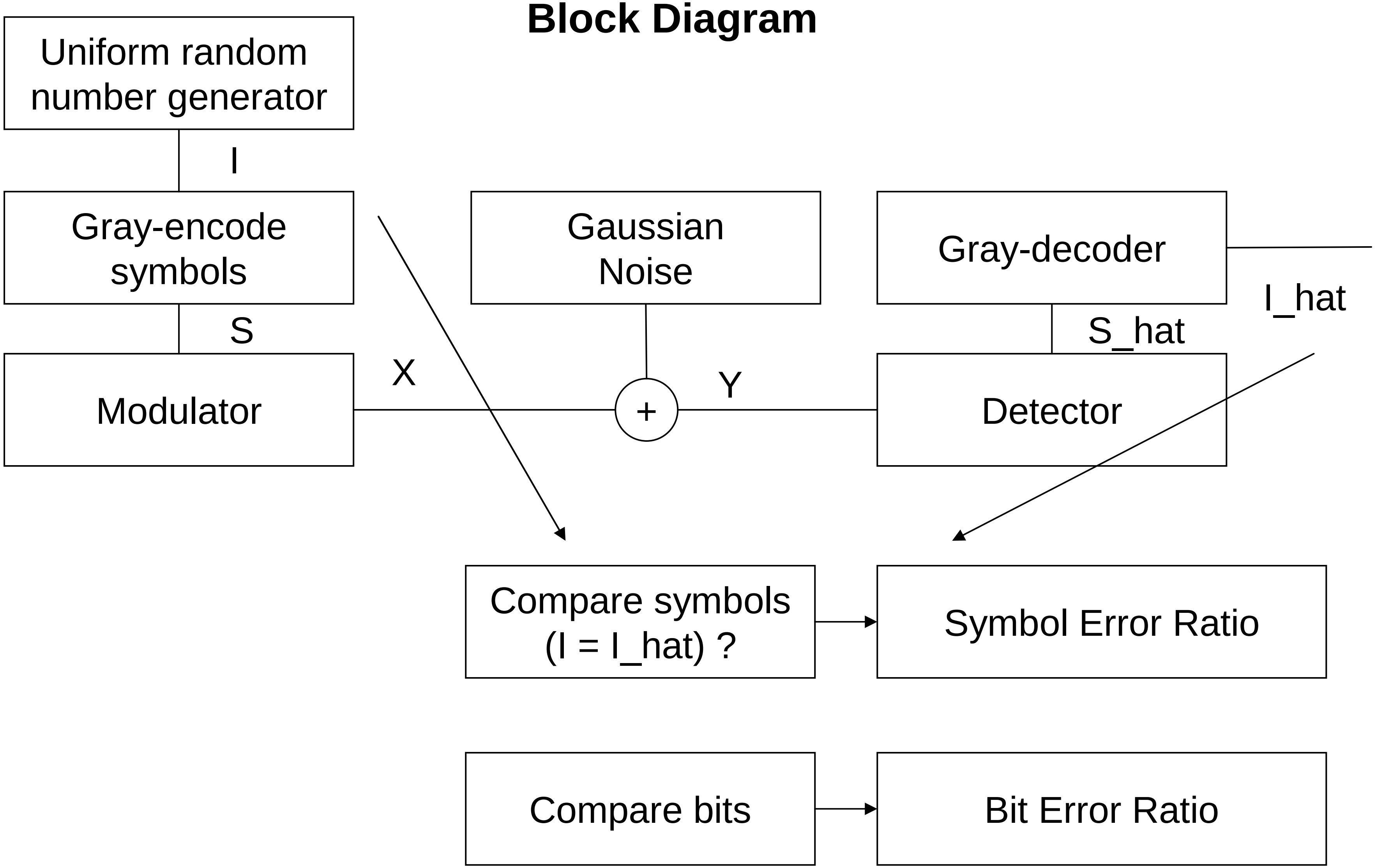
Exercise.1 8 PAM – BER/SER Monte Carlo Simulation

- Simulate a 8 level PAM communication system and calculate bit and symbol error ratios (BER/SER).
- Plot the calculated and simulated SER and BER curves.
- Plot the theoretical SER and BER curves.
- Calculate the average energy.
- Plot the constellation and histogram of samples.

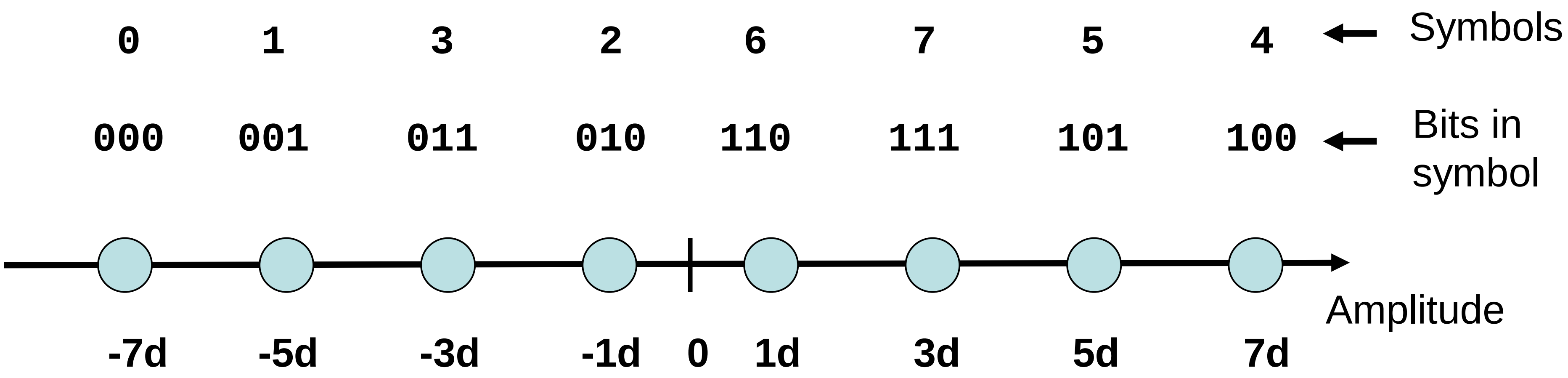
Exercise.1 8 PAM – BER/SER Monte Carlo Simulation

Block Diagram



8 PAM System – Signal constellation

Gray encoding



d : scaling

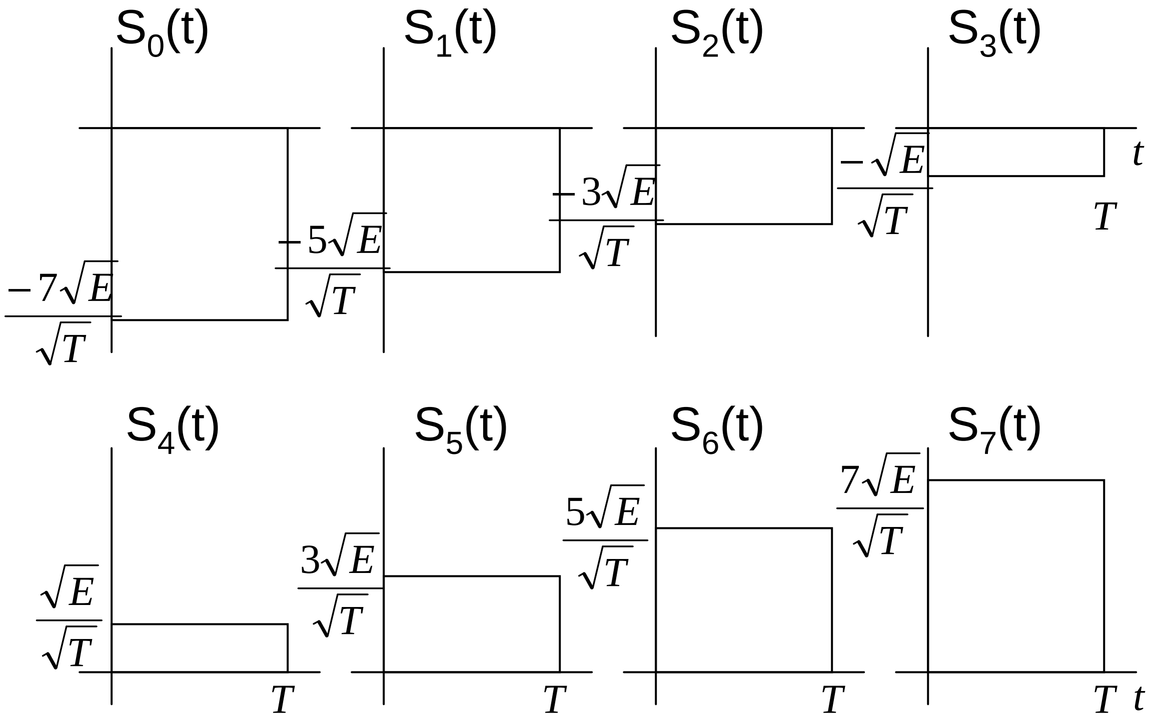
Gray encoding: Slide 65 (Lecture_3_2007_HypothesisTesting_SignalSpace.pdf)

8 PAM System – Derivation of standard deviation

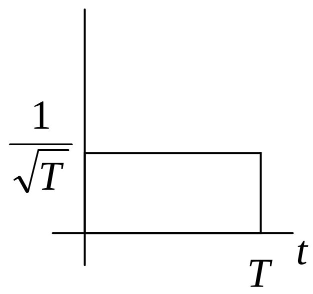
Average transmitted signal energy

$$E_{av} = \frac{1}{M}\sum_{k=1}^{M}\left(\int_0^T s_k(t)\phi(t)dt\right)^2$$

$$E_{av} = \frac{1}{8}\sum_{k=1}^{8}\left(\int_0^T s_k(t)\phi(t)dt\right)^2 = \frac{1}{8}2E\left(7^2+5^2+3^2+1^2\right) = 21E = 21d^2$$



Basis function $\phi(t)$



8 PAM System – Derivation of standard deviation

Transmitted average energy per bit

$$E_{av\_bit} = \frac{E_{av}}{\log_2 M} = \frac{E_{av}}{3}$$

Average bit SNR

$$SNR_{bit} = \frac{E_{av\_bit}}{N_0} = \frac{E_{av}}{3N_0} = \frac{21d^2}{3 \cdot 2\sigma^2} = \frac{7d^2}{2\sigma^2}$$

Standard deviation used in the simulation

$$\sigma = \sqrt{\frac{7d^2}{2SNR_{bit}}}$$

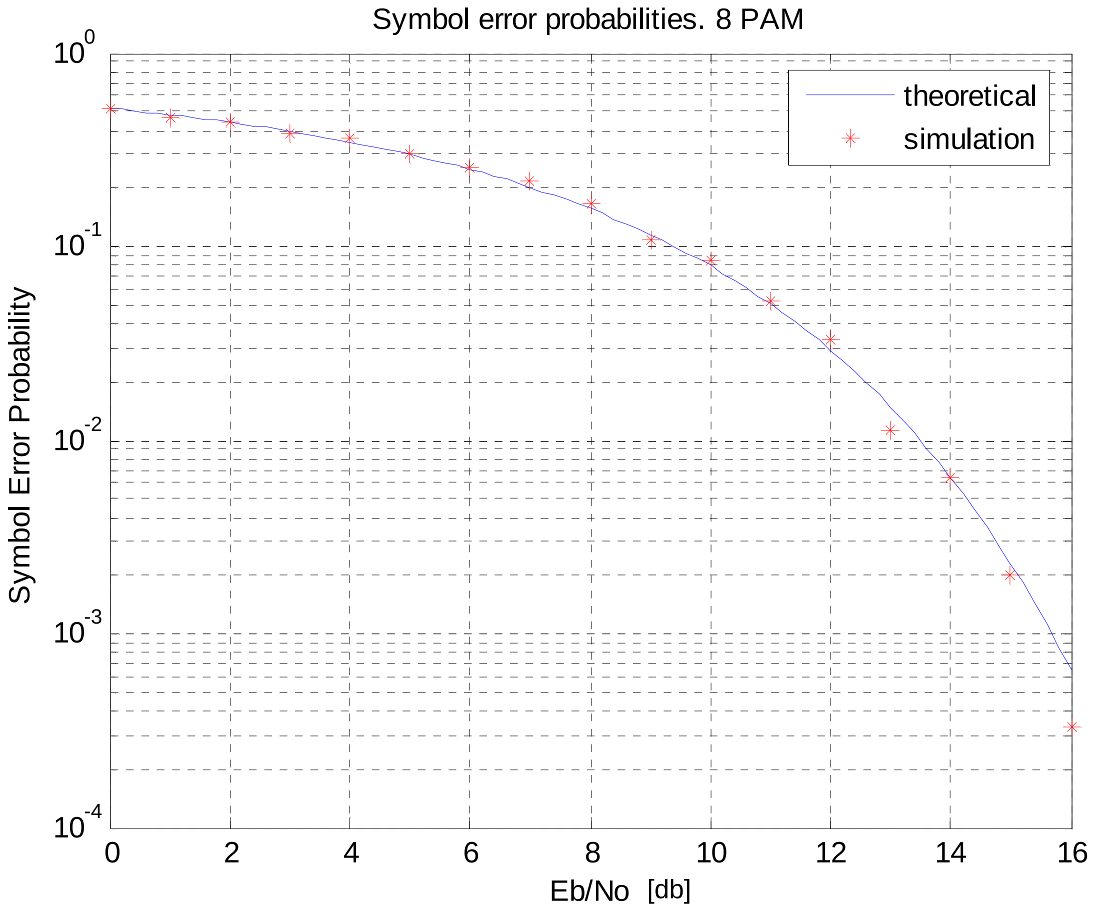
Note that some books use different notation

$$SNR_{bit} = \frac{E_{av\_bit}}{N_0} = \frac{E_b}{N_0} = \frac{E_b}{No} = EbNo$$

8 PAM System – Theoretical symbol error probability

SEP = (2*(M-1)/M)*qfunc(sqrt((6*log2(M)/(M^2-1))*SNR_bit_abs));

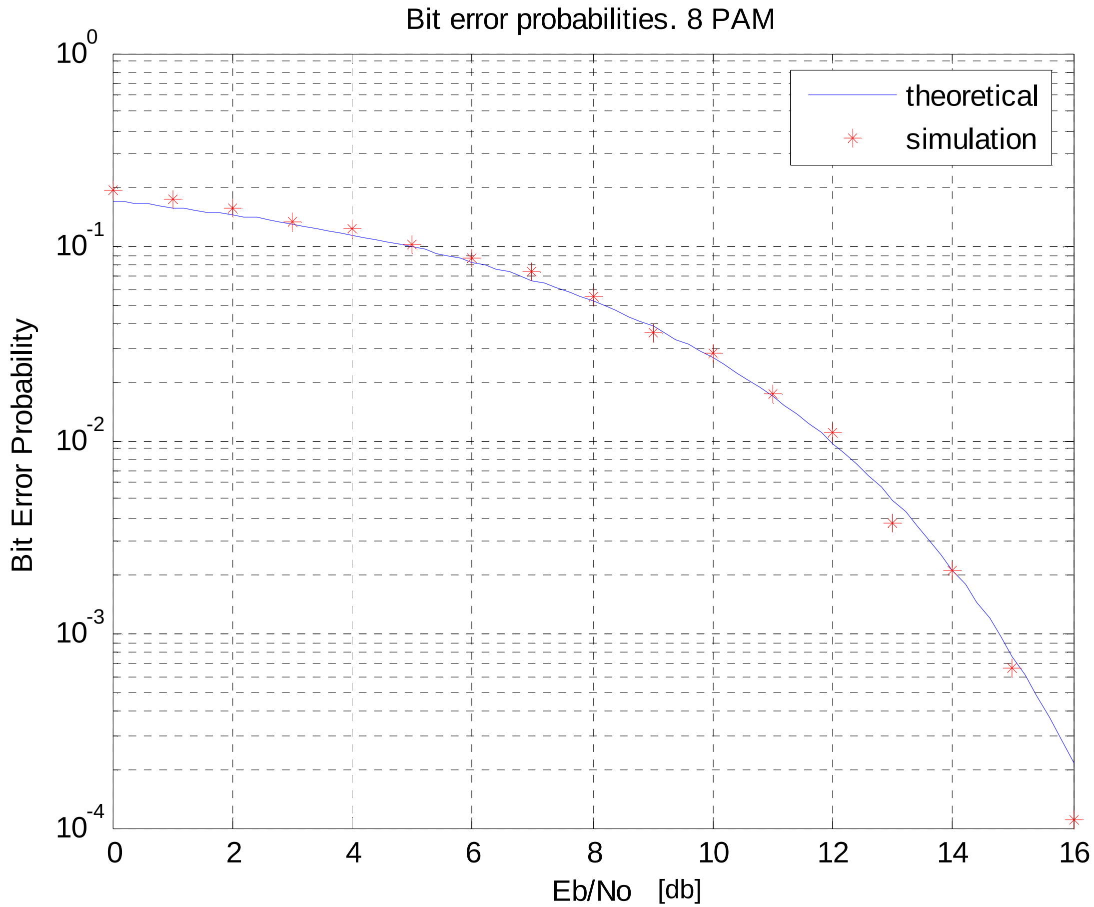
M = 8



SEP: Slide 63 (Lecture_3_2007_HypothesisTesting_SignalSpace.pdf)

8 PAM System – Theoretical bit error probability

approximation for low BEP : $BEP = (1/\log_2 M)\ SEP$



BEP: Slide 68 (Lecture_3_2007_HypothesisTesting_SignalSpace.pdf)

8 PAM System – Matlab exercise

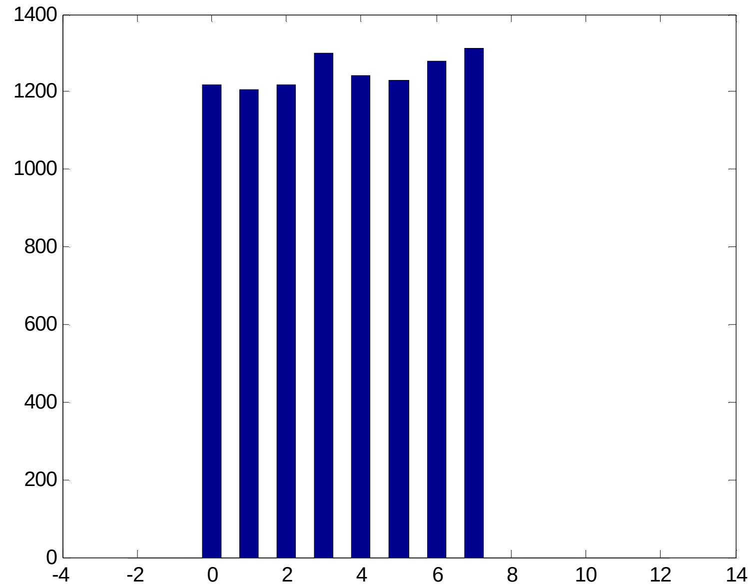
-Download the file from the course web page and open with an editor

STEP 1 : transform SNR [dB] to SNR [abs]

STEP 2: complete standard deviation for 8 PAM

STEP 3: generate *N* integer random numbers in the interval [0,7] and verify the distribution with a histogram

Histogram of symbols



8 PAM System – Matlab

STEP 4 : Add noise to symbols, use randn()

STEP 5: Calculate the symbol error ratio

STEP 6: Calculate the bit error ratio

STEP 7: Repeat the simulation for different SNR

STEP 8: Plot using the code in the file. Observe:
- symbol error probabilities and compare to theoretical results
- bit error probabilities and compare to theoretical results
- constellation
- histogram of the samples

STEP 9: Observe the average energy

STEP 10: Optional. Modulate using pammod() from the Communications Toolbox

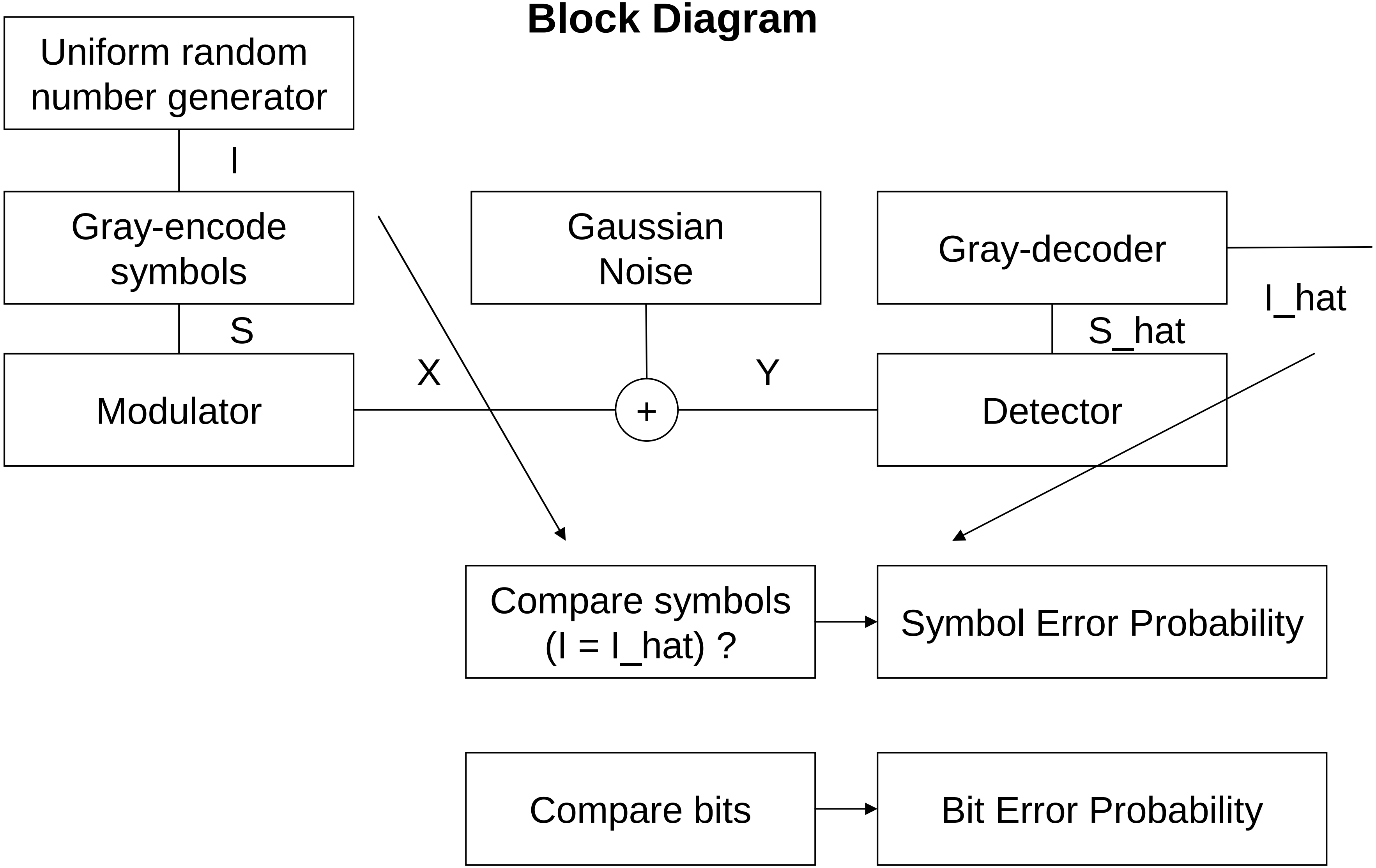
Exercise 2 BPSK / QPSK – BER/SER Simulation

-Simulate a communication system using BPSK/QPSK modulation and calculate bit and symbol error ratios.

- Set BPSK modulation.
- Plot the calculated and simulated SER and BER curves.
- Plot the theoretical SER and BER curves.
- Change modulation to QPSK
- Plot the calculated and simulated SER and BER curves.
- Plot the theoretical SER and BER curves.
- Introduce a rotation in the channel and equalize the channel.

Exercise 2 BPSK / QPSK – BER/SER Simulation

Block Diagram



BPSK

Transmitted average energy per bit

$$E_{av\_bit} = E_{av\_s}$$

SNR bit

$$SNR_{bit} = \frac{Eb}{No} = \frac{E_{av\_bit}}{N_0} = \frac{E_{av\_s}}{N_0} = \frac{E}{2\sigma^2} = \frac{E}{2\sigma^2} \quad \rightarrow \sigma = \sqrt{\frac{E}{2SNR_{bit}}}$$

Theoretical Bit Error Probability for BPSK

$$P_b = Q\left(\sqrt{2SNR_{bit}}\right)$$

Symbol Error Probability = Bit Error Probability in BPSK (1bit=1symbol)

$$P_s = Q\left(\sqrt{2SNR_{bit}}\right)$$

QPSK

Transmitted average energy per bit

$$E_{av\_bit} = \frac{E_{av\_s}}{\log_2 M} = \frac{E_{av\_s}}{2} \qquad M = 4$$

SNR bit

$$SNR_{bit} = \frac{Eb}{No} = \frac{E_{av\_bit}}{N_0} = \frac{E_{av\_s}}{2N_0} = \frac{E}{2 \cdot 2\sigma^2} = \frac{E}{4\sigma^2} \qquad \rightarrow \sigma = \sqrt{\frac{E}{4SNR_{bit}}}$$

Theoretical Bit Error Probability for BPSK = QPSK

$$P_b = Q\left(\sqrt{2SNR_{bit}}\right)$$

Approximation of Theoretical Symbol Error Probability for M >= 4

$$P_s = 2 \cdot Q\left(\sqrt{2SNR_{bit}} \cdot \sin\frac{\pi}{M}\right)$$

BPSK/QPSK – Matlab exercise

-Download the file from the course web page and open with an editor

STEP 1 : Set modulation to BPSK

STEP 2: Gray-encode symbols, use bin2gray()

STEP 3: Demodulate PSK by using pskdemod()

STEP 4: Plot using the code in the file. Observe:

- symbol error probabilities and compare to theoretical results
- bit error probabilities and compare to theoretical results

STEP 5: Change modulation to QPSK

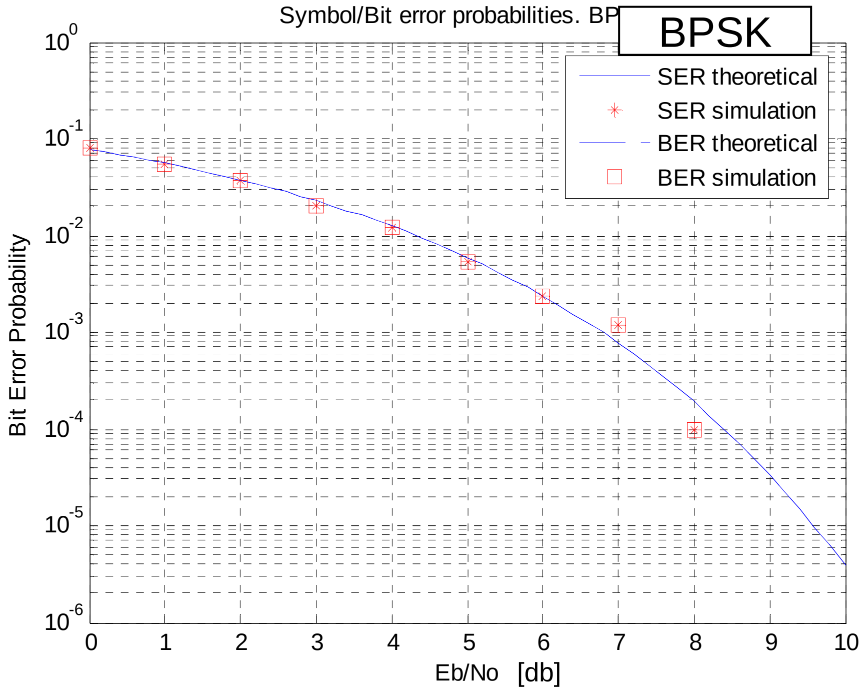
STEP 6: Introduce a rotation in the channel

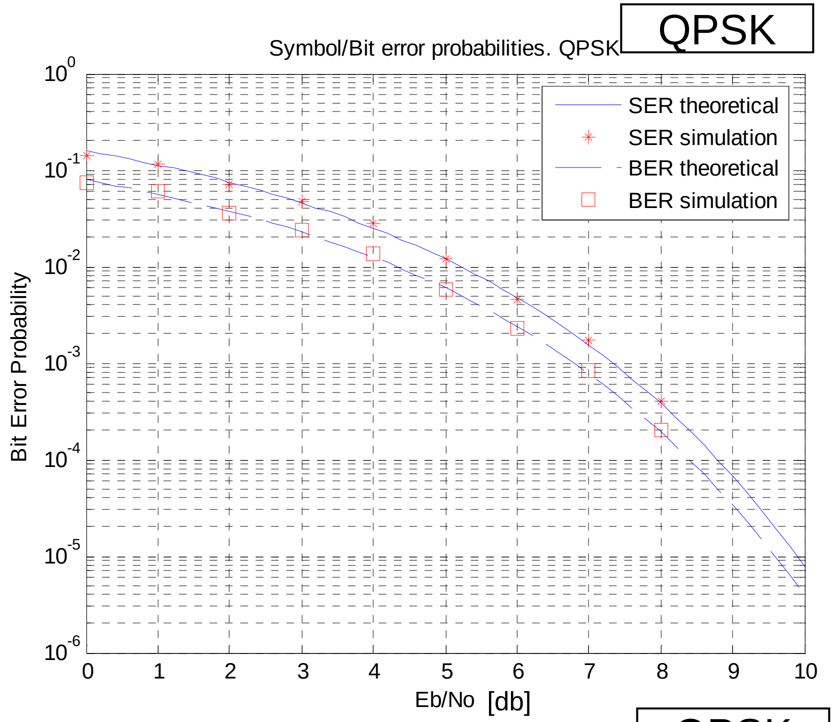
STEP 7: Equalize the channel

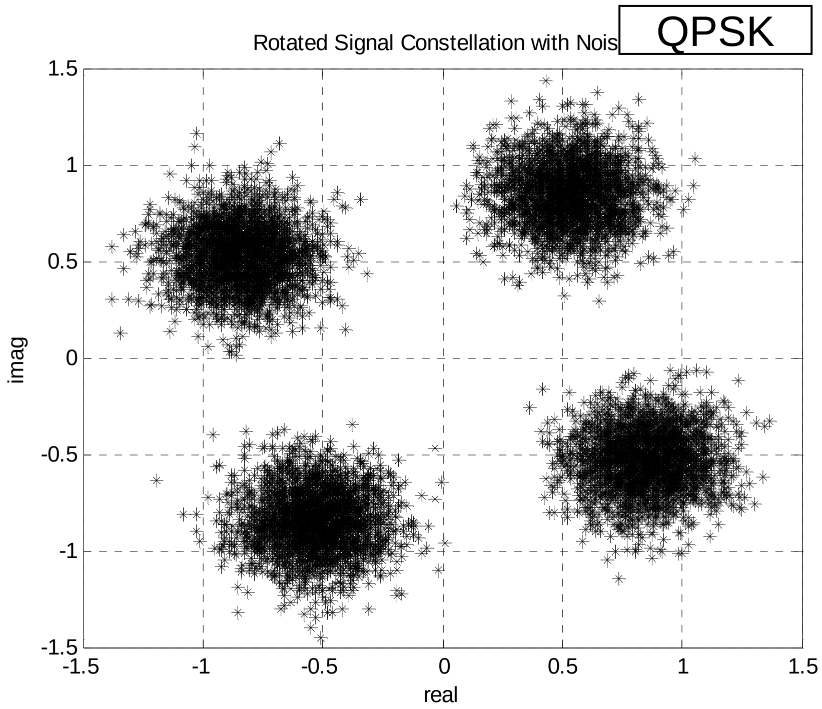
BPSK/QPSK – Matlab exercise

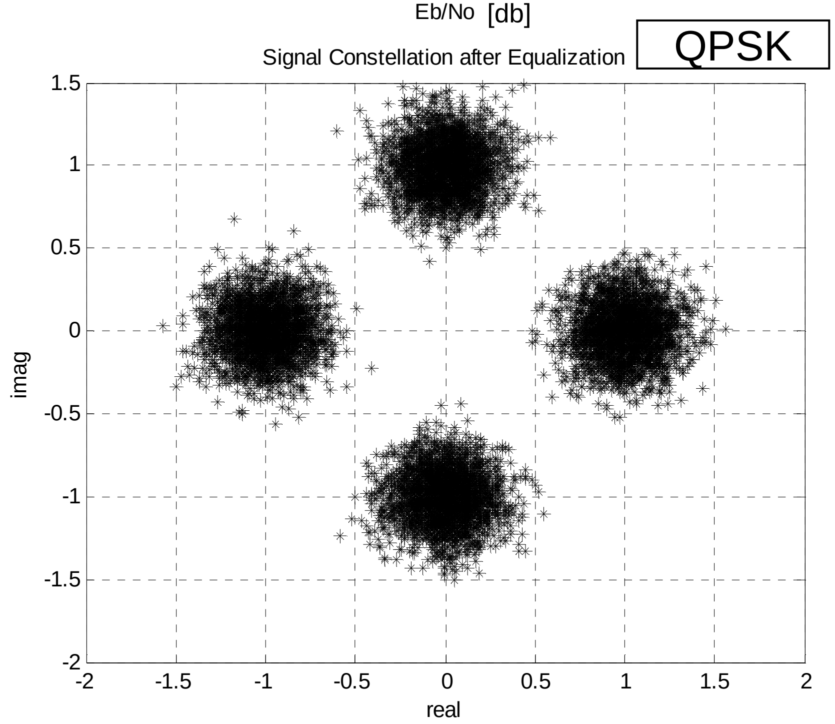
STEP 8 : Plot using the code in the file. Observe:

- The symbols rotated
- The symbols rotated + noise
- The symbols after the equalizer


Symbol/Bit error probabilities. BP
BPSK
SER theoretical
SER simulation
BER theoretical
BER simulation
Bit Error Probability
10^{0}
10^{-1}
10^{-2}
10^{-3}
10^{-4}
10^{-5}
10^{-6}
0 1 2 3 4 5 6 7 8 9 10
Eb/No [db]


Symbol/Bit error probabilities. QPSK
QPSK
SER theoretical
SER simulation
BER theoretical
BER simulation
Bit Error Probability
10^{0}
10^{-1}
10^{-2}
10^{-3}
10^{-4}
10^{-5}
10^{-6}
0 1 2 3 4 5 6 7 8 9 10
Eb/No [db]

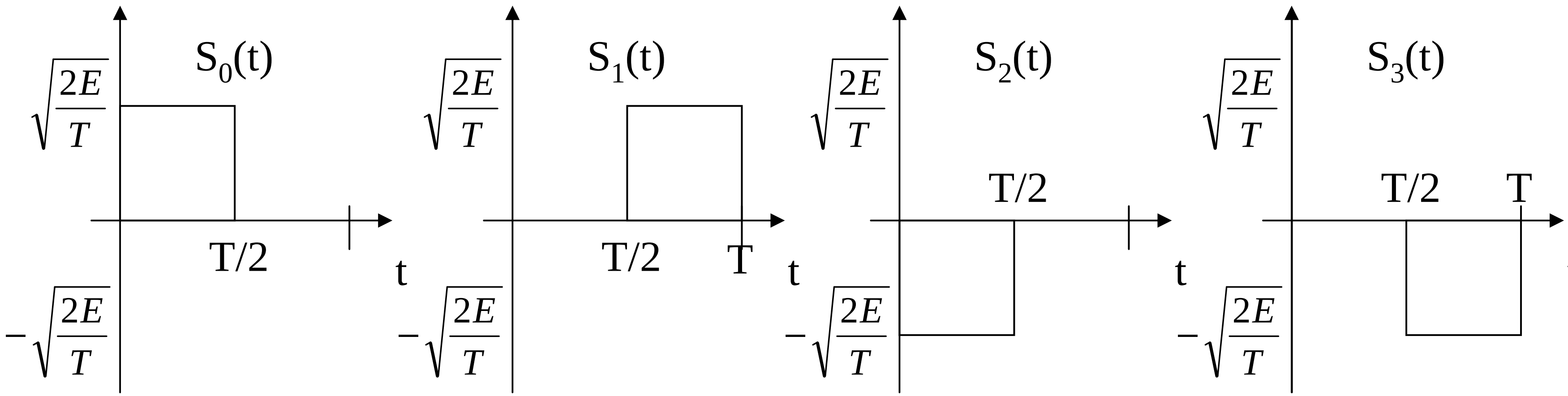

Rotated Signal Constellation with Nois
QPSK
imag
1.5
1
0.5
0
-0.5
-1
-1.5
-1.5 -1 -0.5 0 0.5 1 1.5
real


Signal Constellation after Equalization
QPSK
imag
1.5
1
0.5
0
-0.5
-1
-1.5
-2
-2 -1.5 -1 -0.5 0 0.5 1 1.5 2
real

Exercise 3   System using Biorthogonal Signals

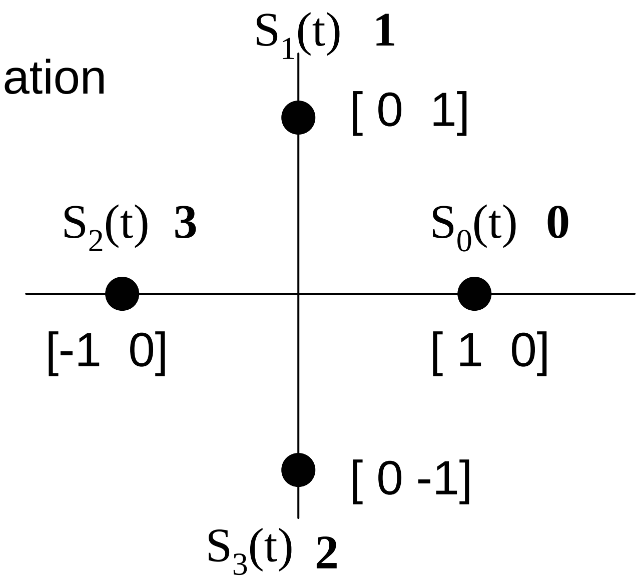
-Simulate a communication system using biorthogonal waveforms
and calculate bit and symbol error ratios.
-Plot the calculated and simulated SER and BER curves.
-Modify the simulator to use orthogonal waveforms.
-Plot the calculated and simulated SER and BER curves.

Exercise 3 System using Biorthogonal Signals



Symbols	**0** 00	**1** 01	**3** 11	**2** 10

Signal constellation



System using Biorthogonal Signals

- Average transmitted signal energy

$$E_{av\_s} = E$$

- Transmitted average energy per bit

$$E_{av\_bit} = \frac{E_{av\_s}}{\log_2 M} = \frac{E_{av\_s}}{2}$$

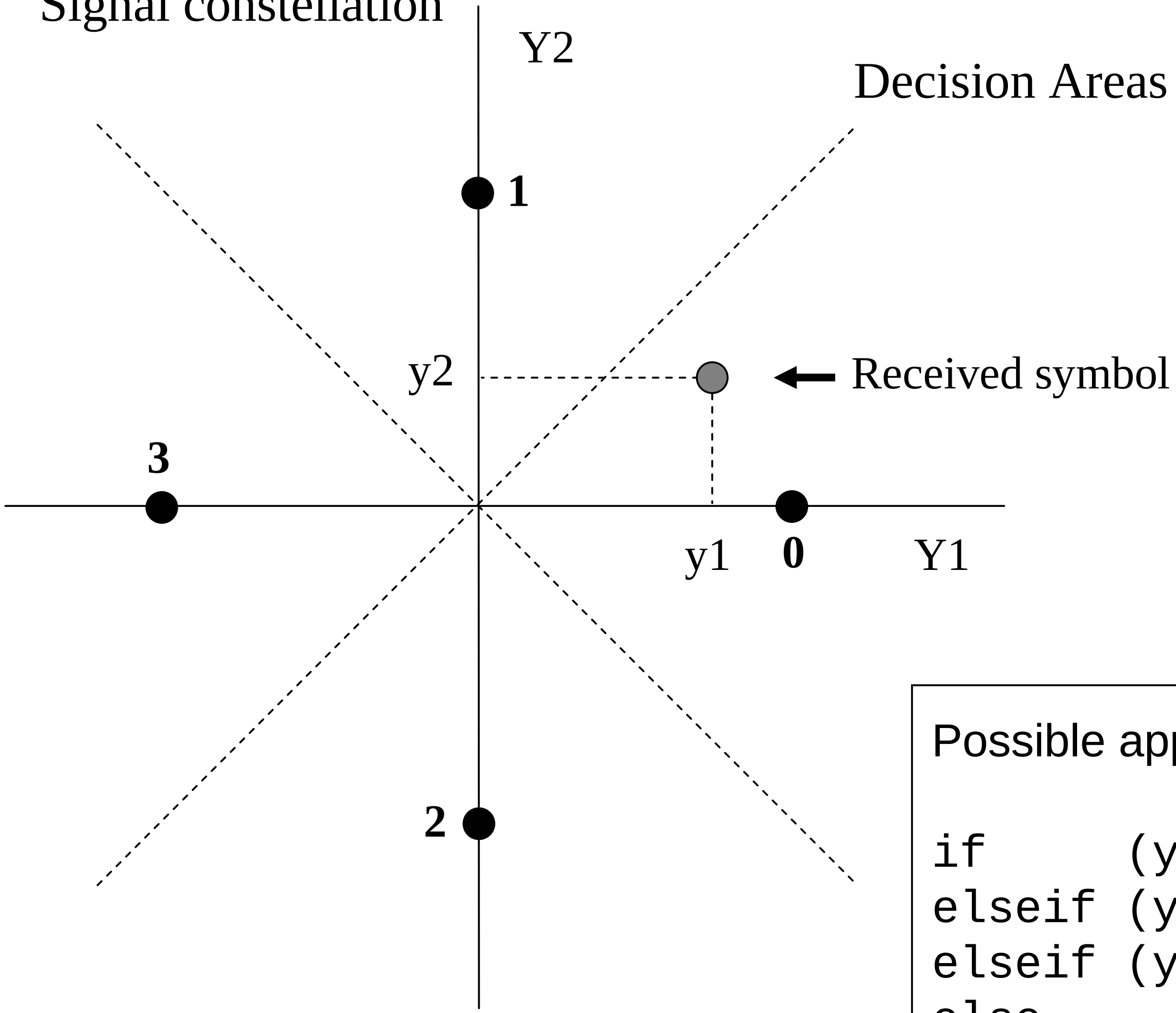
- SNR bit

$$SNR_{bit} = \frac{Eb}{No} = \frac{E_{av\_bit}}{N_0} = \frac{E_{av\_s}}{2N_0} = \frac{E}{2 \cdot 2\sigma^2} = \frac{E}{4\sigma^2}$$

$$\rightarrow \sigma = \sqrt{\frac{E}{4Eb/No}}$$

System using Biorthogonal Signals - Detector

Signal constellation



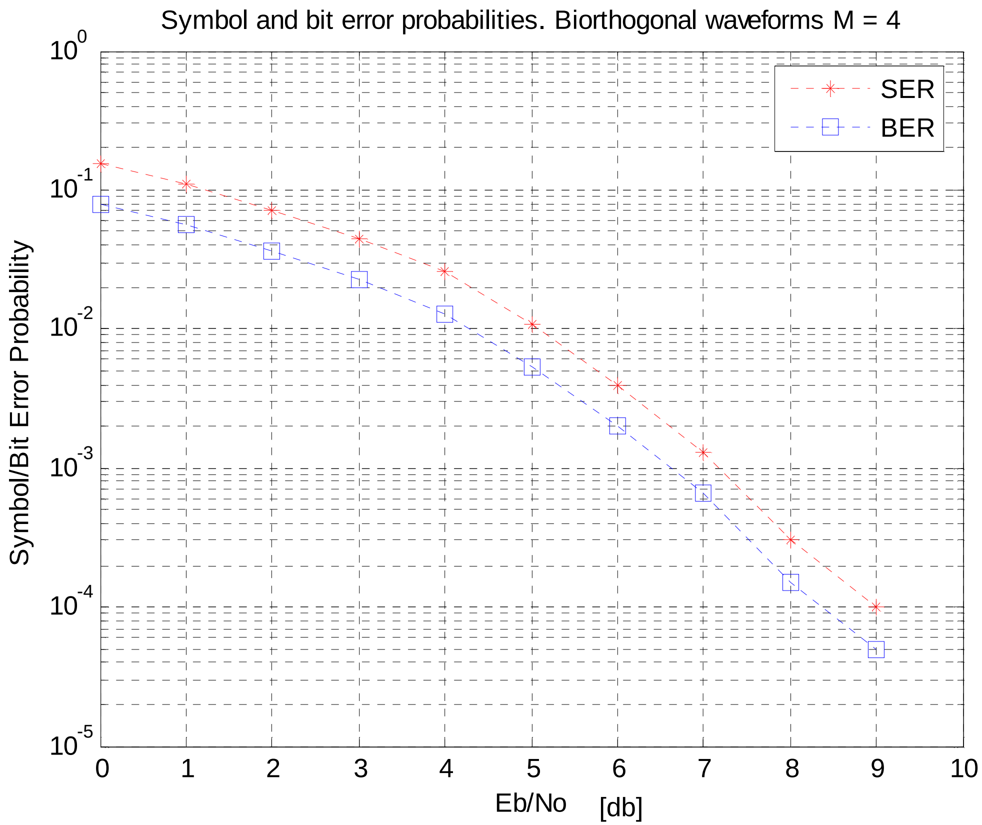
Possible approach to make decisions

```
if     (y1 > abs(y2), S = 0;
elseif (y2 > abs(y1), S = 1;
elseif (y1 <-abs(y2), S = 3;
else                  S = 2;
```

Biorthogonal Signals – Matlab exercise

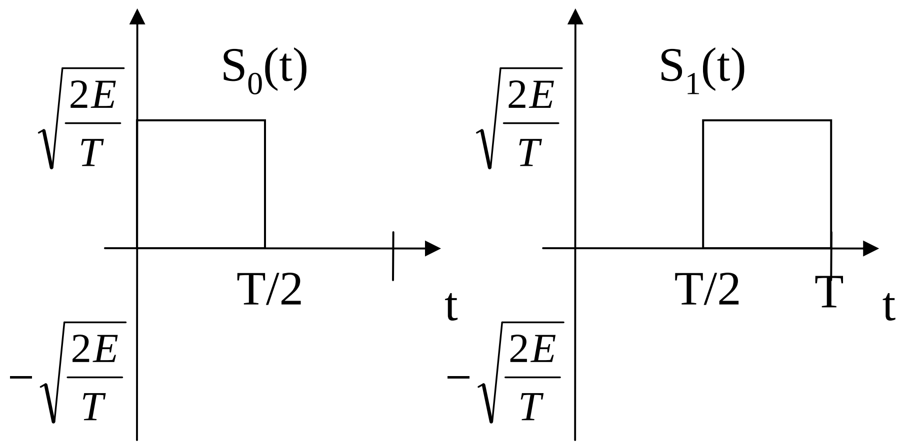
Task : Download the file from the course web page and open with an editor
This code simulates biorthogonal signals and is ready to run.
Compare the Symbol and Bit error probabilities obtained here to the results obtained in the QPSK exercise.

SER / BER



<u>Orthogonal</u> Signals – Matlab exercise

Task : The code provided simulates <u>biorthogonal</u> signals.
Modify the code to simulate the <u>orthogonal</u> signals shown below (M = 2).

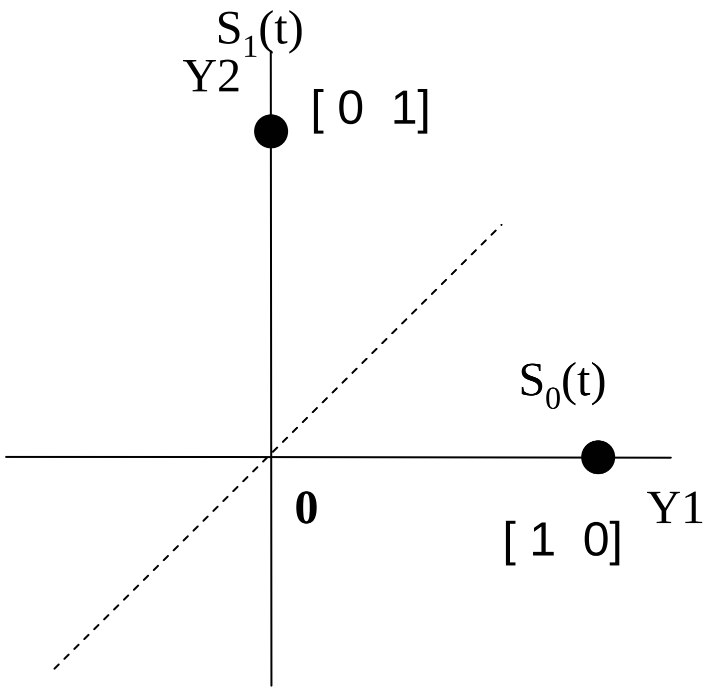


STEP 1: Set M = 2

STEP 2: Change standard deviation

STEP 3: Set modulation

STEP 4: Set the decision areas

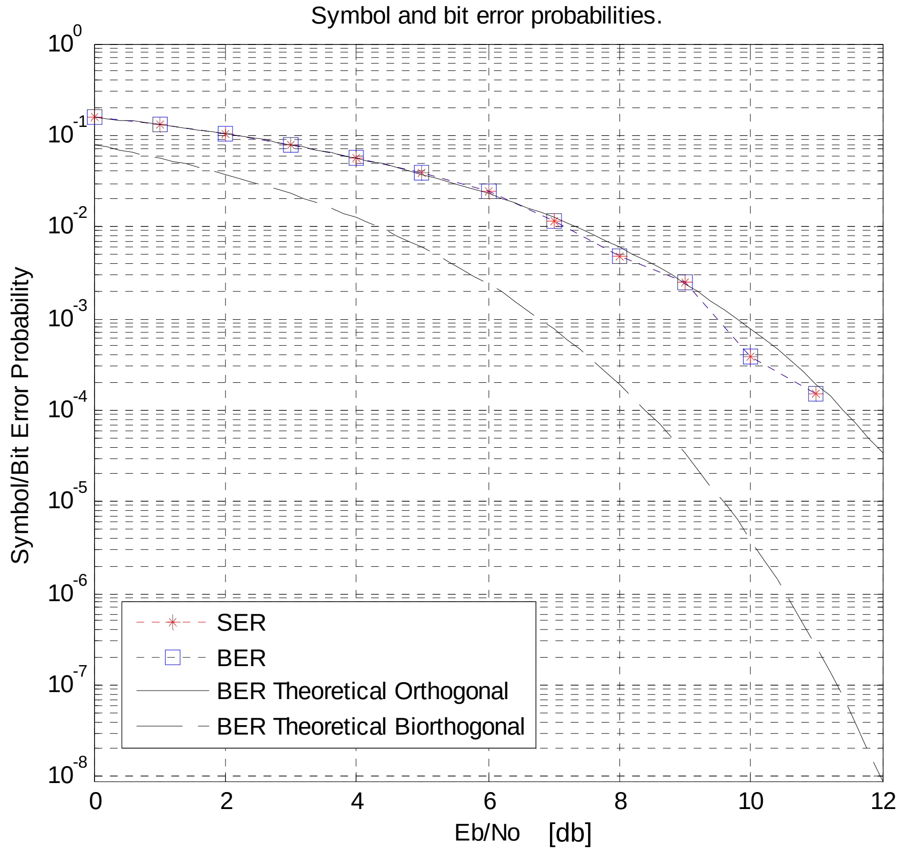


Signal constellation

<u>Orthogonal</u> Signals – Matlab exercise

STEP 5: Observe symbol and bit error rates

STEP 6: Plot theoretical bit error probabilities,
for orthogonal and biorthogonal cases.
Observe 3dB difference.



Exercise 4 Power Spectrum Density

Consider a four-phase PSK represented by the following equivalent low pass signal:

$$u(t) = \sum_n I_n g(t - nT)$$

where I_n takes one of the four possible values: $\sqrt{1/2}(\pm 1 \pm j)$ with equal probability.

a. Determine and sketch the power spectrum density of $u(t)$ when:

$$g(t) = \begin{cases} A & 0 \le t \le T \\ 0 & otherwise \end{cases}$$

b. Repeat **a.** when

$$g(t) = \begin{cases} A\sin\left(\frac{\pi t}{T}\right) & 0 \le t \le T \\ 0 & otherwise \end{cases}$$

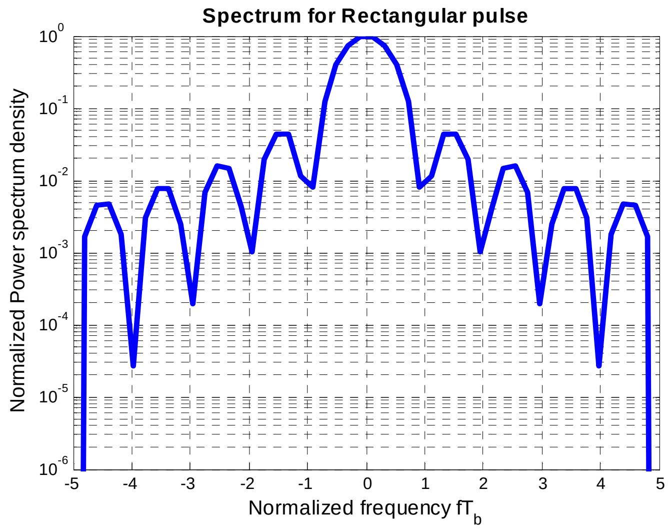
c. Compare the spectra obtained in **a.** and **b.** in terms of the 3dB bandwidth and the bandwidth to the first spectral zero.

Exercise 4

a).

In PAM the I_n coefficients are real but in PSK this is not the case since the signal space is not uni-dimensional.

$$g(t) = \begin{cases} A & 0 \le t \le T \\ 0 & otherwise \end{cases} \quad \xLeftrightarrow{\text{FT}} \quad G(f) = AT\frac{\sin(\pi fT)}{\pi fT}$$



The power spectrum density is calculated as:

$$\Phi_{uu}(f) = \frac{1}{T}\left|G(f)\right|^2$$

$$\left|G(f)\right|^2 = A^2T^2\frac{\sin^2(\pi ft)}{(\pi ft)^2} \Rightarrow \Phi_{uu}(f) = A^2T\frac{\sin^2(\pi ft)}{(\pi ft)^2}$$

First Zero at 1/T

Exercise 4

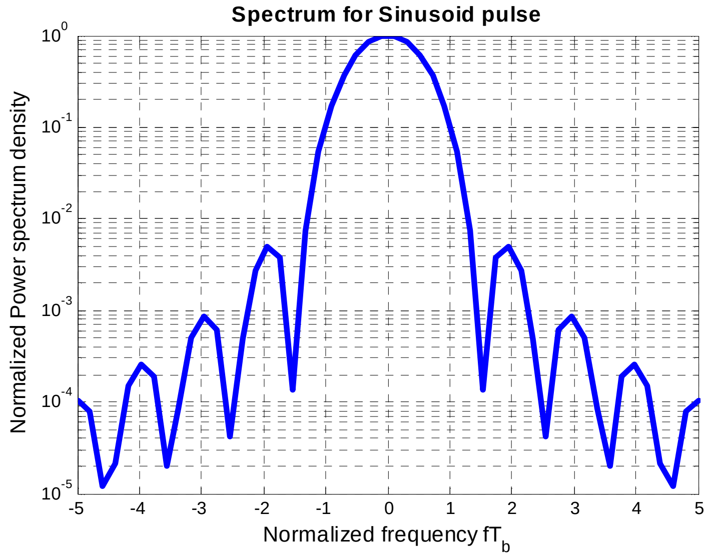
b).

$$g(t)=\begin{cases} A\sin\left(\dfrac{\pi t}{T}\right) & 0\le t\le T \\ 0 & otherwise \end{cases} \quad \overset{\textbf{FT}}{\Longleftrightarrow} \quad G(f)=\frac{2AT}{\pi}\cdot\frac{\cos(\pi Tf)}{1-4T^2f^2}e^{-j\pi Tf}$$

The power spectrum density is calculated:

$$\Phi_{uu}(f)=\frac{1}{T}\left|G(f)\right|^2=\left(\frac{2A}{T}\right)^2 T\frac{\cos^2(\pi Tf)}{\left(1-4T^2f^2\right)^2}$$

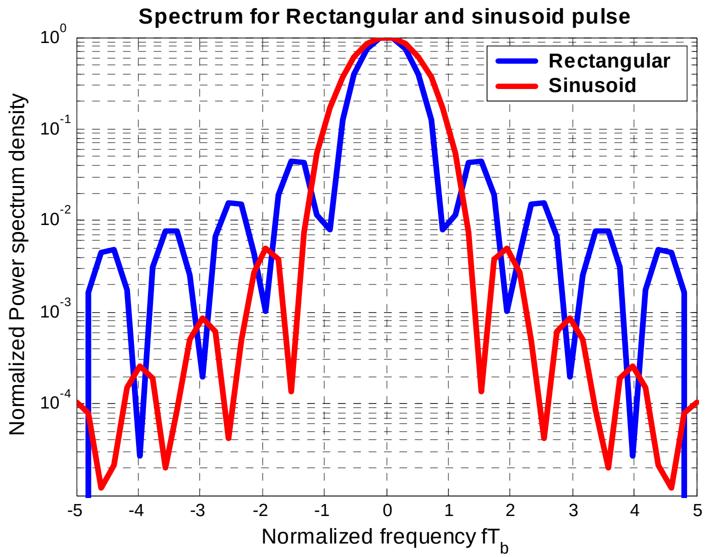
First Zero at 1.5/T



Exercise 4

c).

The power spectrum for the rectangular pulse has narrower mainlobe but higher sidelobes



The sinusoid pulse demands 50% more bandwidth compared to the rectangular pulse

$$\text{rectangular:}\quad \frac{\sin^2(\pi f_{3db} T)}{(\pi f_{3db} T)^2} = \frac{1}{2} \Rightarrow f_{3dB} = \frac{0.44}{T}$$

$$\text{sinusoid:}\quad \frac{\cos^2(\pi T f)}{(1 - 4T^2 f^2)^2} = \frac{1}{2} \Rightarrow f_{3dB} = \frac{0.59}{T}$$

Exercise 4

• To determine the power spectrum density we used the Fourier transform of a proposed signal.
• But the Fourier transform is available only for deterministic signals.
• With random message signals we can not find the Fourier transform.
• Nevertheless we can determine the autocorrelation function of these signals from their statistical information.
• Then we find the power spectrum density from the Fourier transform of the autocorrelation function:

$$\Phi_{uu}(f) = \Im\{\psi(\tau)\}$$

Where $\psi(\tau)$ is the autocorrelation function

$$\psi(\tau) = \int_{-\infty}^{\infty} u^*(t)u\ (t+\tau)\, dt$$

In Matlab we use *xcorr(x)* (crosscorrelation of vector *x* with itself)

Exercise 4

TASK : Simulate a communication system for the given problem using Matlab and determine the Power Spectrum Density when *g(t)* is a rectangular and sinusoidal pulse.