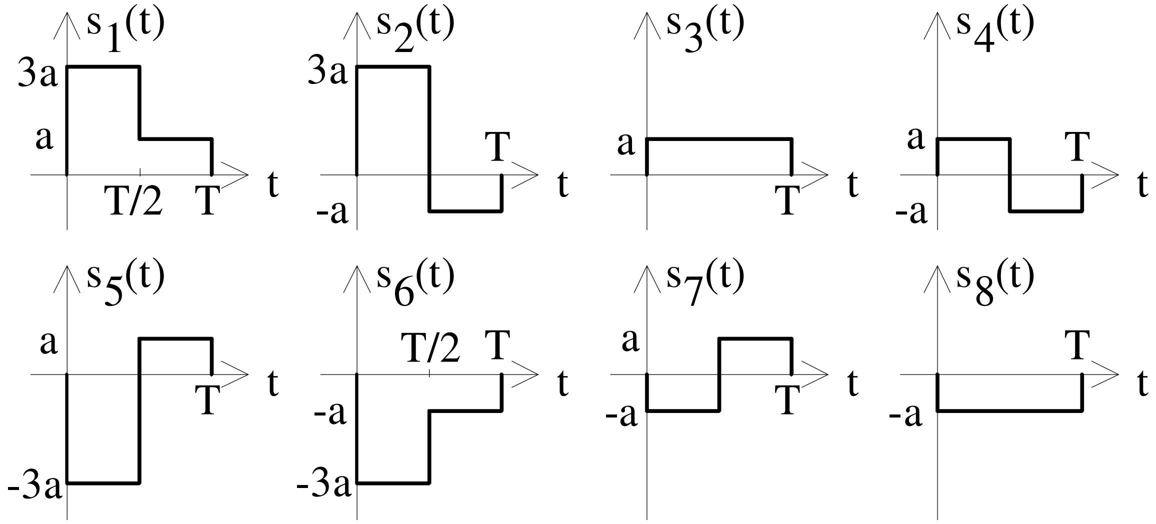
# S-72.2205 Digital Transmission Methods
## Homeworks 2007

**Submit your solution at latest on 13th December, 2007**

## Homework 1.




In the transmission of a symbol taking 8 different values, the pulse shapes shown in the above figure are used.

a) Select the minimum number (=2) of base function and determine and draw the constellation diagram including the receiver decision areas, when all symbol values have equal probability of occurrence.

b) Derive the expression of symbol error probability as a function of average $E_b/N_o$.

c) How many dB is this transmission system better than a bipolar 8PAM-system when the required average energy/bit for achieving the symbol error probability $10^{-6}$ are compared?

## Homework 2

A memoryless discrete source produces six symbols (A,B,C,D,E,F) with the occurrence probabilities P(A)=1/2, P(B)=1/4, P(C)=1/8, P(D)=P(E)=1/20, P(F)=1/40. The information of a symbol i according to Shannon's definition is

$$I = \log_2\left(\frac{1}{P(i)}\right)$$

a) Determine the information of the messages ABABBA and FDDFDF.

b) Compare the results with the average information of a 6-symbol message (=entropy).

## Homework 3

The bit error probability of differentially decoded BPSK in the AWGN-channel is $BEP = 0.5\exp\left(-E/N_o\right) = 0.5\exp\left(-\gamma\right)$.

a) Derive the average error probability expression in the slowly flat fading Rayleigh-channel, wher the probability density function of the signal to noise ratio is $p(\gamma) = \left[\exp\left(-\gamma/\gamma_m\right)/\gamma_m\right]\mathrm{u}(\gamma)$.

b) Calculate how many dB must the average signal to noise ratio $\gamma_m$ be increased due to Rayleigh-fading on the BEP-values $10^{-2}$ and $10^{-4}$.

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## Homework 4

In a radio communication system the received signal is slowly Rayleigh-fading and the disturbances are of AWGN-type. The performance target is to have signal level more than 10 dB 95 % of the time.

a) How large should the average signal to noise ratio (dB) be if no diversity is used?

b) In the receiver two-branch diversity with selection combining is used. The two branches fade independently and have the same average signal to noise ratio. How many dB can the average signal to noise requirement be lowered compared to the single branch receiver?

## Home work 5.

After selection combining the probability that the signal to noise ratio (SNR) drops below a given value with $M$ independent diversity branches is $P\{\gamma < \gamma\} = \prod_{k=1}^{M} \left(1 - \exp\left(-\gamma/\gamma_{m_k}\right)\right)$, where $\gamma_{m_k}$ is the average SNR of the $k^{\text{th}}$ diversity branch. Here two-branch diversity is investigated.

a) How large is the diversity improvement at 20 dB fade (i.e. the instantaneous SNR is 20 dB less than the average SNR, when the branches are equally strong in average?

b) The second diversity branch is in average 10 dB weaker than the first branch. How large is now the diversity improvement at 20 dB fade?

## Home work 6.

The p.d.f of the signal to noise ratio after M-fold diversity with maximum ratio combining is

$$p_c(\gamma) = \frac{1}{(M-1)!} \cdot \frac{\gamma^{M-1}}{\gamma_m^M} \cdot e^{-\gamma/\gamma_m}\,\mathrm{u}(\gamma)$$

a) Derive an expression for the average bit error probability of DPSK in the rapidly fading Rayleigh-channel when two-fold diversity is applied.

b) Calculate the diversity gain at the bit error probability level 0.001.

## Home work 7.

Compare selection combining and maximum ratio combining of two identically and independently Rayleigh distributed diversity branches in terms of diversity gain at the average bit error probability (BEP) levels $10^{-2}$ and $10^{-4}$ when the modulation method is differential BPSK, instantaneous BEP:

$P_b(\gamma) = 0.5\exp(-\gamma)$.