_{i=1}^G c_k(i)g_c(t - iT_c) \quad 0 \leq t \leq T$$

is assigned to each user k . The sequence $[c_k(i)]$ is the spreading code or signature for the user k . The approximate orthogonality of $g_j(t)$ and $g_k(t)$ for different time offsets τ can be represented as the requirement

$$R_{jk}(\tau) = \int_{-\infty}^{\infty} g_j(t + \tau)g_k^*(t)dt \approx 0 \quad \text{for } j \neq k$$

where it is assumed that a matched filter is used at the receiver. Consequently as spreading codes it should be used such codes that have good crosscorrelation properties. An additional requirement is that the code is also self-orthogonal $R_{kk} \approx 0$ for all $\tau > 0$. The degree to which these are satisfied depends on the choice of the spreading codes.

1.1 Some system level issues on CDMA

In downlink direction, all channels that are emitted by the same base station can be synchronized and thus it is possible to select codes that result in zero multiple access interference.

In the uplink, the the users can not be synchronized and consequently there is multiple access interference. The interference change as the number of active users N change. This results in cell breathing. In a CDMA-system the interference margin is

$$IM = 10 \log_{10} \frac{1}{1 - \eta}$$

that depends on the up-link fractional load η . The up-link fractional load can be calculated as follows

$$\eta = (1 + f) \sum_{i=1}^N \frac{\rho_i \gamma_i}{G_i}$$

where f is other-cell to own cell interference factor, ρ is the activity of a user, γ is the SNR of the user and G is the spreading factor.

1. How many speech user ($\rho_i = 0.4, 10 \log_{10} \gamma = 8dB, G_i = 265$) can be served when the other cell to own cell interference ratio is $f = 0.75$, and the fractional load *target* is 0.7?

Solution

The fractional load is a function of the number of users N

$$\eta = (1 + f) \sum_{i=1}^N \frac{\rho_i \gamma_i}{G_i} = (1 + 0.75)N \frac{0.4 \cdot 10^{0.8}}{265} = 0.7$$

Solve for N keeping the target load

$$N = \frac{256 \cdot 0.7}{1.75 \cdot 0.4 \cdot 10^{0.8}} = 40.57$$

From this we get $N = 40$.

2. A new data user ($\rho_{N+1} = 1, 10 \log_{10} \gamma_{N+1} = 8dB, G_i = 32$) is admitted in the own cell. How many dB must the interference margin be increased from the value of the previous task to maintain all the connections?

Solution

The resulting fractional load with 40 speech users is

$$\eta = (1 + 0.75)40 \frac{0.4 \cdot 10^{0.8}}{256} = 0.6900$$

Now we have 40 speech users with the parameters given in previous task and one data user with the above parameters and the fractional load becomes

$$\eta = (1 + f) \sum_{i=1}^N \frac{\rho_i \gamma_i}{G_i} = (1 + 0.75) \left(40 \frac{0.4 \cdot 10^{0.8}}{256} + \frac{1 \cdot 10^{0.4}}{32} \right) = 0.8274$$

Next the interference margin difference can be calculated

$$\Delta IM = 10 \log_{10} \frac{1 - \eta_a}{1 - \eta_b} = 10 \log_{10} \frac{1 - 0.6900}{1 - 0.8274} = 2.54dB$$

3. In WCDMA the chip rate is 3.84 Mchip/s. How many users in a cell can theoretically be simultaneously served in the up-link direction of a *single cell* system, when the user bit rate after channel coding is 15 kbit/s and the E_b/I_o requirement for proper reception is 5 dB. The user activity factor is 0.4 and noise is not considered.

Solution

When a single service with constant rate is used the capacity in number of users is obtained from the expression for E_b/I_o

$$\frac{G}{\rho(1 + f)(N - 1)} = \frac{E_b}{I_o}$$

Solve for N

$$N = \frac{G}{(1 + f)\rho \frac{E_b}{I_o}} + 1 = \frac{3840/15}{(1 + 0)0.4 \cdot 10^{5/10}} + 1 = 203.4 \rightarrow 203$$

4. Repeat the calculation for a multicell system, when the other to own cell interference ratio is 0.6.

Solution

Observing the other cell to own cell interference ratio we get

$$N = \frac{3840/15}{(1 + 0.6)0.4 \cdot 10^{5/10}} + 1 = 127.5 \rightarrow 127$$

5. How many users received with 10 times higher power can exist in a cell before the total number of users is halved? Do the estimation both for a single-cell network and a multi-cell network.

Solution

Assuming M users with 10 times higher received power the basic expression is

$$\frac{G}{\rho(1 + f)((N - M - 1) + 10M)} = \frac{E_b}{N_o}$$

From this we can solve for M

$$M = \frac{1}{9} \left(\frac{G}{\rho(1 + f) \frac{E_b}{N_o}} - N + 1 \right)$$

Note that now N has the value half of the values calculated in two previous tasks. In single cell network we have

$$M = \frac{1}{9} \left(\frac{G}{0.4(1 + 0)10^{0.5}} - 203/2 + 1 \right) = 11.4 \rightarrow 11$$

and in multicell network we have

$$M = \frac{1}{9} \left(\frac{G}{0.4(1 + 0.6)10^{0.5}} - 203/2 + 1 \right) = 7.11 \rightarrow 7$$

2 OFDM

Orthogonal frequency division multiplexing is a special case of multicarrier transmission method. Multicarrier transmission means that the assigned frequency band is divided into subchannels with subcarriers and each subcarrier is modulated by lower rate data streams. Modulation methods used are QAM and PSK. These lower rate data streams are then simultaneously transmitted over a number of subcarriers resulting in a high speed data transmission.

The carrier orthogonality is achieved by spacing subcarriers in frequency domain at integer multiples of symbol frequency of a single subcarrier. Fig. 1 demonstrates orthogonal carriers in frequency domain. It can be seen from the figure that the carriers have zeros at integer multiples of symbol frequencies

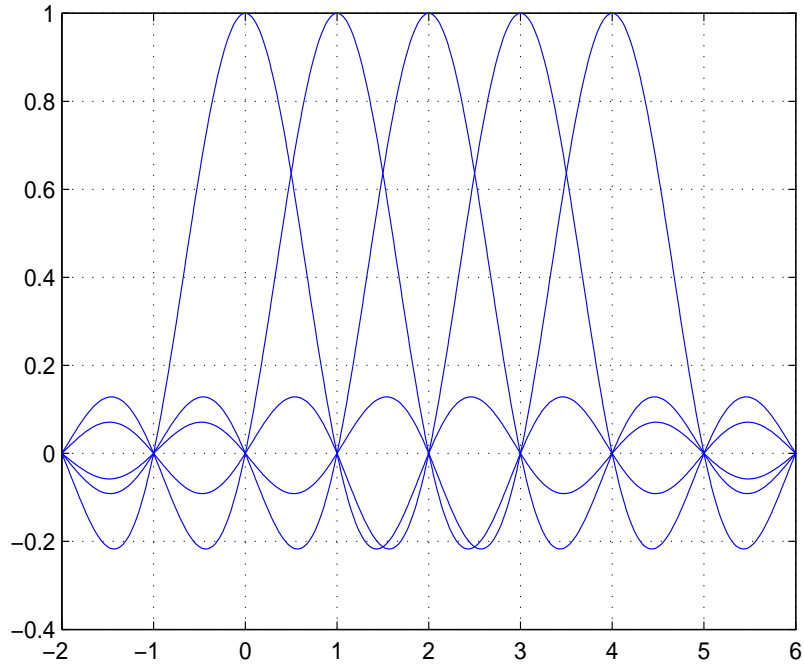


Figure 1: Orthogonal subcarriers

which means that the other carriers are zero when one carrier takes on its maximum value.

The complex, frequency domain (QAM/PSK) symbols at the transmitter are denoted as $A_k, k = 0, \dots, N - 1$, where N is the number of carriers. To get the time domain samples of the signal an IFFT is made

$$s_n^m = \frac{1}{N} \sum_{k=0}^{N-1} A_k e^{jnk2\pi/N}$$

1. Show how adding cyclic prefix transforms the linear convolution with the channel into a cyclic one.

Solution

After IFFT the cyclic prefix (CP) is added to the signal. If the CP length is L you take L samples from the end of the signal $[s_{N-L} \dots s_N]$ and place it in the beginning of the signal. Then the signal that can be described as length $P = N + L$ vector \mathbf{s}^m which is then convolved with the multipath channel with L taps.

The convolution can be written in terms of matrix multiplication $\mathbf{x} = \mathbf{H}\mathbf{s}$ where the L tap channel h is put into a $P \times P$ matrix \mathbf{H} as

$$\mathbf{H} = \begin{pmatrix} h(0) & 0 & 0 & \dots & 0 \\ \vdots & h(0) & 0 & \dots & 0 \\ h(L) & \dots & \ddots & \dots & \vdots \\ \vdots & \ddots & \dots & \ddots & 0 \\ 0 & \dots & h(L) & \dots & h(0) \end{pmatrix}$$

At the received the CP is removed from \mathbf{x} before the FFT. This procedure transforms the linear convolution into cyclic (or circular) convolution as can be seen from the following.

$$\begin{pmatrix} x(N) \\ \vdots \\ x(N-L) \\ x(1) \\ \vdots \\ x(N) \end{pmatrix} = \begin{pmatrix} h(0) & 0 & 0 & 0 & \dots & 0 \\ h(1) & h(0) & 0 & 0 & \dots & 0 \\ \vdots & h(1) & h(0) & 0 & \dots & 0 \\ h(L) & \dots & \ddots & \ddots & \dots & \vdots \\ \vdots & \ddots & \dots & \dots & \ddots & 0 \\ 0 & \dots & h(L) & h(L-1) & \dots & h(0) \end{pmatrix} \begin{pmatrix} s(N) \\ \vdots \\ s(N-L) \\ s(1) \\ \vdots \\ s(N) \end{pmatrix}$$

Which is after CP removal equivalent to

$$\begin{pmatrix} x(1) \\ \vdots \\ x(N) \end{pmatrix} = \begin{pmatrix} h(0) & 0 & h(L) & \dots & h(1) \\ \vdots & h(0) & 0 & \ddots & \vdots \\ h(L) & \dots & \ddots & \dots & h(L) \\ \vdots & \ddots & \dots & \ddots & 0 \\ 0 & \dots & h(L) & \dots & h(0) \end{pmatrix} \begin{pmatrix} s(1) \\ \vdots \\ s(N) \end{pmatrix} = \mathbf{H}_{eq}\mathbf{s}$$

where \mathbf{H}_{eq} is the $N \times N$ equivalent channel matrix that is circulant.

2. By using FFT matrices show how CP-OFDM transforms the frequency selective multipath channel into N flat fading narrowband channels.

Solution

An FFT matrix is an (unitary Vandermonde) $N \times N$ matrix \mathbf{F} with entries $\frac{1}{\sqrt{N}}e^{-j2\pi nk/N}$ and the IFFT counterpart is \mathbf{F}' . The circulant channel matrix \mathbf{H}_{eq} may be diagonalized by the FFT matrices because every circulant matrix is diagonalized by FFT matrices.

$$\mathbf{H}_{eq} = \mathbf{F}'\mathbf{D}\mathbf{F}$$

where \mathbf{D} has the channel frequency domain components on its diagonal.

$$\mathbf{D} =: [H(j0)H(j2\pi\frac{1}{N}), \dots, H(j2\pi\frac{N-1}{N})] = \mathbf{F}\mathbf{H}_{eq}\mathbf{F}'.$$

The absolute values of the diagonal elements are the singular values of the channel and the squares of those are naturally the eigenvalues which tell the channel gains per tone.

The received estimates \hat{A}_k (in Y) of the transmitted symbols A_k (in X) are given by the signal model

$$Y = \mathbf{F}\mathbf{H}_{eq}\mathbf{F}'X + \mathbf{F}n = \mathbf{D}X + \mathbf{F}n$$

From there it can be seen that the multipath channel has been transformed into N flat fading channels.

The division of input data stream to N subcarriers enlarges the symbol duration by N . Keeping in mind the relationship between delay spread and coherence bandwidth it can be understood that, in case of ISI reduction, it would be preferable to have long symbols compared to delay spread. The shorter the length L CP is compared to the symbol duration the more efficient the transmission becomes because CP is redundant information.

3. Assume 4 subcarriers and a three tap channel. What is the SNR on each subcarrier after FFT? The channel has snapshot values $h(0) = -0.6033 - 0.0177i$, $h(1) = -0.2757 - 0.0323i$ and $h(2) = -0.0051 - 0.0044i$ and transmit SNR is 15dB. Assume block fading.

Solution

Length 4 FFT of the channel gives the following values

$$H(0) = -0.8841 - 0.0544j,$$

$$H(j\pi\frac{1}{4}) = -0.6305 + 0.2624j,$$

$$H(j\pi\frac{1}{2}) = -0.3327 + 0.0102j \text{ and}$$

$$H(j\pi\frac{3}{4}) = -0.5659 - 0.2890j$$

which give the channel gains for the subcarriers during this OFDM symbol 0.7846, 0.4664, 0.1108 and 0.4038 and in dB

-1.0536, -3.3126, -9.5549 and -3.9387 dB.

The resulting SNRs are

13.9, 11.69, 5.45 and 11.06dB.

As can be seen, there is a deep fade on the third subcarrier. This is the situation before the frequency domain equalizer. The frequency domain equalizer is very simple since the channel is flat fading on each carrier. The equalizer is simply \mathbf{D}^{-1} . Problems occur when there is null channel on some subcarrier. Then \mathbf{D} becomes singular and is not invertible.

4. Name and discuss two disadvantages of OFDM signal.

Solution

Sensitivity to carrier frequency offsets caused by Doppler shifts and carrier frequency mismatches between transmit and receive oscillators. The division of input data stream to N subcarriers enlarges the symbol duration by N . Enlarging the symbol duration exposes the signal to ICI due to relation between Doppler spread and coherence time of the channel. The coherence time is approximately the inverse of the Doppler spread and they describe the time selectivity of the channel. When the coherence time is short compared to the symbol duration the channel is fast fading and the channel varies during one OFDM symbol. The time variations of the channel results in spectral spreading from one carrier to another.

Another serious drawback of OFDM is the high peak-to-average power ratio (PAPR). Any multicarrier signal with a large number of subcarriers may have a high PAPR due to occasional constructive addition of subcarriers. High power peaks are problem at the transmitting amplifier because they either lie in the nonlinear range of the amplifier, resulting in a performance degradation at the receiver, or force the amplifier to work with large input back-off, meaning that the input power of the signal is reduced, which results in low power efficiency.

Note that also the possibly singular channel matrix is an disadvantage.

A transmission scheme that overcomes these disadvantages but still preserves the advantage of simple frequency domain equalization is single carrier block transmission with CP addition. It transmits blocks of QAM/PSK symbols with CP. It is not multicarrier thus it does not have problems with ICI and the PAPR is the PAPR of the modulation. The equivalent channel matrix becomes Toeplitz that does not have the rank deficient problem (it is always full rank). The performance of OFDM and single carrier block transmission with CP addition are similar. This scheme has been adopted to LTE uplink standard.