# Channel convolution, ISI and Pulse Shaping

## Band-limited channels

- The channel may be band-limited
  - due to channel characteristics
    - effect: signal is distorted in frequencies not supported by the channel
  - due to wish of effective bandwith utilization
    - dividing frequency into nearly orthogonal parts – i.e. parts that can be separately used
    - effect: out-of band emissions are not desired

## Rectangular pulse

- Simplest transmission pulse: rectangular
- large out-of band emissions
  - more involved pulse shaping methods are needed
- Simplest receiver orthogonal waveform: rectangular
  - orthogonality guarantees AWGN in signal space
  - more involved receiver pulses may be used, depending on transmission pulse & need to suppress out-of band interference

## Transmitted Base band signal

- Follow path of individual symbol from transmitter, through air interface to base band receiver.

Notation:

- Transmitter pulse shaping filter $f_{\mathrm{t}}(t)$
- Receiver (pulse shaping) filter $f_{\mathrm{r}}(t)$
- channel impulse response $c(t)$
- symbol interval $T$

Transmitted base band (equivalent low-pass) signal,
independent symbols transmitted with interval T using pulse shape $f_{\mathrm{t}}$

$$x(t) = \sqrt{P} \sum_m x_m \ f_{\mathrm{t}}(t - mT)$$

- transmit power $P$, symbols $x_m$ normalized to unit power
- transmit (and receive) filter should have compact support - causality

## Received base band signal

$$y_b(t) = \int_{-\infty}^{\infty} d\tau \; x(t-\tau)\; c(\tau) + n(t) = (x * c)(t) + n(t)$$

- Received BB signal is a convolution of channel and transmitted signal
- at each time instant
  - received signal sum of multiple delayed copies of transmitted signals
  - delays caused by channel impulse response
- channel impulse response is causal
- propagation time difference absorbed into definition of c(t)

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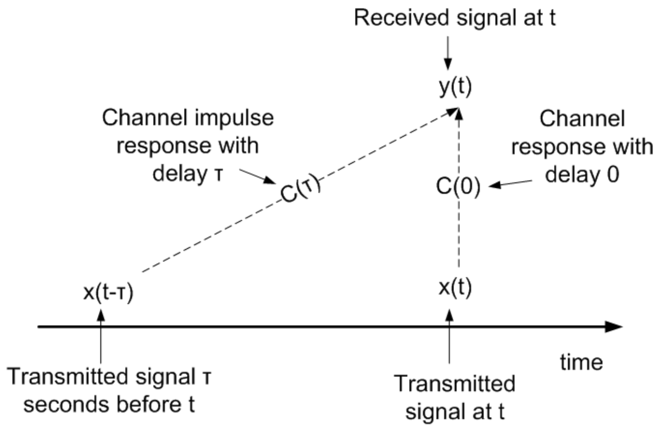
## Channel convolution



Received signal at t

y(t)

Channel impulse response with delay τ

C(τ)

Channel response with delay 0

C(0)

x(t-τ)

x(t)

time

Transmitted signal τ seconds before t

Transmitted signal at t

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## AWGN process

- AWGN process $n(t)$ has the auto-correlation

$$\mathrm{E}\left\{n(t)\; n(t')^*\right\} = \mathrm{N}_0\; \delta(t-t')$$

  - $\delta(t-t')$ is the Dirac delta-function:

$$\int_{-\infty}^{\infty} dt\; f(t)\; \delta(t-t') = f(t')$$

- below, limits of integrals generally dropped

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## Example: Flat fading (single-tap) channel

- channel impulse response

$$c(t) = \delta(t-d)$$

  - $d$ is propagation delay between transmitter and receiver
- received signal becomes

$$y_b(t) = x(t-d) + n(t)$$

  - delayed copy of transmitted signal

## Receive (pulse shaping) filtered signal

- received signal after filtering

$$\begin{aligned} y(t) &= \int d\tau'\; y_b(t-\tau') f_{\mathrm{r}}(\tau') \\ &= \int d\tau \underbrace{\int d\tau'\; f_{\mathrm{r}}(\tau')\; x(t-\tau'-\tau)}_{\text{Rx filtered Tx signal } x_{\mathrm{r}}(t-\tau)}\; c(\tau) + \underbrace{\int d\tau'\; n(t-\tau') f_{\mathrm{r}}(\tau')}_{\text{Rx filtered noise } n_{\mathrm{r}}(t)} \end{aligned}$$

- the part due to Rx filtered Tx signal:

$$x_{\mathrm{r}}(t) = \sqrt{P} \sum_m x_m \int d\tau'\; f_{\mathrm{r}}(\tau')\; f_{\mathrm{t}}(t-mT-\tau') \equiv \sqrt{P} \sum_m x_m\; f(t-mT)$$

  - convolution of the Rx and Tx filters: combined Rx-Tx pulse shaping filter

$$f(t) = (f_{\mathrm{r}} * f_{\mathrm{t}})\,(t) = \int d\tau'\; f_{\mathrm{r}}(\tau')\; f_{\mathrm{t}}(t-\tau')$$

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## Receive (pulse shaping) filtered signal II

- The received signal after pulse shaping is

$$y(t) = \sum_m x_m \sqrt{P} \int d\tau\; f(t-mT-\tau)\; c(\tau) + n_{\mathrm{r}}(t) \equiv \sum_m x_m\; h(t-mT) + n_{\mathrm{r}}(t)$$

- convolution of channel impulse response and combined Tx-Rx pulse shaping filters is

$$h(t) = \sqrt{P}\;\; (f_{\mathrm{t}} * f_{\mathrm{r}} * c)\,(t) = \sqrt{P} \int d\tau \int d\tau'\; f_{\mathrm{t}}(t-\tau-\tau')\; f_{\mathrm{r}}(\tau')\; c(\tau)$$

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## Sampling

- After Rx filtering, signal is sampled:

$$\begin{aligned} y_k &= y(kT'+\Delta) = \sum_m x_m\; h(kT'-mT+\Delta) \;+\; n_{\mathrm{r}}(kT'+\Delta) \\ &= \sum_m h_{km}\; x_m \;+\; n_k \end{aligned}$$

- sampling interval $T' \leq T$
  - symbol-spaced Tapped Delay Line: $T' = T$
  - fractionally spaced TDL: $T' < T$
- sampling instances $kT' + \Delta$,
  - $\Delta$ reflects the insecurity in timing
- channel coefficients $h_{km}$
- noise samples $n_k$

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## Symbol-spaced sampling

- One received sample per transmitted symbol

$$y_k = h_{kk} x_k + \underbrace{\sum_{m \neq k} h_{km}\; x_m}_{\text{ISI}} \;+\; n_k$$

- if $h_{km} \neq h\delta_{km}$, there is Inter-Symbol-Interference (ISI)
- With symbol-spaced sampling, the channel taps are

$$h_{km} = \sqrt{P} \int d\tau\; f\Big((k-m)T + \Delta - \tau\Big)\; c(\tau)$$

- In flat fading $h_{km} = \sqrt{P}\; f\Big((k-m)T + \Delta - d\Big)$
- Nyqvist criterion: $f\Big((k-m)T + \Delta - d\Big) = \delta_{km}$

## Tx-Rx filter design

- The combined Tx & Rx filters should be designed to minimize ISI
- The Tx filter should be designed to optimize bandwidth usage
- If $f_t$ is rectangular pulse, there is no ISI
  - ⇒ extensive out-of band emissions at Tx
  - ⇒ pulse shaping at Tx is required
- family of pulse shapes f
  satisfying Nyqvist criterion:
  Raised Cosine pulses

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## Raised Cosine (RC) pulse shaping

$$f_{\mathrm{RC}}(t) = \frac{\sin(\pi t/T)}{\pi t/T} \cdot \frac{\cos(\alpha \pi t/T)}{1-(2\pi t/T)^2}$$

- ★ If $f$ is a Raised Cosine pulse, and $\Delta = d$
  - ▶ no ISI
  - ▶ $h_{km} = \sqrt{P}\delta_{km}$.
- ★ vulnerability to sampling inaccuracy
- ★ When non roll-off $(\alpha = 0)$ $f$ is sinc-pulse
  - ▶ ISI with non-exact timing infinite
- ★ With roll-off $(\alpha > 0)$,
  - ▶ RC tolerates some timing inaccuracy ("eye open").

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## Bandwidth usage of Raised Cosine Puleses

- sinc-pulse: "brick-wall" spectrum
- Raised Cosine:
  - rolloff determines excess bandwidth

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## Distributing combined waveform between Tx and Rx

- Tx:
  - we are interested in transmitting a band-limited signal
- Rx:
  - we are interested in expanding the received signal in an orthogonal basis
    - noise is white
  - We may be interested to have a band-limited Rx filter
    - to filter out out-of band interference
  - We may be interested in collecting all of the transmitted power per symbols
- Combined waveform should fulfill Nyqvist criterion
  - Examples:
    - RC at Tx, Rectangular pulse at Rx
      - Rx filter has wide frequency response
      - power used to transmit symbol outside of main symbol period is discarded
    - Rectangular at Tx, RC at Rx
      - Tx filter casuses excessive out-of-band emissions
    - Root-raised cosine (RRC) at Tx and Rx
      - limited out-of band emissions, fnite Rx frequency response

## Root-Raised Cosine (RRC)

★ noise power at Rx is minimized if $f$ distributed evenly between Tx and Rx: root-raised cosine $f_{\mathrm{t}} = f_{\mathrm{r}} = f_{\mathrm{RRC}}$

$$\begin{aligned} f_{\mathrm{RRC}} &= \frac{\sqrt{1/T}}{1-(4\alpha t/T)^2}\left(\frac{\sin(\pi(1-\alpha)t/T)}{\pi t/T} + \frac{4\alpha}{\pi}\cos(\pi(1+\alpha)t/T)\right) \\ f_{\mathrm{RRC}} &* f_{\mathrm{RRC}} = f_{\mathrm{RC}} \end{aligned}$$

- RRCs with delay kT are orthogonal functions
- with RRC at Tx \& Rx
  - matched filter:
    - receiver matched to orthogonal Tx waveform
      - collects all transmitted power per symbol
      - no noise colouring (see two slides ahead)

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## Noise sample covariance

★ sampled, Rx-filtered noise process

$$n_k = n_{\mathrm{r}}(kT+\Delta) = (n * f_{\mathrm{r}})\,(kT+\Delta)$$

★ noise covariance

$$\begin{aligned} \mathrm{E}\left\{n_m n_k^*\right\} &= \int d\tau \int d\tau'\; f_{\mathrm{r}}(\tau)\; f_{\mathrm{r}}(\tau')\; \underbrace{\mathrm{E}\left\{n(mT+\Delta-\tau)\; n^*(kT+\Delta-\tau')\right\}}_{=\mathrm{N}_0\;\delta\left((m-k)T\right)} \\ &= \mathrm{N}_0\,(f_{\mathrm{r}} * f_{\mathrm{r}}^*)\left((m-k)T\right) \end{aligned}$$

★ If $f_{\mathrm{r}}$ is RRC, we have

$$\mathrm{E}\left\{n_m n_k^*\right\} = \mathrm{N}_0\;\delta_{mk}$$

- RRC filtering at Rx does not color the noise

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## Optimal filtering in flat fading

- In frequency flat channel, RRC filtering at Tx and Rx is optimum
  - removes ISI
  - maximizes SNR at the output of the sampler
  - does not color noise
  - consequence of RRC orthogonality
- If some other Rx filter is used, one may have to whiten the noise before further processing.

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## Fractional spaced sampling

- Symbol-spaced sampling works when the receiver knows the sampling time perfectly.
- If imperfect or random sampling timing
  - oversampling (at least Nyqvist rate) required to fully be able to reproduce the band-limited signal.
  - the additional samples of Nyqvist sampling compared to symbol-spaced sampling can be understood to be required to estimate transmitter pulse timing
- receivers for oversampled signals: fractional spaced equalizers
  - not discussed here

## Pulse shaping, summary

- sinc pulse (α=0)
  - brick wall spectrum utilization
    - non-implementable
  - no ISI (pulse = 0 at other sampling instances)
  - slow decay in time (1/t)
    - in case of non-exact sampling, ISI decays slowly
      - or in case of non-constant frequency response
  - non-causal (pulse starts at t=-infinity)
    - in practice pulse has to be started at a cut-off time
    - this causes out-of band emissions
- raised cosine pulse
  - roll-off factor α >0
    - excess bandwidth = tradeoff between excess bandwidth and ISI and out-of band emissions due to slower decay
- root-raised cosine
  - spectrum is square root of raised cosine spectrum
  - root raised cosine at Rx: orthogonal waveforms

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## S-72.2205 Digital Transmission Methods

# Continuous Phase Modulation

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## Problems with linear modulation methods

- linear modulation methods (QAM, PSK) have high out-of band emissions
  - caused by discontinuities in phase/amplitude
  - these are cured by applying suitable pulse shaping
    - some bandwidth has to be sacrificed to keep out-of band emissions in control
- Linear modulations have high peak-to-average power ratios (PAR)
  - Require higher back-off in PA, shorter MS battery life time
  - Caused by amplitude changes in QAM with M>4
  - Caused by transitions between constellation points in opposite quadrants
  - Especially transitions to opposite points are problematic
    - Transition goes through “0”
    - These are cured by ”offsetting” the constellation

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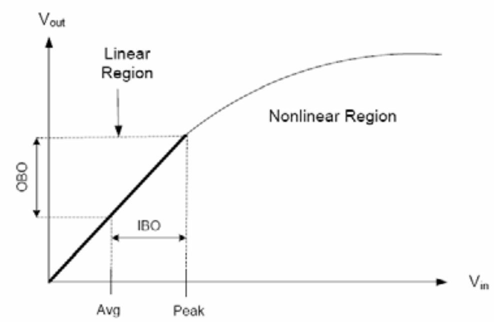
## Power Amplifier (PA) Characteristics



- To avoid distortion of the output signal, the average input power should be such that the peak power is in the linearity region of the
- PA power consumption is defined by the maximum output power
- Outout Back Off (OBO) induced by PAR means wasted power

# CPM

- **Continuous Phase Modulation**
  - **constant amplitude modulations where bits stored in continuous changes of the phase**
- **discontinuities in phase removed**
- **discontinuities in derivative of phase may be removed as well**
  - **significant decrease in out-of band emissions**
  - **low backoff, high PA efficiency in MS transmitter**
- **downside:**
  - **non-linear modulation**
    - **joint detection of bits (MLSE)**
  - **only phase used to store information**
    - **bad minimum distance properties for higher order modulation**

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# CPM II

$$s(t)=\mathrm{Re}\left\{\sqrt{2P_{Tx}}\,e^{j2\pi f_c t}\,e^{j2\pi h\sum_{k=-\infty}^{\infty}a_k x(t-kT)}\right\}$$

- $a_k=\pm1,\pm3,\ldots,\pm(M-1)$
- **x(t) is the phase waveform**
- **x'(t) is the frequency pulse**
- ***h* is the modulation index**

**Minimum Shift Keying**

- **binary (M=2), rectangular pulse of length T, h=1/2**

$$x'(t)=\frac{1}{2T}rect\left(\frac{t-T/2}{T}\right)$$

- **a continuous-phase Frequency shift keying**
- **in one symbol period T, the signal space constellation shifts 90 degrees (i.e. from one QPSK point to one of its neighbors)**

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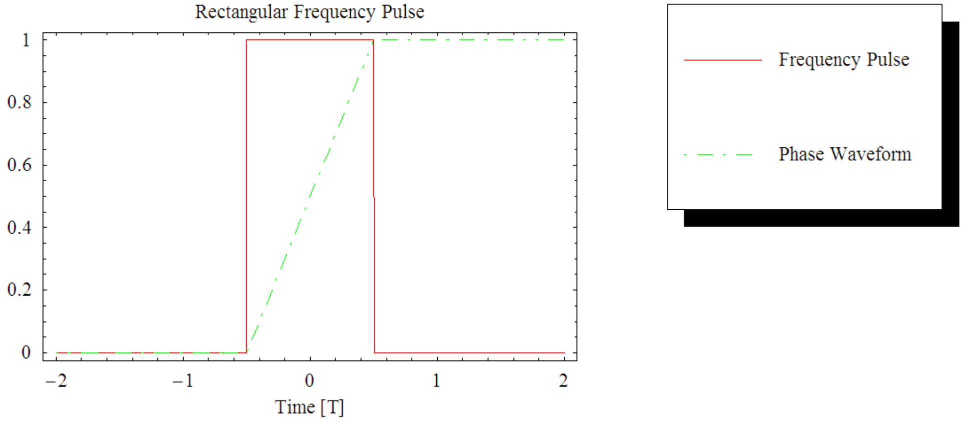
# CPM III

- **The phase wave form is the integral of the frequency pulse.**
- **The phase changes its level according to the modulation index h during a frequency pulse**



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# GMSK

- **Gaussian Minimum Shift Keying**
  - **Frequency pulse is a convoluted rectangular and Gaussian pulse**
  - **bandwidth parameter *BT***
  - ○

$$g(t)=x'(t)=rect\left(\frac{t-0,5T}{T}\right)*\frac{2\pi B}{\sqrt{2\pi\ln 2}}\exp\left(-\frac{4\pi^2B^2t^2}{\ln 2}\right)$$

$$=\frac{1}{2T}\left[Q\left(\frac{2\pi B(t-T)}{\sqrt{\ln 2}}\right)-Q\left(\frac{2\pi Bt}{\sqrt{\ln 2}}\right)\right]$$

$$x(t)=\int_{-\infty}^{t}\frac{1}{2T}\left[Q\left(\frac{2\pi B(u-T)}{\sqrt{\ln 2}}\right)-Q\left(\frac{2\pi Bu}{\sqrt{\ln 2}}\right)\right]du$$

- The smaller BT is, the smoother x(t) is
  - More bits interfere with each other
  - Less bandwidth occupied (smaller sidelobes)

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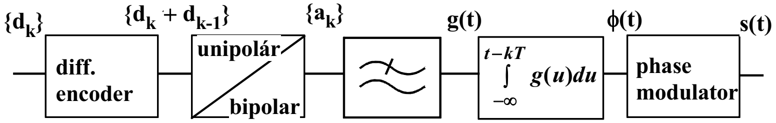
## GSM-modulation, Gaussian Minimum Shift Keying, GMSK



**frequency pulse**

$$g(t)=\frac{\exp\left(-t^2/2\delta^2T^2\right)}{\sqrt{2\pi}\delta T}\otimes \mathrm{rect}\left(\frac{t}{T}\right)=\mathrm{Q}\left(\frac{t-T/2}{\delta T}\right)-\mathrm{Q}\left(\frac{t+T/2}{\delta T}\right)$$

$$\delta=\frac{\sqrt{\ln 2}}{2\pi BT} \qquad BT=0.3$$

**carrier phase** $\varphi(t)=\sum_k a_k\pi h\int_{-\infty}^{t-kT} g(u)du$ $h=$ modulation index $=0.5$

**transmitted signal** $s(t)=\sqrt{\frac{2E_c}{T}}\cos\left(2\pi f_c t+\varphi(t)+\varphi_o\right)$

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## (G)MSK Performance in AWGN

$$P_b=Q\left(d_{\min}\sqrt{\gamma}\right); \qquad \gamma=\frac{P_{Rx}}{N_oR}$$

- MSK: $d_{\min}^2=2$
  - Same probability of error as BPSK
- GMSK with GSM modulation parameters $d_{\min}^2=1{,}78$

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## Modulation

**Desirable characteristics of modulation for Mobile Communication Systems**

- Negligible out-of band emissions
  - To keep frequency orthogonal
  - Band-limited transmission
- High spectral efficiency
  - High number of bits transmitted per symbol period
  - efficient spectrum usage
    - efficient pulse shaping, small excess bandwidth
- Good error performance
  - Minimum distance

**Abrupt changes in the signal broaden the spectrum**

- Discontinuity in phase/amplitude
- Discontinuity in derivative of phase/amplitude

## Modulation Where and Why

- **2G (GSM): GMSK**
  - **Continuous phase: good PA efficiency**
  - **Pulse shaping: low out-of band emissions**
- **3G (CDMA): QPSK**
  - **Non-linear modulation non-compatible with the principle of DS-CDMA**
  - **Pulse shaping (channel selection filter) to cut out-of band emissions**
  - **Power efficiency compromised**
    - **In BS: multicode transmission ==> very high PAR**
    - **In MS: transitions through zero==> some PAR**
- **Wimax, Flash-OFDM: OFDM**
  - **Very bandwidth efficient**
    - **OFDM waveforms + channel selection filters**
  - **Power efficiency very much compromised**
    - **OFDM has very high PAR, both in UL & DL**
- **LTE: DL OFDM, UL: BPSK, QPSK, 16-QAM**
  - **Lower PAR and in UL, better PA efficiency**

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## S-72.2205 Digital Transmission Methods

# Fading Multipath Channel

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## Channel

- From perspective of digital baseband, channel consists of
  - effects of physical medium
  - effects of analog circuitry at Tx and Rx (e.g. RF)
- Such effects are
  - Delay
  - Attenuation
    - Large scale effects: caused by propagation distance
    - Short scale effects: caused by fading
  - Non-linear dispersion effects
    - Dispersion of power in time (multipath, echoes)
      - Causes frequency-selective fading
    - Dispersion of power in frequency (Doppler spread)
      - Causes time-selective fading
  - Erasure fading
    - Channel may vanish completely

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## Examples of Physical Channels

- Mobile radio channel
  - Time and frequency selective fading
- Cable (PSTN, ADSL)
  - Frequency selective, static
- Power line communication
  - Frequency and time selective fading
- Magnetic tape
  - Time and frequency selective fading
- Optical media (CD, DVD)
  - Erasure fading from scratches, frequency flat

# Channel Characterization

- properties of static channels (or instantaneous channel properties)
  - impulse response
    - the time domain response of an impulse
  - power delay profile
    - The absolute square of the impulse response
    - describes dispersion of signal power in time(multipath propagation)
  - frequency response
    - frequency domain response to white noise
    - describes frequency selective fading
- properties of dynamic channels
  - Doppler spread
    - Frequency domain response to sine wave
    - Describes dispersion of signal power in frequency

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# The Mobile Radio Channel

- Multipath progation
  - Each multipath component characterized by a delay, an attenuation, a phase shift
- Time selectivity due to mobile station motion

## Propagation mechanisms in the mobile radio channel

- Free space propagation
- Ground reflection
  - Interference between direct path and reflected path
- Reflection
- Diffraction
- Scattering
- Absorption
- Wall attenuation etc

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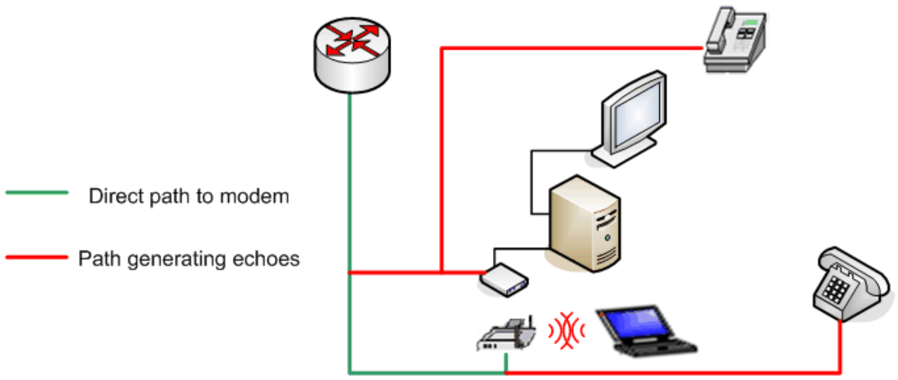
# Cable channel

- Multipath propagation due to branches in cable
  - Each multipath component characterized by a delay, an attenuation, a phase shift



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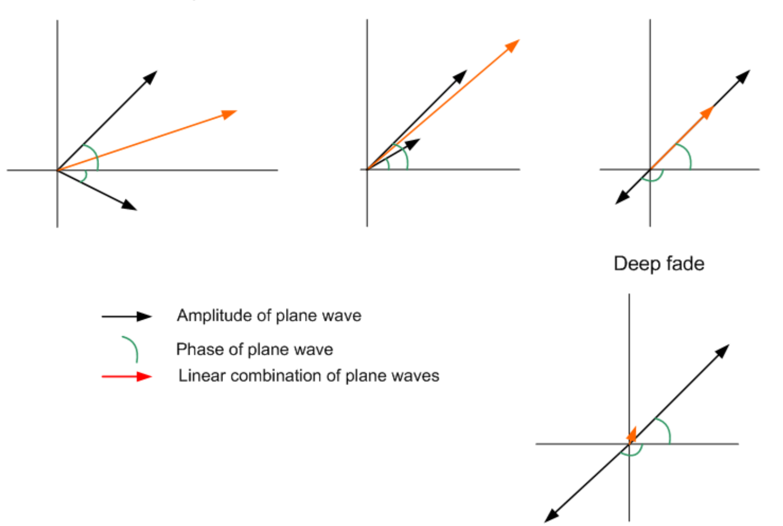
# Multipath propagation & fading

- Non-separable multipath components
  - Arrive virtually simultaneously
    - Path interarrival time << inverse sampling rate
  - From the perspective of base band, non-measurable
  - Give rise to fading
    - Combining multiple plane waves causes large changes in signal (channel) amplitude (and power)
    - Constructive and destructive interference
- Separable multipath components
    - Path interarrival time ~> inverse sampling rate
  - Can be measured
    - Separate fading characteristics & channel coefficient for each separable path
  - Cause inter-symbol-interference

# Destructive and Constructive Interference and Fading



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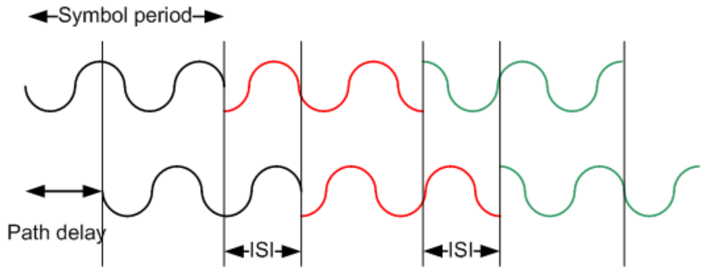
# Multipath Propagation and ISI

- Separable multipath components arrive at different delays
- Delayed copies of transmitted signal cause Inter-Symbol-Interference (ISI) at the receiver



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# Impulse Response and Frequency Response

- The frequency response is the Fourier transform of the Impulse response
- At a given frequency, a linear combination of multiple time domain paths with different delays is seen
- This causes frequency-selective fading
- Two sides of the same coin: multipath propagation induces
  - ○ Time dispersion
    ➔ Inter-Symbol Interference
  - ○ Frequency selectivity
    ➔ fading changes in the frequency domain

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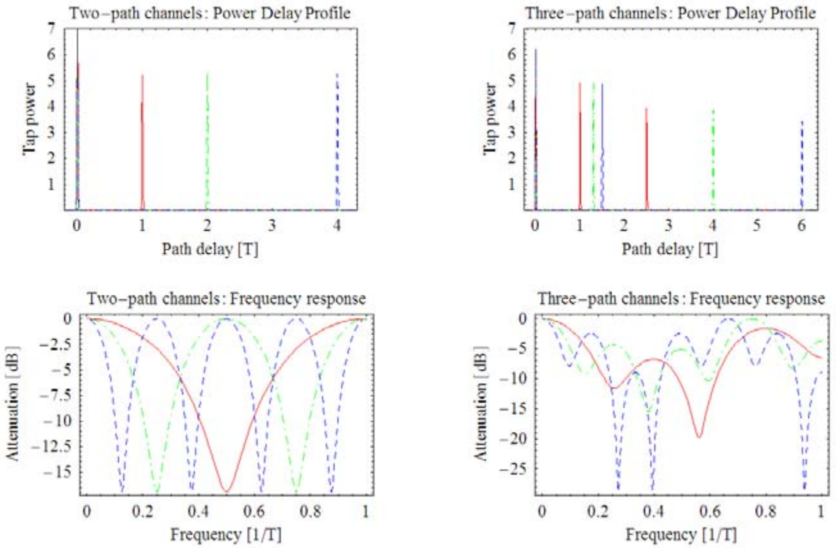
# Examples of 2- and 3-path channels



# Delay Spread and Coherence Bandwidth

- The power delay profile is characterized by the delay spread
  - ○ The delay spread (maximum excess delay) $\tau_s$ is the difference between the first and last path arrival times
  - ○ The RMS delay spread is the square root of the mean square path delay
- The delay spread gives the speed of change of the mobile channel in the frequency domain
- The channel coherence bandwidth is proportional to the inverse delay spread $B_{Coh} \sim 1/\tau_s$
  - ○ The channel at two frequencies separated more than the channel coherence bandwidth are ~uncorrelated

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# Wide vs. narrowband signal

- Characterization by delay spread, $\tau_s$ / coherence bandwidth
- (Frequency) flat fading
  - ○ narrowband signal, $B\tau_s < 1$
  - ○ When there are no separable multipath components (with the accuracy given by the sampling rate of the receiver bandwidth)
  - ○ Single-tap channel
  - ○ Simple receiver operation
- Frequency selective fading
  - ○ wideband signal, $B\tau_s$ >~ 1
  - ○ When there are separable multipath components
  - ○ multi-tap channel
  - ○ channel equalization required

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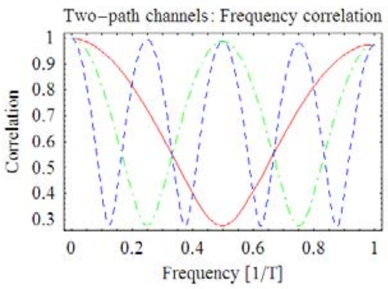
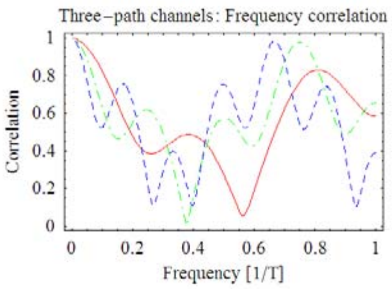
# Frequency Correlation

- modelling the channel with a few taps causes periodic frequency correlation
  - ○ interpretation of coherence bandwidth unclear





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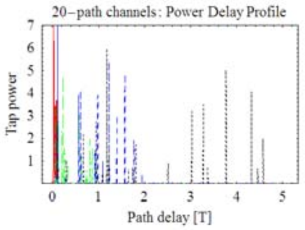
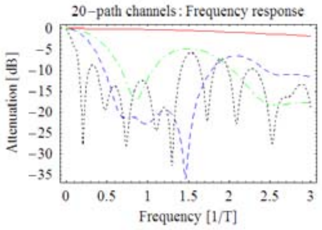
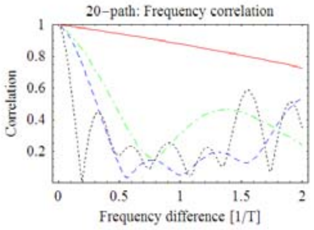
# Channel with Large Number of Paths

- 20 paths with delay spreads 1/8 (NB), 1, 2, 5 (WB) =>~realistic frequency correlation properties







# Time selective fading and Doppler shift

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## Interference Pattern in 2D plane II

- Different fading realization at different spatial location
- Caused by multiple copies of the same signal, arriving essentially at the same time, constructive or destructive interference
  - ○ Reflection
  - ○ Diffraction
  - ○ Scattering
- Length scale between destructing interferences ~wavelength
  - ○ One wavelength difference in propagation path lengh cause full 2pi (360 degree) phase shift
  - ○ In one wavelength spatial separation, phases of scattered, diffracted etc copies change
- A MS moves in a 2D interference pattern (fading profile)

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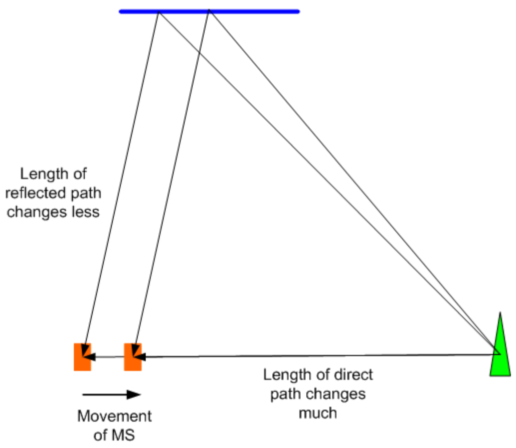
## Doppler Shift

- From perspective of Mobile Station, 2D fading profile caused by Doppler Shifts
- Different waves arrive from different directions, path lengths change differently, relative phases change
- Doppler shift

  $f_D = (v/c) f_c \cos\alpha_k$

  - $v$ is speed of mobile station
  - $c$ is speed of light
  - $f_c$ is the carrier frequency
  - $\alpha_k$ is the angle between the k:th propagation path and the mobile station velocity vector



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## Doppler Spread

- Two sides of the same coin: the Doppler shift induces
  - ○ Frequency dispersion
    - ➔ Inter-Carrier Interference (ICI)
  - ○ Time selectivity
    - ➔ fading changes in the time domain
- The maximum difference between Doppler shifts of two paths is the Doppler spread $B_D = 2(v/c) f_c$
- rapidly fading signal when $B_d$T > 1 (T is the symbol period)
  - ○ Channel estimation not possible
  - ○ non-coherent detection
  - ○ Differential modulation
- slowly fading signal when $B_d$T < 1
  - ○ Channel possible to estimate
  - ○ coherent detection
  - ○ linear modulation (QPSK, 16-QAM etc)

## Doppler spread and coherence time

- The Doppler spread gives the largest frequency of change in the mobile channel
- The channel coherence time is proportional to the inverse Doppler spread $T_{Coh} \sim 1/B_D$
  - ○ The channel at two time instances separated more than the channel coherence time are uncorrelated

---

# Channel Model

$$h(\lambda,t)=\sum_{k=0}^{M-1} h_k e^{j2\pi\nu_k t}\delta\left(\lambda-\tau_k\right)$$

- time-variant impulse response of discrete M-path channel
  - ○ path gains $h_k$
  - ○ constant delays $\tau_k$
  - ○ constant speed ➔ constant per path Doppler-shift $\nu_k=\frac{v}{c}f_c\cos\left(\alpha_k\right)$
- in the small region where the mobile station moves under one transmission frame $h_k$, $\tau_k$, $\alpha_k$, and $M$ are approximately constant
- paths are discrete but time is continuous
  - ○ separable vs non-separable multipath not yet distinguished
- *Two time domains:*
  - ○ Path delay time $\lambda$
  - ○ Mobile station movement time (Doppler-time) $t$

---

## Power Delay Profile

$$p(\lambda)=\left|h(\lambda,t)\right|^2=\sum_{k=0}^{M-1} h_k{}^2\delta\left(\lambda-\tau_k\right)$$

- The distribution of power in the delay domain

## Fourier transforms of impulse response

- time-variant transfer function:

$$H(f,t)=\mathrm{F}_\lambda\left\{h(\lambda,t)\right\}=\sum_{k=0}^{M-1} h_k e^{j2\pi\nu_k t}e^{-j2\pi f\tau_k}$$

  - ○ Fourier transform in the signal delay domain
  - ○ fading profile as a function of signal frequency & MS movement time
- the delay-Doppler-spread function:

$$S(\lambda,\nu)=\mathrm{F}_t\left\{h(\lambda,t)\right\}=\sum_{k=0}^{M-1} h_k\delta\left(\nu-\nu_k\right)\delta\left(\lambda-\tau_k\right)$$

  - ○ Fourier transform in the MS movement domain
  - ○ Channel characteristics as function of Doppler-frequency & path delay

---

## Tapped delay line

- Model of N-tap delay line:

$$h(\lambda,t)=\sum_{n=1}^{N} h_n(t)\delta\left(\lambda-(n-1)\tau\right)$$

  - ○ The power delay profile sampled according to the channel sampling frequency
  - ○ Note that channel sampling frequency may be higher than signal bandwidth ➔ oversampling
- Discrete time $\tau_k = n\tau$
  - ○ One channel tap per sampling instance $n$
- Non-separable multipaths combine to channel coefficient within tap

$$h_n(t)=\sum_{k=0}^{M_n-1} h_k e^{j2\pi\nu_k t}$$

  - ○ The complex coefficients $h_n$ are linear combinations of multiple paths, and fade as function of time
  - ○ with pulse shaping, coefficients would be weighted by pulse shape
- Often approximation: one channel sample per symbol time, $\tau = T$

# Statistical Channel Modeling: Large and Small scale effects of Channel

**Large scale effects:**

1. Average path loss as function of distance
2. Shadow fading due to large obstacles ("slow fading")

-System level effects, not discussed here

**Small scale effects:**

3. (Fast) fading & Multipath effects
    a. Separable Multipath fading
    b. Fast fading non-separable multipath propagation

- The effects seen at baseband

---

## Complex Gaussian Channel Taps

- In a tapped delay line $h(\lambda,t)=\sum_{n=1}^{N} h_n(t)\delta\left(\lambda-(n-1)\tau\right)$
    - ○ The channel coefficients per tap $h_n(t)=\sum_{k=0}^{M_n-1} h_k e^{j2\pi\nu_k t}$ are assumed to consists of $M_n \to \infty$ non-separable paths
- The channel coefficients are modelled as random variables
    - ○ Uniformly distributed angles of arrival
    - ○ Path Gains uniformly distributed, or constant (in AoA)
    - ➔Central limit theorem: $h_n(t)$ has zero mean complex Gaussian distribution

$$p(h)=\frac{1}{2\pi\sigma^2}e^{-|h|^2/2\sigma^2}$$

---

## Rayleigh distribution

- Channel amplitude and phase $h=ae^{i\varphi}$
- When $h$ is complex Gaussian, $a$ is Rayleigh distributed

$$p(a)=\frac{a}{\sigma^2}e^{-a^2/2\sigma^2}$$

  - ○ Note: amplitude is positive semi-definite. In lecture notes step function $u(a)$ denotes this
- When $h$ is complex Gaussian, $\varphi$ is uniformly distributed between $[-\pi,\pi]$
- Channel power $P=|h|^2=a^2$ has exponential distribution

$$p(P)=\frac{1}{\overline{P}}e^{-\frac{P}{\overline{P}}}$$

  - ○ The average power: $\overline{P}=2\sigma^2$

---

## Rice distribution

- When there is a dominant path
    - ○ Path gain distribution in angle-of-arrival is non-uniform
    - ○ $h$ is not zero mean, but has real mean value $a_0$
- the amplitude is Rice-distributed:

$$p(a)=\frac{a}{\sigma^2}e^{-\left(a^2+a_o^2\right)/2\sigma^2}I_0\left(\frac{aa_o}{\sigma^2}\right)$$

- $I_0(x)$ is zeroth-order modified Bessel function
- power distribution is

$$p(P|K)=\frac{(1+K)e^{-K}}{\overline{P}}e^{-(1+K)\frac{P}{\overline{P}}}I_0\left(\sqrt{4K(1+K)\frac{P}{\overline{P}}}\right)$$

  - ○ where $K$ is the power ratio of the specular and random components
      - ▪ the "Ricean K-factor"

# Temporal Fading Characteristics

- The time development of the fading coefficient $h_n(t)$ is given by the Doppler shifts of its non-separable multipath components
- Doppler spectrum tells the distribution of power of the components in $h_n(t)$ with different Doppler shift
- With uniform angular distribution and constant gain of the non-separable multipath components, “classical Jakes” Doppler spectrum

$$S_D(\nu) = \frac{2}{\pi\nu_{\max}} \frac{P_{Rx}}{\sqrt{1-\left(\nu/\nu_{\max}\right)^2}}$$

---

# Wideband Channel Modeling

- Wideband channels modelled by a Tapped Delay line
- Examples of channel models used in GSM development

- Bad urban

| i | 1 | 2 | 3 | 4 | 5 | 6 |
|---|---|---|---|---|---|---|
| $\tau_i$/µs | 0 | 0.3 | 1.0 | 1.6 | 5.0 | 6.6 |
| $P_{im}$/dB | –2.5 | 0 | –3.0 | –5.0 | –2.0 | –4.0 |

- Typical urban

| i | 1 | 2 | 3 | 4 | 5 | 6 |
|---|---|---|---|---|---|---|
| $\tau_i$/µs | 0 | 0.2 | 0.5 | 1.6 | 2.3 | 5.0 |
| $P_{im}$/dB | –3.0 | 0 | –2.0 | –6.0 | –8.0 | –10.0 |

- Hilly terrain

| i | 1 | 2 | 3 | 4 | 5 | 6 |
|---|---|---|---|---|---|---|
| $\tau_i$/µs | 0 | 0.1 | 0.3 | 0.5 | 15.0 | 17.2 |
| $P_{im}$/dB | 0 | –1.5 | –4.5 | –7.5 | –8.0 | –17.7 |

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## S-72.2205 Digital Transmission Methods

# Diversity Combining and Fading Countermeasures

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# Problems due to Fading

- Temporal fading
  - The quality of the channel changes in time, part of the transmission may vanish
- Frequency fading
  - If multicarrier transmission (a wide band divided into multiple separately utilized subcarriers)
    ➔ the quality of the subcarriers is different
  - If single carrier transmission
    ➔ ISI

# How to combat Fading

1. Diversity methods
    - discussed below
2. Link adaptation methods
    - Transmitter knows the quality of the channel
    - adapts transmission rate accordingly
    - Temporal fading:
        - Transmit more when the channel is good
    - Frequency fading, narrowband signals
        - Transmit more, or only, on the better subcarriers
    - Frequency fading, wideband signals
        - pre-equalize channel so that Rx sees only one tap
3. Receiver processing
    - Frequency fading, wideband signals: equalize or combine power from different taps
        - These are actually diversity combining methods

---

# Diversity

- Several copies of the signal are available at the receiver
    - ○ Combining the signals gives a better quality signal
        - Combining may be performed at RF or baseband
    - ○ Different copies transmitted over different channels, different fading realizations
    - ○ it is unlikely that all channels fade simultaneously
- Different kinds of diversity differ in the way that power is distributed among the diversity branches
- Diversity gain depends on number of diversity branches and how uncorrelated the branches are
- Explicit diversity:
    - ○ Explicit copies of each symbol or bit available at the receiver
- Implicit diversity:
    - ○ Parity bits constructed with Forward Error Correction coding transmitted over different channels

---

# Sources of Diversity

- Space diversity:
    - ○ Receive diversity
        - Multiple Rx antennas. Each doubling of the number of Rx antennas doubles the Rx power
        - Antennas should be sufficiently far apart to be uncorrelated
            - MS: wavelength/2 or more
            - BS: several wavelengths
    - ○ Transmit diversity
        - Multiple Tx antennas. Power divided between antennas
    - ○ Macro diversity
        - Spatial diversity with multiple BSs. Always uncorrelated
        - Soft handover
- Polarization diversity
    - ○ As spatial diversity
- Multipath diversity
    - ○ Power of different paths collected
    - ○ Multipath components typically uncorrelated

---

- Frequency diversity
    - ○ Parts of a coding block transmitted over carriers with different frequency
    - ○ Full diversity gain: Carrier separation > coherence bandwidth
    - ○ No additional power – implicit diversity copies of the bits constructed at transmitter
- Time diversity
    - ○ Implicit: Parts of a coding block transmitted over time instances separated more than the coherence time
        - No additional power – implicit diversity copies of the bits constructed at transmitter
    - ○ Explicit: ARQ/HARQ
        - additional power – explicit diversity copies with new transmission and new power

## Combining methods

### Selection combining

- the branch with the best SNR is chosen
- SINR of the combined signal is $\gamma_c = MAX\{\gamma_i\}, \quad i=1,2,\ldots M$
- if noise/interference power is equal in all branches the branch with maximum power is chosen. Simplifies combiner.

### Equal-gain combining

- branches co-phased and summed $y(t)=\sum_{i=1}^{M} x_i(t)e^{j\varphi_i}$
- SNR of combined signal is $\gamma_c = \frac{\left(\sum_{i=1}^{M}\sqrt{P_i}\right)^2}{M\sigma_n^2}$

---

## Maximum Ratio Combining (MRC)

- the branches are co-phased and summed with a proper weighting

$$y(t)=\sum_{i=1}^{M} g_i x_i(t)e^{j\varphi_i}$$

- weights $g_i$ are chosen to maximize the SNR of the combined signal
- if noise/interference power is same in all branches the SNR of the combined signal is

$$\gamma_c = \frac{\left(\sum_{i=1}^{M} g_i\sqrt{P_i}\right)^2}{\sigma_n^2\sum_{i=1}^{M} g_i^2} \le \frac{\sum_{i=1}^{M} g_i^2 \sum_{j=1}^{M} P_j}{\sigma_n^2\sum_{i=1}^{M} g_i^2} = \frac{\sum_{i=1}^{M} P_i}{\sigma_n^2} = \sum_{i=1}^{M}\gamma_i$$

First Schwarz' inequality. The equality holds when the weights are the

MRC weights: $g_i \sim \sqrt{P_i}$

---

## Diversity Improvement and Diversity Gain

- diversity improvement $I_{div} = \frac{P(\gamma_1 < \gamma)}{P(\gamma_c < \gamma)}$
    - ○ $\gamma_1$ is SNR of one branch
    - ○ $\gamma_c$ is SNR of the combined signal
    - ○ property of fading PDF
    - ○ measures decrease of probability deep fade with diversity
    - ○ E.g. the probability of a -20dB fade may decrease from 0.01 to 0.0001
- Diversity gain
    - ○ Property of performance measure in fading channel
    - ○ Difference in SNR for reaching an operation point in a given error measure
    - ○ how many dB's diversity improves the link budget
    - ○ E.g. at BLER 10^-2, required SNR with diversity may be 5 dB less than without: diversity gain at BLER 10^-2 is 5 dB

---

## Fading Distribution with Diversity (Rayleigh Fading)

### Rayleigh PDF distribution

- ○ SNR is exponentially distributed, PDF is $p(\gamma) = \frac{1}{\gamma_m}e^{-\gamma/\gamma_m}$
    - ○ $\gamma_m$ is the average SNR
- ○ CDF is $P(\gamma_1 < \gamma) = 1 - e^{-\gamma/\gamma_m}$

### Selection combining:

- ○ If all branches have the same average SNR,

$$P(\gamma_c < \gamma) = \prod_{i=1}^{M}\left(1 - e^{-\gamma/\gamma_{m_i}}\right) = \left(1 - e^{-\gamma/\gamma_m}\right)^M$$

# MRC:

$$\gamma_c = \sum_{i=1}^{M} \gamma_i$$

- sum of exponentially distributed SNRs is gamma-distributed:

$$p_c(\gamma) = \frac{1}{(M-1)!} \cdot \frac{\gamma^{M-1}}{\gamma_m^{M}} \cdot e^{-\gamma/\gamma_m}$$

- CDF:

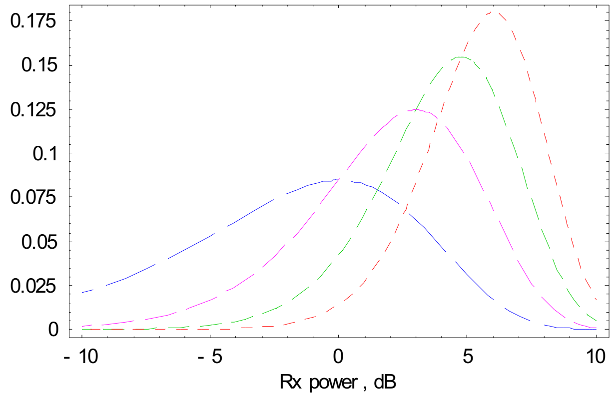
$$P(\gamma_c < \gamma) = 1 - e^{-\gamma/\gamma_m} \sum_{i=1}^{M} \frac{1}{(i-1)!} \cdot \left(\frac{\gamma}{\gamma_m}\right)^{i-1}$$

- **with 2 branches independent Rayleigh fading, MRC offers ~1.6dB better Diversity gain than selection combining and ~0.4 dB better than Equal Gain Combining**

---

# PDF of SNR after Rx-diversity combining

- **1,2,3,4 diversity branches**
- **MRC receive diversity**
  - **Average power after combining: 0, 3, 4.77, 6 dB**
  - **Antennas of same size and same antenna gain, adding Rx antennas adds Rx power**



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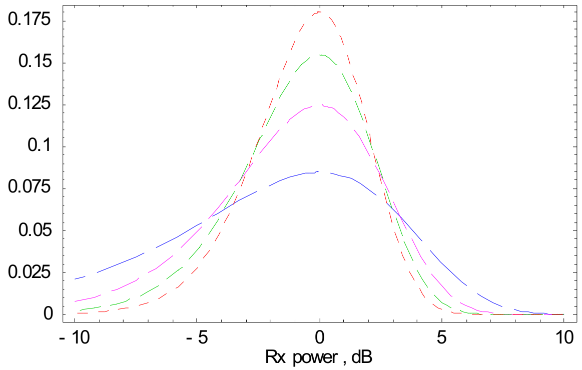
# PDF of SNR after Tx-diversity combining

- **1,2,3,4 diversity branches**
- **MRC combined Transmit diversity**
  - **Average power after combining: 0, 0, 0, 0 dB**
  - **Adding Tx antennas keeping Tx power constant does not add average Rx power**
  - **Diversity gain: reduced power fluctuations**



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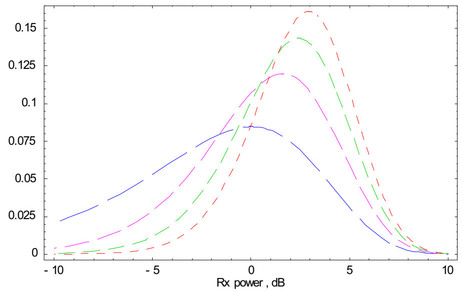
# PDF of SNR after selection combining

- **1,2,3,4 diversity branches**
- **Selection combined diversity**
  - **Rx diversity: receive with better of N**
  - **Tx diversity: transmit with better of N antennas antennas**
  - **Average power after combining: 0, 1.76, 2.63, 3.19 dB**
  - **Selection of better antenna both at Tx & Rx end increases average Rx power. In FDD, selection at Tx requires feedback.**



## Diversity Gains, 2-branch diversity

- ○ gain depends on the operation point
  - ○ at BER 10^-2, gain 9 dB, at BER 10^-4, 18.6 dB

---

## Diversity Why and Where

- 2G (GSM)
  - ○ Rx-diversity at BS to improve UL coverage
    - ▪ UL coverage is bottleneck especially in voice system
  - ○ Frequency diversity through frequency hopping
  - ○ Time diversity from code + interleaving
- 3G (WCDMA)
  - ○ Rx-diversity at BS
  - ○ Time diversity from code + interleaving
  - ○ Multipath (Frequency) diversity from wide band
  - ○ Macro diversity (soft handover) in DL & UL
  - ○ Tx-diversity at BS to improve DL coverage in environments with low multipath diversity
- Rx diversity in terminals is coming
  - ○ Terminal class in WCDMA
  - ○ Baseline in LTE

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# S-72.2205 Digital Transmission Methods

# Basic principles of DS-CDMA

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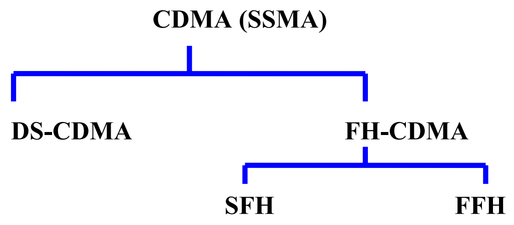
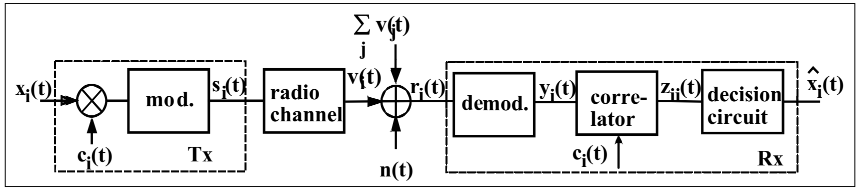
## Taxonomy of spread-spectrum methods



**CDMA, Code Division Multiple Access**
**SSMA, Spread Spectrum Multiple Access**
**DS-CDMA, Direct Spread CDMA**
**FH-CDMA, Frequency Hopping CDMA**
**SFH, Slow Frequency Hopping**
**FFH, Fast Frequency Hopping**

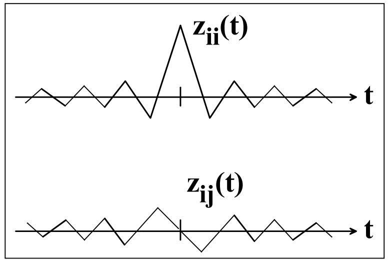
## Direct Sequence (DS) Spread Spectrum Principle



$x_i(t)$ user i information signal
$c_i(t)$ user i spreading code
$s_i(t)$ user i transmit signal
n(t) noise
$r_i(t)$ user i receive signal
$z_{ii}(t)$ user i correlation signal
$\hat{x}(t)$ user i output information signal

$$z_{ii}(t) = \int_T y_i(u) c_i(u+t)\,du$$

T = symbol duration



With orthogonal spreading codes and perfect timing the cross-correlation between different codes is zero

---

## Chips and Bits

- A chip is the shortest modulated signal in a CDMA system
    - ○ Chip duration ~ inverse bandwidth
    - ○ Chip rate ~ signal bandwidth
- A bit (or symbol) is spread over multiple chips
- The Spreading Factor (SF) tells how many chips are used to transmit one symbol

$$SF = N = \frac{R_c}{R_s} = \frac{\text{chip rate}}{\text{symbol rate}} = \text{spreading factor}$$

- ○ SF is a spectrum spreading factor: bit rate ➔ chip rate
- ○ With orthogonal codes (synchronous CDMA), the channel can be divided into SF orthogonal code channels, each with a bit rate 1/SF of the chip rate
- ○ Just as with TDMA, one can divide the chip rate into SF orthogonal time domain channels with rate 1/SF

- With a fixed bandwidth & chip rate, CDMA with large SF offers less complex receiver processing against multipath interference than TDMA

---

## Orthogonal and non-orthogonal codes

- A spreading code with lengh SF chips can be described in discrete time as a vector in SF-dimensional space
- A family of spreading codes is orthogonal if

$$\mathbf{c}_i^T \mathbf{c}_j = SF\,\delta_{ij}$$

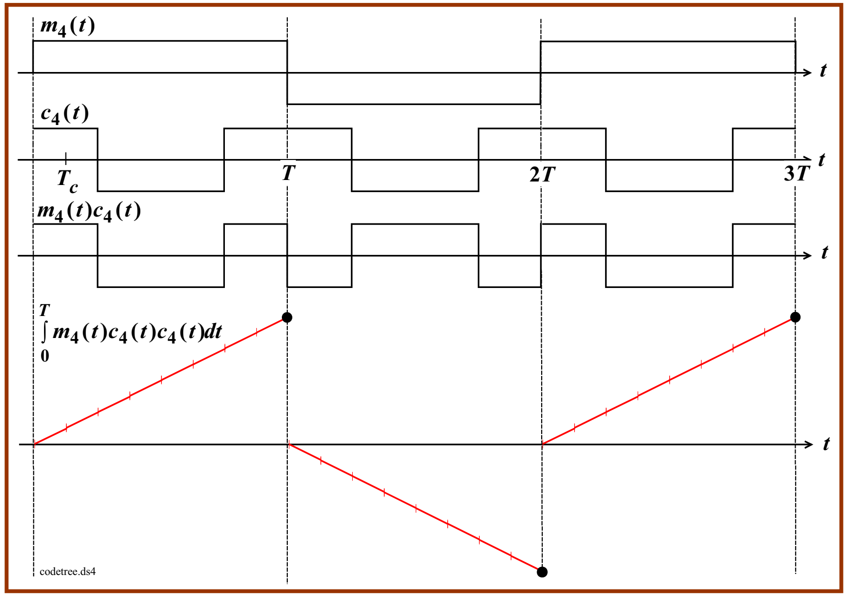
- Orthogonal codes are a very specific class of codes
- Orthogonality is lost if timing changes
    - ○ Example: $\mathbf{c}_1 = [1 \;\; 1 \;\; -1 \;\; -1]^T$ and $\mathbf{c}_2 = [1 \;\; -1 \;\; -1 \;\; 1]^T$ are orthogonal whereas for $\mathbf{c}_1 = [1 \;\; 1 \;\; -1 \;\; -1 \;\; 0]^T$ and the delayed copy $\mathbf{c}_2{}^d = [0 \;\; 1 \;\; -1 \;\; -1 \;\; 1]^T$ we have $\mathbf{c}_1^T \mathbf{c}_2{}^d = 3$
    - ○ Autocorrelation properties of orthogonal codes are poor
- For random spreading codes,

$$E\left(\mathbf{c}_i^T \mathbf{c}_j\right) = SF\,\delta_{ij} + \sqrt{SF}\,(1-\delta_{ij})$$

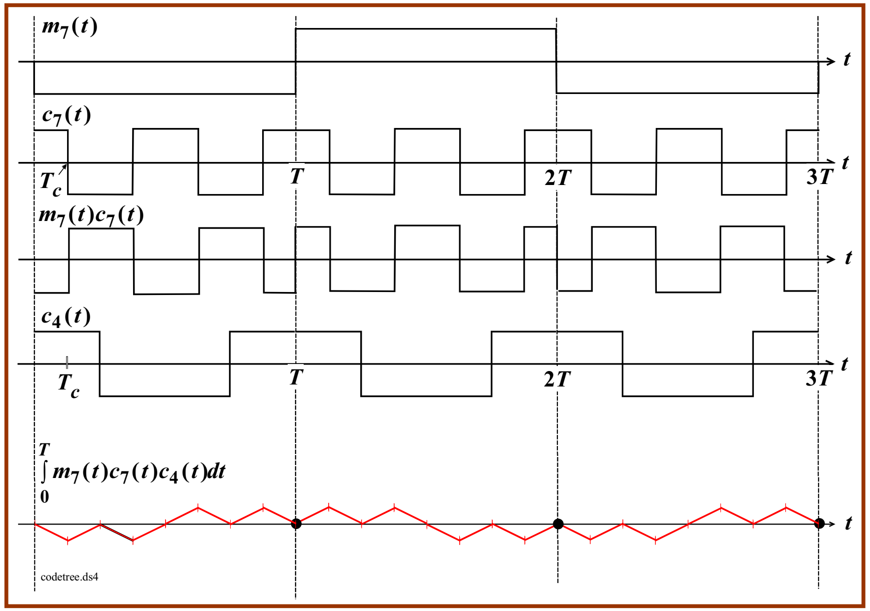
that is, the expected cross-correlation of random codes is $\sqrt{SF}$

---

## Despreading of the desired signal



# Cancellation of a synchronous interfering signal, orthog codes



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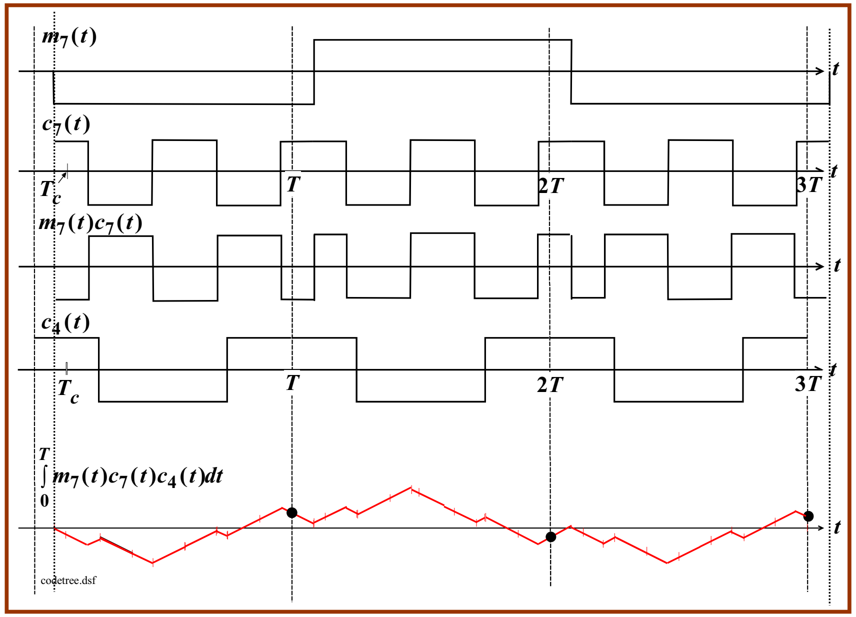
# Impact of a non-synchronous interfering signal



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# Processing gain

- **Due to spreading, multiple copies (the chips) of the modulated signal are received**
- **When despreading, these copies are coherently combined by correlating with the spreading code and integrating the result in the receiver**

$$z(T)=\int_0^T \sqrt{P_k}\, m_k s_k(t)s_k(t)dt+\int_0^T \sum_{\substack{i=0\\ i\neq k}}^{M} \sqrt{P_i}e^{i\theta_i} m_i s_i(t)s_k(t)dt+\int_0^T n_o(t)s_k(t)dt$$

$$=GT_c\sqrt{P_k}\, m_k+\sum_{\substack{i=0\\ i\neq k}}^{M} T_c\sqrt{P_i}e^{i\theta_i} m_i \sum_{c=1}^{G} s_{ic}s_{kc}+T_c\sum_{c=1}^{G} n_c$$

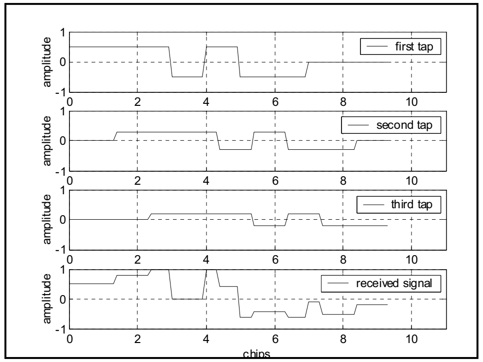
- $P_j$ **is received power of user *j***
- $m_j$ **is the transmitted *complex* symbol of user *j*, expected amplitude 1**
- $s_j$ **is the spreading code of user *j*,**
- ***M* is the number of users in the cell,**
- ***k* is the desired user**
- $n_o(t)$ **is the random noise process**
- $\theta_i$ **is the c channel phase difference between user *i* and *k***
- $T_c$ **is the chip period**
- ***G* is the spreading factor**
- $s_{jc}$ **is the value of the spreading code *j* in chip *c*, assumed +-1**
- $n_c$ **is a noise sample in chip *c***

---

- **the common factor $T_c$ may be omitted (units are powers, energy per chip period)**
- ***G* (the SF) times the chip amplitude is coherently combined for the wanted signal**
  - **➔ *G*^2 times the chip power**
- **Noise and random interference are non-coherently combined in the receiver:**
  - ○ **When calculating the noise+interference covariance, only terms with the same *i* and *c* count**
  - ○ **The expected noise + interference power becomes** $G\sum_{\substack{i=0\\ i\neq k}}^{M} P_i+GP_n$
- **Post-despreading SINR is** $\mathrm{SINR}=\dfrac{GP_k}{\sum_{\substack{i=0\\ i\neq k}}^{M} P_i+P_n}$
- **Processing gain against random noise and interference equals *G*, the spreading factor**

# Impact of a multipath channel

- Impulse response of a multipath channel: $h(\lambda,t)=\sum_{k=0}^{M-1} h_k e^{j2\pi\nu_k t}\delta(\lambda-\tau_k)$
- Received signal is convolution of the received signal and the channel impulse response.
- Multipath destroys code orthogonality:
  - The spreading codes are orthogonal if they are synchronised, i.e. they start at the same time instant
  - If the codes are not synchronised their cross correlation is not zero.
  - In a multipath channel signal components arrive at different time instants.
  - Assume that the correlation receiver is synchronised to a certain tap. The integration covers part of the previous symbol and next symbol from an another tap



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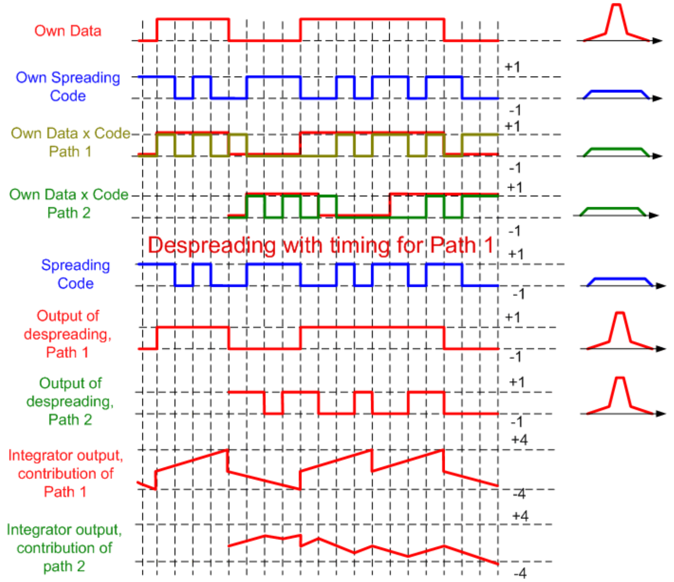
# Loss of Orthogonality in multipath channel



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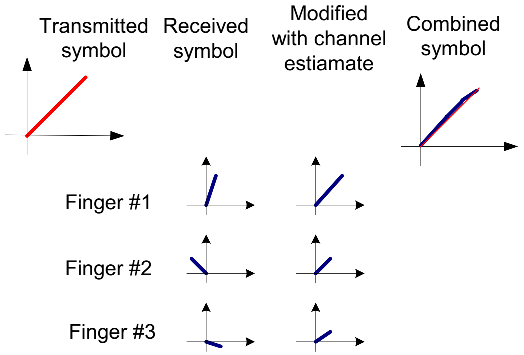
# Maximum ratio “RAKE” combining of multipath components



- Channel can rotate the signal to any phase and to any amplitude.
- PSK & QAM symbols carry information in phase.
- Energy split to many fingers → multipath diversity
- Maximum ratio combining corrects channel phase rotation and weights components with channel amplitude estimate

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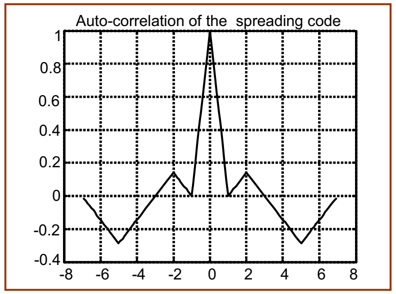
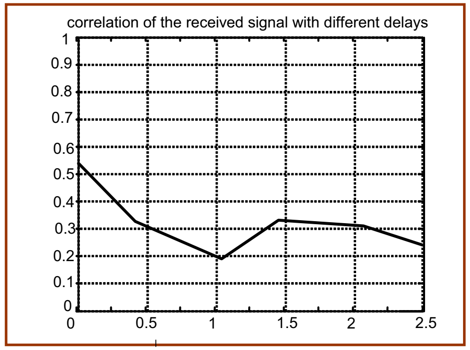
# RAKE receiver

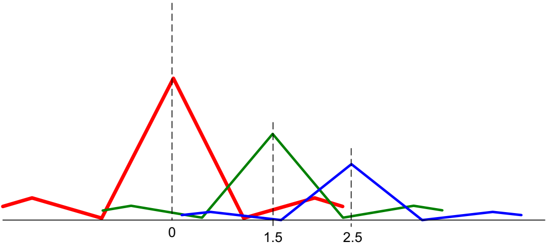
- **Requires chip-level synchronization**
- **Power delay profile estimation – one channel estimate for each finger**

# Inter-path interference from Auto-corrlation





- Correlation of a spreading sequence with delayed copies of itself: auto-correlation
- Auto-correlation functions are not delta functions.
- Auto-correlation properties of the spreading code determine the inter-path-interference between multipath components



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# Near-far effect I

**Recall that** $$\text{SINR}=\frac{GP_k}{\sum_{\substack{i=0\\ i\neq k}}^{M} P_i + P_n}$$

**With the same Tx power, the received power differences of UL users due to path loss may be up to 90 dB. To overcome this, a user in a disadvantaged position would need a processing gain of the same magnitude, i.e. *G* of the order of a billion.**

**This *near-far effect* causes a significant reduction of the capacity and requires very effective up-link *power control* in DS-CDMA systems**

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# Near-far effect II

**Uplink: Because of different attenuation signals to/from users nearer to BS are stronger than signals to/from further located users.**

**Downlink: Because of the nature of attenuation at the cell border the users experience higher interference than near to the BS. They have high level of interfering signals from own BS and from other BS.**

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# Purpose of Power Control in WCDMA

- **Removes near far effect.**
- **Mitigates fading.**
- **Compensates changes in propagation conditions.**
- **In the system level**
  - **decrease interference from other users**
  - **increase capacity of the system**

- **Uplink**
  **Power control in uplink must make signal powers from different users nearly equal in order to maximise the total capacity in the cell.**
- **Downlink**
  **In downlink the power control keeps the signal at minimum required level in order to decrease the interference to users in other cells.**

# Advantages of DS-CDMA

- *Multiple access capability*

  This is based on low crosscorrelation between the spreading codes
- *Protection against intersymbol interference caused by multipath propagation*

  This is based on good autocorrelation properties of the spreading codes, as long as the spreading factor is large
- *Multipath diversity can be utilised by the RAKE-receiver*
- *Narrowband interference rejection*

  A narrowband signal will be spread in the correlation receiver
- *Silence periods in the transmitted signal don't consume any system capacity*
- *Privacy*

  The signal can be detected only if the spreading code is known by the receiver
- *In a hostile environment the good anti-jamming properties and low probability of interception*

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# Disadvantages of DS-CDMA

- *It is difficult to find spreading codes which have simultaneously good autocorrelation and cross-correlation properties*
- *In UL chip synchronisation between the users is impossible due to different propagation delays → interference between user signals*
- *In DL, multipath propagation will reduce the orthogonality between the users*
- *Accurate power control needed to avoid near-far problems which put the distant users in an unfavourable situation*