Review of channel coding methods

Channel coding

- Purpose: To improve the error performance in symbol transmission over noisy channels

- Coding schemes:

- Forward Error Correction (FEC) where controlled redundant information is added to the symbol stream. The redundant information can be utilized for error detection and error correction. No return link required
- Automatic Repeat-reQuest (ARQ) where the symbol frame is retransmitted if symbol errors are detected in the reception. Only error detection necessary. A return link is required. Introduces latency to the transmission.
- Hybrid ARQ combines ARQ and FEC. Return link necessary, lover latency than pure ARQ.

❑Coding methods:

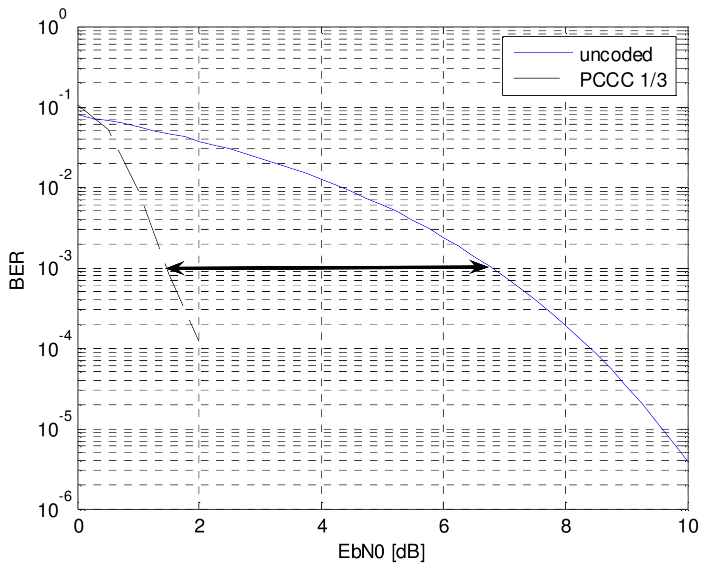
- Block coding where information symbols are segmented into frames to which redundant symbols are added to form code words.
- Convolutional coding where the redundancy is generated by feeding the original symbol stream through a shift register and the outputs corresponding to the used coding polynomial are multiplexed in time.
- Concatened codes with outer and inner coding, e.g. first block coding and the block coded symbol stream is then convolutionally coded.
- Code interleaving where the subsequent coded symbols are spread (interleaved) over several frames. Deinterleaving in the receiver will then spread error bursts in the transmitted stream and make them appear more randomly before channel decoding, thus preserving the code gain which otherwise suffers from error bursts
- Rate matching with puncturing (removal of bits from the encoded bit stream) or repetition (insertion of repeated bits from the encoded bit stream)

❑ Terminology

- Code rate = number of bit in/number of bits out
- Coding gain: the reduction in SNR due to coding to obtain a given performance level. Usually expressed with Eb/No-values, where Eb is the energy used for transmission of a information bit (energy in redundant bits included) No is the one-sided power spectrar density of white noise
- Hard decision decoding: The decoder operates with the symbol sequence produced by the channel demodulator
- Soft decision decoding: The decoder operates with more levels than the symbol levels and e.g. the probability of each such level. Applicable to convolutional decoding; enhances error performance, computationally more complex than hard decision decoding
- Code puncturing: rate matching by removing encoded symbols
- Code repetition: rate matching by unequal coded symbol repetition

Code performance measure

Coding gain is the measure in the difference between the signal to noise ratio (SNR) levels between the uncoded system and coded system required to reach the same bit error rate (BER) levels when used with the error correcting code (ECC). (from Wikipedia)



Coding gain for parallel concatenated turbo code rate 1/3 generator polynomial [5 7] 300 information bits. Gain at BER 10^-3 is about 5 dB.

Channel capacity limits in radio channels

Shannon capacity of the ideal memoryless AWGN-channel:

$$C = W\log_2\left(1+\gamma\right) = W\log_2\left(1+\frac{P_{rx}}{N_o W}\right)$$

C = channel capacity, (*bit/s*)
W = (one-sided) channel bandwidth, (*Hz*)
γ = average signal power to average noise power ratio, (*absolute value*)
P_{rx} = receiver average input power
N_o = one-sided white Gaussian noise power spectral density (*W/Hz*)

Shannon capacity of the flat fading Rayleigh-channel with AWGN-channel will vary randomly
The average (ergodic) capacity is

$$C_m = W\int_0^\infty p(\gamma)\log_2\left(1+\gamma\right)d\gamma = \frac{W}{\gamma_m \ln 2}\int_0^\infty e^{-\gamma/\gamma_m}\ln\left(1+\gamma\right)d\gamma$$

$$= \frac{W}{\gamma_m \ln 2} \cdot \gamma_m \, e^{1/\gamma_m} \, \mathrm{Ei}\left(-\frac{1}{\gamma_m}\right) = \frac{W}{\ln 2} \cdot e^{1/\gamma_m} \, \mathrm{Ei}\left(-\frac{1}{\gamma_m}\right)$$

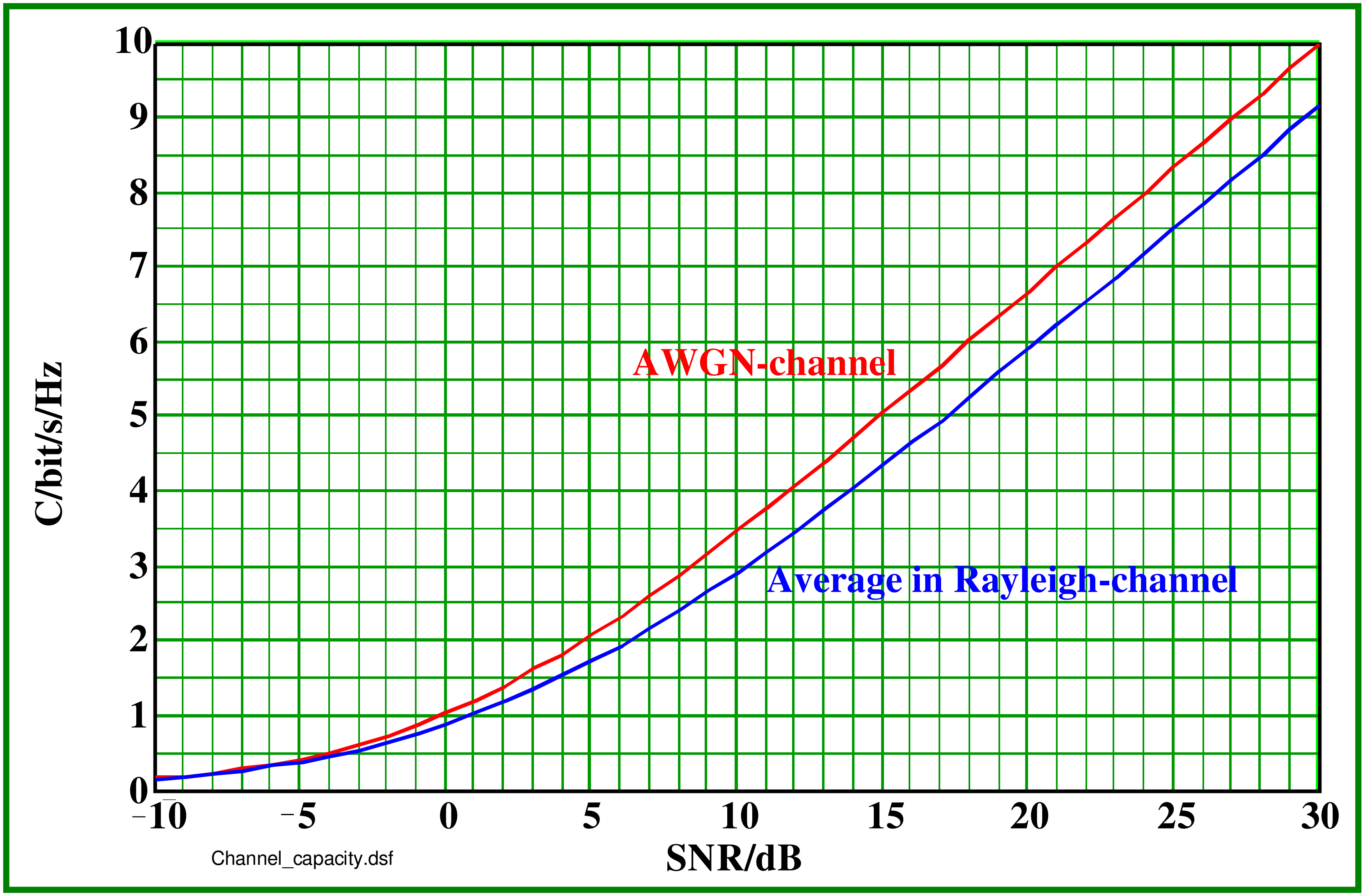
[Gradsteyn and Ryzik: 4.337.1]

The exponential integral is defined by

$$\mathrm{Ei}(x) = -\int_x^{\infty} \frac{e^{-x}}{x} dx = C + \ln x - x + \frac{1}{2}\frac{x^2}{2!} - \frac{1}{3}\frac{x^3}{3!} +$$

where Euler's constant is defined by

$$C = -\int_0^{\infty} e^{x} \ln x dx \cong 0.577216$$


10
9
8
7
6
5
4
3
2
1
0
C/bit/s/Hz
AWGN-channel
Average in Rayleigh-channel
−10
−5
0
5
10
15
20
25
30
SNR/dB
Channel_capacity.dsf

The concept of capacity distribution and capacity outage

In the fading radio channel the "instantaneous" capacity is a random variable C

Definition of capacity outage: $P\{\mathrm{C} \le C\}$, where C is a target capacity

In the flat-fading Rayleigh-channel $C = C(\gamma)$, and the probability distribution of the channel capacity can be obtained in the following way:

$$P\{\mathrm{C} \le C(\gamma)\} = P\{\gamma \le \gamma(C)\} = 1 - e^{-\gamma(C)/\gamma_m}$$

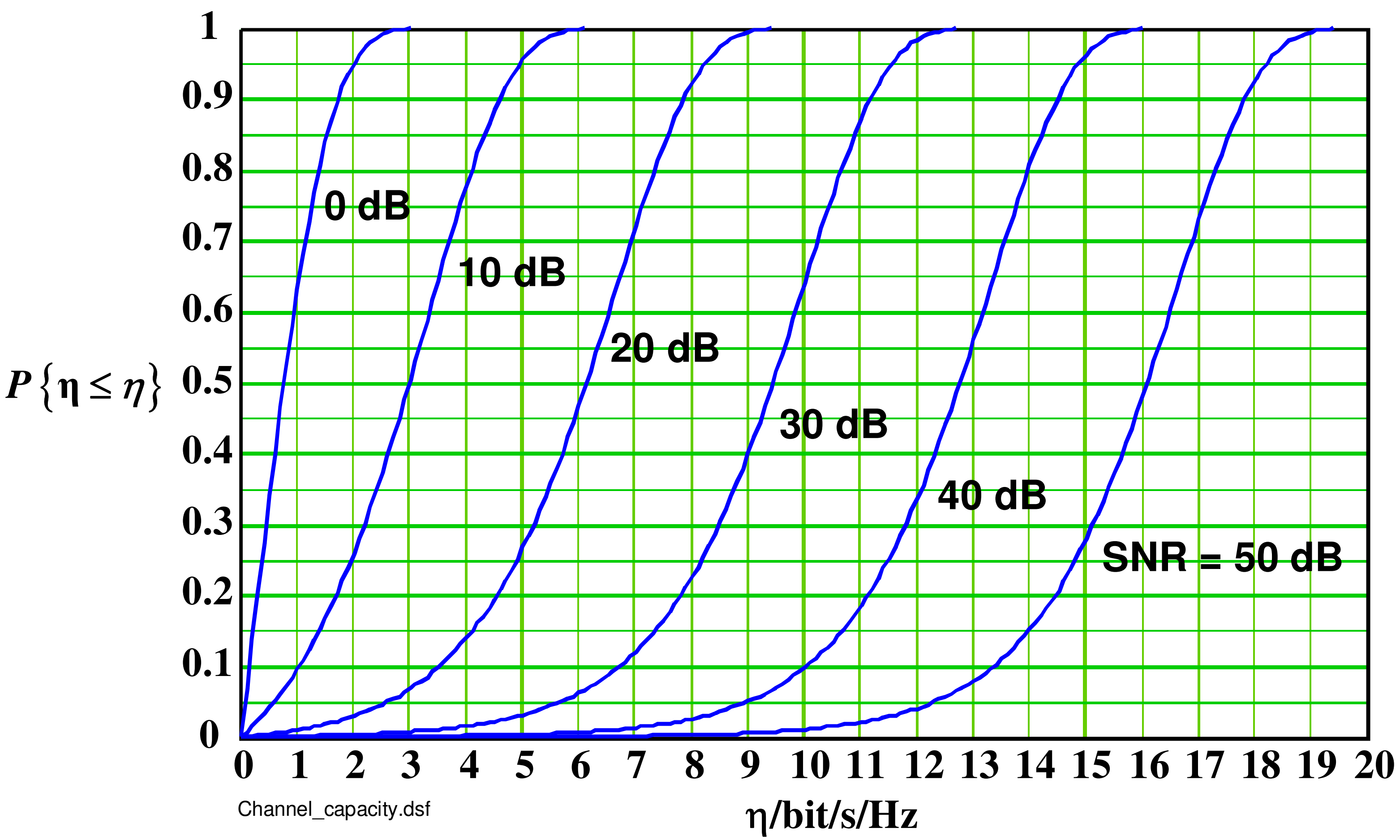
$$C = W \log_2(1+\gamma) \rightarrow \gamma(C) = 2^{\frac{C}{W}} - 1 \rightarrow \gamma(\eta) = 2^{\eta} - 1, \; \eta = C/W$$

$$\rightarrow P\{\eta \le \eta(\gamma)\} = 1 - \mathrm{e}^{-\gamma(\eta)/\gamma_m} = 1 - \exp\left(\frac{-\left(2^{\eta} - 1\right)}{\gamma_m}\right) = 1 - \exp\left(\frac{-\left(\exp(\eta \ln 2) - 1\right)}{\gamma_m}\right)$$

The probability density function is

$$\frac{d}{d\eta}P\{\mathtt{h} \le \eta(\gamma)\} = -\exp\left(\frac{1-\exp(\eta\ln 2)}{\gamma_m}\right)\left(-\exp(\eta\ln 2)\frac{\ln 2}{\gamma_m}\right)$$

$$= \frac{\ln 2}{\gamma_m}\exp\left(\frac{1-\exp(\eta\ln 2)}{\gamma_m} + \eta\ln 2\right)$$


1
0.9
0.8
0.7
0.6
0.5
0.4
0.3
0.2
0.1
0
$P\{\eta \le \eta\}$
0 dB
10 dB
20 dB
30 dB
40 dB
SNR = 50 dB
0 1 2 3 4 5 6 7 8 9 10 11 12 13 14 15 16 17 18 19 20
Channel_capacity.dsf
η/bit/s/Hz

Shannon capacity of the frequency selective AWGN-channel:

$$C = \int_{-W}^{W} \log_2\left(1 + \frac{\left|H_c(f)\right|^2 S_{tx}(f)}{N_o}\right) df$$

the average received power is given by

$$P_{rx} = \int_{-W}^{W} \left|H_c(f)\right|^2 S_{tx}(f) df$$

$H_c(f)$ = the complex low-pass transfer function of the radio channel
$S_{tx}(f)$ = the power spectral density of the transmitted complex low-pass signal

The capacity can be maximised by choosing the best possible transmit spectrum according to the channel power transfer function (water filling principle)

Assuming for simplicity an ideal complex low-pass transmit power spectral density

$$S_{tx}(f) = \frac{P_{tx}}{W} \cdot \mathrm{rect}\left(\frac{f}{W}\right)$$

the received power is

$$P_{rx} = \int_{-W/2}^{W/2} \left|H_c(f)\right|^2 \frac{P_{tx}}{W} df = \frac{P_{tx}}{W} \int_{-W/2}^{W/2} \left|H_c(f)\right|^2 df$$

This results in the following capacity expression:

$$C = \int_{-W/2}^{W/2} \log_2\left(1 + \frac{P_{rx}}{N_o} \frac{|H_c(f)|^2}{\int_{-W/2}^{W/2} |H_c(f)|^2 \, df}\right) df$$

Example

Assume a static two-path channel:

$$h_c(t) = h_1\delta(t) + h_2\delta(t-T)$$

$$\rightarrow H_c(f) = h_1 + h_2\,\mathrm{e}^{-j2\pi fT} = h_1\left(1 + \frac{h_2}{h_1}\mathrm{e}^{-j2\pi fT}\right)$$

$$= h_1\left(1 + \beta\,\mathrm{e}^{-j2\pi(f-f_o)T}\right)$$

$$\rightarrow \left|H_c(f)\right|^2 = \left|h_1\right|^2\left(1 + \beta^2 - 2\beta\cos\left(2\pi\left(f - f_o\right)T\right)\right)$$

$$\beta = \frac{|h_2|}{|h_1|}$$

$f_o = \dfrac{\arg\{h_2\} - \arg\{h_1\}}{2\pi T}$, the fade notch frequency giving a minimum phase response

In this case

$$\int_{-W/2}^{W/2} \left|H_c(f)\right|^2 df = \int_{-W/2}^{W/2} \left|h_1\right|^2 \left(1+\beta^2-2\beta\cos\left(2\pi\left(f-f_o\right)T\right)\right) df \sqrt{2}$$

$$=\left|h_1\right|^2 W\left(1+\beta^2-\beta\left(\frac{\sin\left(\pi\left(W-2f_o\right)T\right)}{\pi WT}+\frac{\sin\left(\pi\left(W+2f_o\right)T\right)}{\pi WT}\right)\right)$$

$$=\left|h_1\right|^2 W\left(1+\beta^2-2\beta\cos\left(2\pi f_o T\right)\mathrm{sinc}\left(WT\right)\right)=\left|h_1\right|^2 W\cdot \mathrm{f}\left(\beta, f_o\right)$$

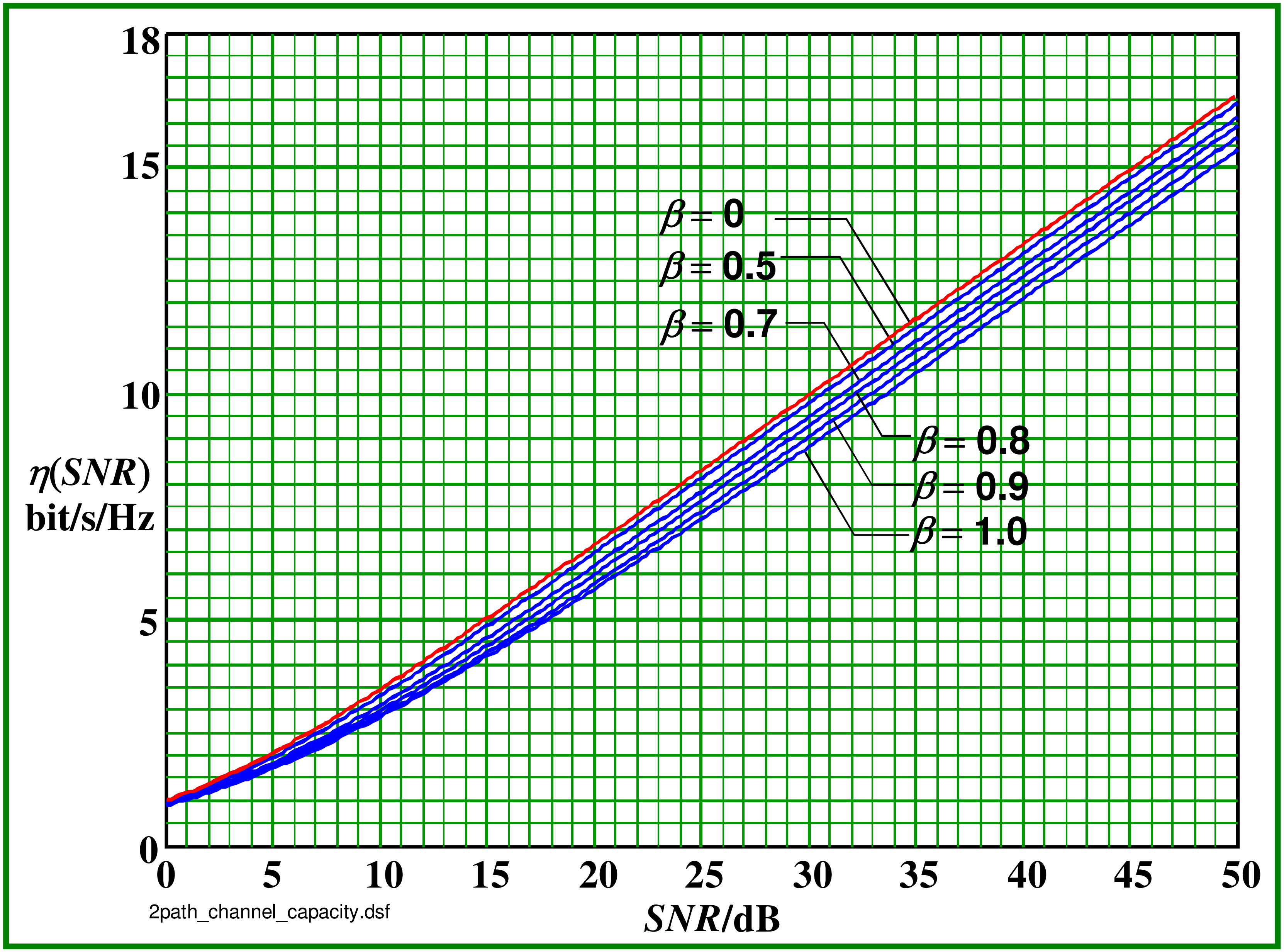
and the channel capacity is

$$C = \int_{-W/2}^{W/2} \log_2\left(1 + \frac{P_{rx}}{N_o}\frac{|h_1|^2\left(1+\beta^2-2\beta\cos\left(2\pi\left(f-f_o\right)T\right)\right)}{|h_1|^2 W\cdot \mathrm{f}\left(\beta,f_o\right)}\right)df$$

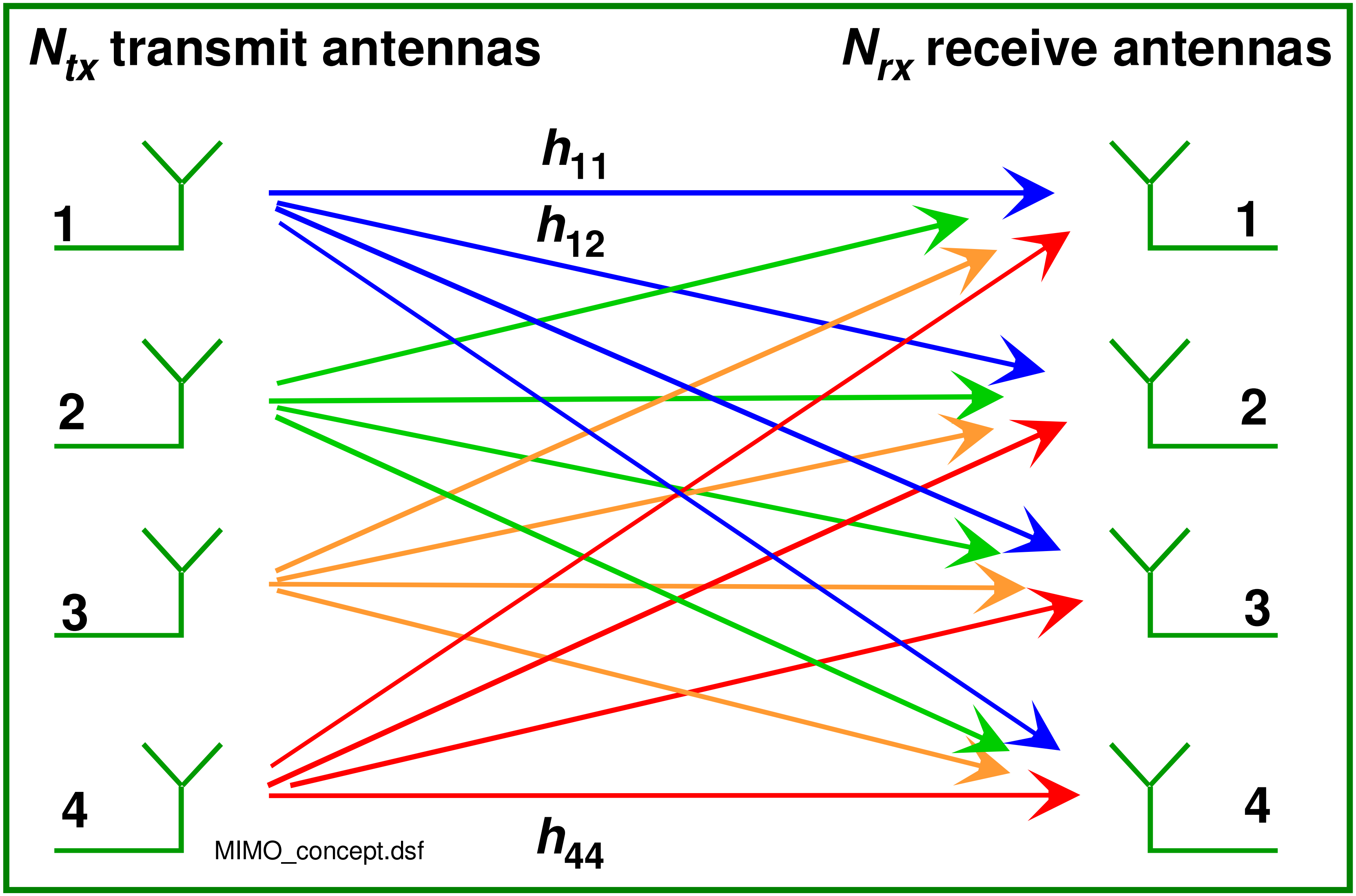
$$= \frac{1}{\ln 2}\int_{-W/2}^{W/2} \ln\left(1 + \frac{P_{rx}}{N_o W}\frac{\left(1+\beta^2-2\beta\cos\left(2\pi\left(f-f_o\right)T\right)\right)}{\mathrm{f}\left(\beta,f_o\right)}\right)df$$

When $x = 2\pi f T$, $\phi_o = 2\pi f_o T$, and $\gamma = \frac{P_{rx}}{N_o W}$, the capacity can be written as

$$C = \frac{W}{2\pi WT\ln 2}\int_{-\pi WT}^{\pi WT} \ln\left(1 + \gamma\frac{\left(1+\beta^2-2\beta\cos\left(x-\phi_o\right)\right)}{\left(1+\beta^2-2\beta\cos\left(\phi_o\right)\mathrm{sinc}\left(WT\right)\right)}\right)dx$$


18
15
10
5
0
η(SNR)
bit/s/Hz
β = 0
β = 0.5
β = 0.7
β = 0.8
β = 0.9
β = 1.0
0
5
10
15
20
25
30
35
40
45
50
SNR/dB
2path_channel_capacity.dsf

Channel capacity of Multiple Input Multiple Output (MIMO) Antenna Systems



Defining the channel gain matrix as:

$$H = \begin{bmatrix} h_{11}(t) & h_{12}(t) & \ldots & \ldots & h_{1N_{tx}}(t) \\ h_{21}(t) & h_{22}(t) & \ldots & \ldots & h_{2N_{tx}}(t) \\ \ldots & \ldots & \ldots & \ldots & \ldots \\ \ldots & \ldots & \ldots & \ldots & \ldots \\ h_{N_{rx}1}(t) & h_{N_{rx}2}(t) & \ldots & \ldots & h_{N_{rx}N_{rx}}(t) \end{bmatrix}$$

gives the ergodic average MIMO-channel capacity:

$$C = W \cdot \mathrm{E}\left\{ \log_2\left(\det\left(I_{N_{rx}} + \frac{\gamma}{N_{tx}} HH^{*T} \right)\right)\right\}$$