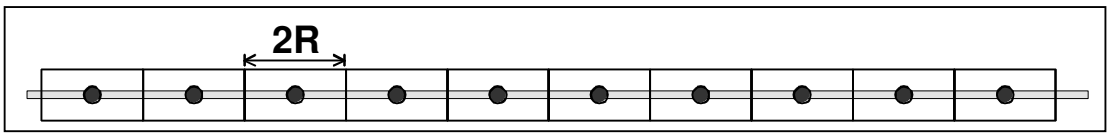
S-72.2210 Home exercises 2005

Home work 1.



On a straight highway the co-channel reuse distance is chosen so, that the CIR-target in GPRS CS1 9.5 dB is reached at the cell border, when a single interferer is considered. The path loss exponent of the average radio path loss is 3.2. All base stations are identical.

a) Determine the minimum normalised reuse distance D/R based on the strongest interferer in the cell configuration given in the above figure. Calculate the achieved CIR-value (dB), when also the interferer in the opposite direction is considered.

b) Check whether GPRS CS4 can be used at the distance R/2 from the base station, when the two strongest interferers are considered, and the CIR-target is now 22 dB.

SOLUTION

a) 3p

A single interferer will cause a CIR-value at the cell border:

$$x = 1.98R$$

where x is the distance from the strongest interferer (closest base station) to the cell border.

With the given cell structure the co-channel interferer must then be at the distance $3R$ from the cell border.

For a single interferer this results in the achieved CIR-value:

$$CIR = 15.27 \text{ dB}$$

When two (both) interferers are considered the CIR-value is calculated as follows:

$CIR = 14.49$ dB

b) 3p

$CIR = 25.43$ dB

As the CIR-value is 3.4 dB higher than the target value the proposed service can be used.

Comment: The previous answer was calculated assuming power control but since it was not mentioned in the exercise this is the correct answer.

Home work 2

How many percent will the coverage area increase if a gain term in the radio link budget is improved with 3 dB, and the path loss exponent of the single slope average loss model is i) 3.0, ii) 4.0, and iii) 5.0?

SOLUTION

n	3	4	5
$A_2/A_1 \rightarrow \Delta A\%$	1.585→58.5%	1.413→41.3%	1.318→31.8%

Home work 3.

After selection combining the probability that the signal to noise ratio (SNR) drops below a given value with M independent diversity branches is $P\{\gamma < \gamma\} = \prod_{k=1}^{M} \left(1 - \exp\left(-\gamma/\gamma_{m_k}\right)\right)$, where γ_{m_k} is the average SNR of the k^{th} diversity branch. Here two-branch diversity is investigated.

a) How large is the diversity improvement at 20 dB fade (i.e. the instantaneous SNR is 20 dB less than the average SNR, when the branches are equally strong in average?

b) The second diversity branch is in average 10 dB weaker than the first branch. How large is now the diversity improvement at 20 dB fade?

SOLUTION

a) 3p $I = 100.5$

b) 3p $I = 10.51$

Home work 4.

The p.d.f of the signal to noise ratio after M-fold diversity with maximum ratio combining is

$$p_c(\gamma) = \frac{1}{(M-1)!} \cdot \frac{\gamma^{M-1}}{\gamma_m^M} \cdot e^{-\gamma/\gamma_m} \, \mathrm{u}(\gamma)$$

a) Derive an expression for the average bit error probability of DPSK in the rapidly fading Rayleigh-channel when two-fold diversity is applied.

b) Calculate the diversity gain at the bit error probability level 0.001.

Solution

a) 3p +1p for good enough explanation for the formula

The average bit error probability is:

$$P_b = \frac{1}{2} \frac{1}{(1+\overline{\gamma})^2}$$

b) 3p

From Problem 11 we have for the system without diversity

$$\overline{\gamma} = 499$$

For the system with diversity

$$\rightarrow G_{div} = 13.68 \text{ dB}$$

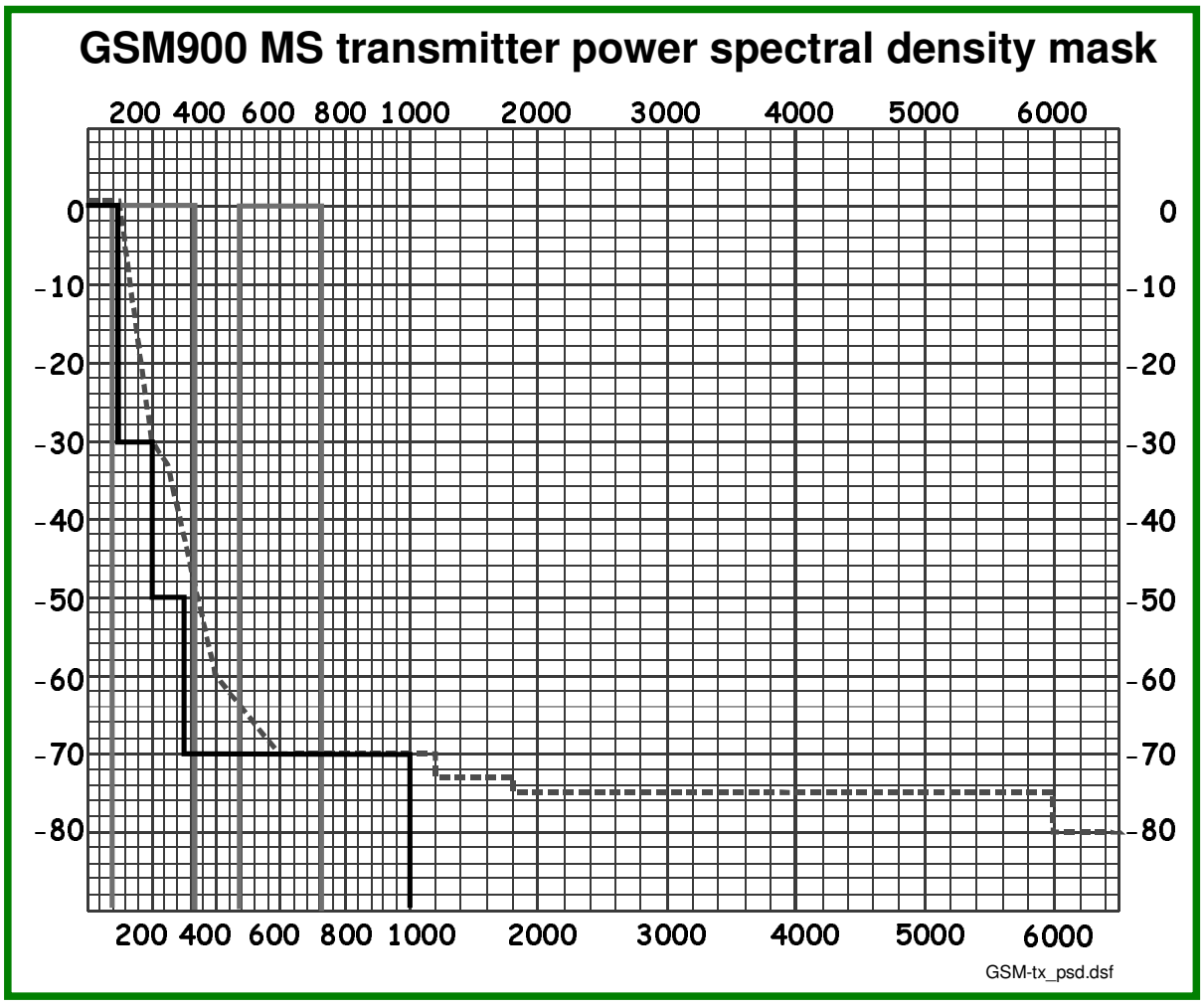
Home work 5.

In the highest power classes the GSM900 BTS power spectrum must be below a spectrum mask shown with a dashed line in the figure. The implemented power spectrum (right part only) is shown with a solid line and is defined as:

$$S_{tx}(f) = \begin{cases} +0.0 \text{ dBc}, & 0 \le f \le 100 \text{ kHz} \\ -30.0 \text{ dBc}, & 100 \le f \le 200 \text{ kHz} \\ -50.0 \text{ dBc}, & 200 \le f \le 300 \text{ kHz}, \\ -70.0 \text{ dBc}, & 300 \le f \le 100 \text{ kHz} \\ -\infty \text{ dBc}, & 1000 \le f \le \infty \text{ kHz} \end{cases}$$

a) Give the piecewise expression of the absolute valued spectral power density defined above and in the figure in logarithmic units.

b) In the mobile phone an ideal band-pass filter with 250 kHz bandwidth is assumed to be used. In the figure this filter is shown shifted to the 1st and 3rd adjacent channels. Under the assumption that BTS transmits the same power on all carriers calculate the signal to interference ratio (dB) on the three first adjacent carriers in



the filter output. The absolute-valued signal and interference powers are defined as the integral $P_o = \int_{-\infty}^{\infty} |H_{rx}(f)|^2 S_{tx,i}(f - n\Delta f_c) df$, where i stands for either signal or interference, n is the order of the adjacent channels ($n = 0$ for the signal channel), and Δf_c is the carrier spacing.

SOLUTION

a) 1p

The spectrum mask must be changed to absolute values

$$S_{tx}(f) = \begin{cases} +0.0 \text{ dBc} \leftrightarrow 1, & 0 \le f \le 100 \text{ kHz} \\ -30.0 \text{ dBc} \leftrightarrow 10^{-3}, & 100 \le f \le 200 \text{ kHz} \\ -50.0 \text{ dBc} \leftrightarrow 10^{-5}, & 200 \le f \le 300 \text{ kHz} \\ -70.0 \text{ dBc} \leftrightarrow 10^{-7}, & 300 \le f \le 100 \text{ kHz} \\ -\infty \text{ dBc} \leftrightarrow 0, & 1000 \le f \le \infty \text{ kHz} \end{cases}$$

b) 5p

Signal power:

$$P_S = 200.05$$

Interference powers:

1st adjacent channel, ideal rx-filter bandwidth 250 kHz

$P_{I1} \cong 25.10$:

$\rightarrow SIR = 9.01$ dB

2nd adjacent channel, ideal rx-filter bandwidth 250 kHz

$P_{I1} = 27.25 \cdot 10^{-5}$

$\rightarrow SIR = 58.66$ dB

3rd adjacent channel, ideal rx-filter bandwidth 250 kHz

$P_{I1} = 2.5 \cdot 10^{-5}$

$\rightarrow SIR = 69.03$ dB

Home work 6.

Estimate the reduction of base station transmit power in dB, when the user density becomes higher near to the base station, having the p.d.f. $p(r,\phi)=\frac{2}{R}\left(1-\frac{r}{R}\right)\frac{1}{2\pi}$. Utilize results from Problem 18.

- ideal power control compensating exactly distance depending path loss is used,
- the cell is dimensioned for 50 % coverage probability at cell border,
- the power control range is i) 10 dB, ii) 20 dB, iii) 30 dB, iv) 40 dB
- the average path loss model is single-slope with the path loss exponent 3.5.

SOLUTION

6p

For instance with $10\lg\left(P_{\min}/P_{\max}\right)=-20$ dB, and $n = 3.5$

$$-10\lg\left(\frac{P_{txm}}{P_{\max}}\right)=10.73 \text{ dB}$$

P-ratio	$n = 3.5$
10 dB	8.41 dB
20 dB	10.73 dB
30 dB	10.92 dB
40 dB	10.93 dB

Home work 7.

The user rate and EGPRS900-BTS receiver sensitivity requirements for different modulation and coding classes in different propagation environments are given in the table below.

Coding class	User net rate kbit/s	BTS-receiver sensitivity/dBm				
		static	TU50	TU50FH	RA250	HT100
MCS1	8.8	–104	–102.5	–103	–103	–102
MCS2	11.2	–104	–100.5	–101	–100.5	–100
MCS3	14.8	–104	–96.5	–96.5	–92.5	–95.5
MCS4	17.6	–101.5	–91	–91	–	–
MCS5	22.4	–101	–96.5	–97	–96	–95
MCS6	29.6	–99.5	–94	–94.5	–91	–91
MCS7	44.8	–96	–89	–88.5	–87	–86
MCS8	54.2	–93	–84	–84	–	–81.5
MCS9	59.2	–91.5	–80	–80	–	–

a) Calculate for the up-link the distance from the BTS where switching from one MCS to another takes place and the average bearer rate when the user is moving along a cell diameter in TU50 propagation environments, when cell size is dimensioned based on –104 dBm sensitivity, and the path loss exponent is 3.5.
b) Calculate under the same conditions the average bit rate, if the cell is dimensioned based on sensitivity value – 90 dBm

Hints: The change in sensitivity will change the effective coverage area of each MCS. Use the radio link budget to calculate the coverage area radius of each MCS, and then the distance along the diameter where each MCS will be used to give the highest rate. In the calculation of the average it can be assumed that the time to change MCS can be neglected.

SOLUTION

a)

3p

Coding class	User net rate kbit/s	BTS-receiver sensitivity difference/dBm				
		static	TU50	TU50 FH	RA250	HT100
MCS1	8.8	0	–1.5	–1	–1	–2
MCS2	11.2	0	–3.5	–3	–3.5	–4
MCS3	14.8	0	–7.5	–7.5	–11.5	–8.5
MCS4	17.6	–2.5	–13	–13	–	–
MCS5	22.4	–3	–7.5	–8	–8	–9
MCS6	29.6	–4.5	–10	–9.5	–13	–13
MCS7	44.8	–8	–15	–15.5	–17	–18
MCS8	54.2	–11	–20	–20	–	–22.5
MCS9	59.2	–12.5	–24	–24	–	–

Coding class	User net rate kbit/s	r/R-ratio, TU50				
		$n = 2$	$n = 3$	$n = 3.5$	$n = 4$	$n = 5$
MCS1	8.8	0.841	0.891	0.906	0.917	0.933
MCS2	11.2	0.668	0.764	0.794	0.818	0.851
MCS3	14.8	0.422	0.562	0.611	0.649	0.708
MCS4	17.6	0.224	0.369	0.425	0.473	0.550
MCS5	22.4	0.422	0.562	0.611	0.649	0.708
MCS6	29.6	0.316	0.464	0.518	0.562	0.631
MCS7	44.8	0.178	0.316	0.373	0.422	0.501
MCS8	54.2	0.100	0.215	0.268	0.316	0.398
MCS9	59.2	0.063	0.158	0.206	0.251	0.331

In the TU50FH-channel:

$n = 3.5 \rightarrow R_{bm} = 29.704$ kbit/s

This is calculated from base station to the cell border when the mobile moves along the cell diameter so no areas are calculated. If you have calculated from cell border to cell border it will also be ok.
If you have calculated the average rate in the whole cell like in Ex 19 it will not be a mistake if you have mentioned that you calculate the average in the whole cell.

b) 3p

Coding class	User net rate kbit/s	BTS-receiver sensitivity difference/dBm				
		TU50 ΔS =0 dB	TU50 ΔS = 10 dB	TU50 ΔS = 14 dB		
MCS1	8.8	–1.5	8.5	12.5		
MCS2	11.2	–3.5	6.5	10.5		
MCS3	14.8	–7.5	2.5	6.5		
MCS4	17.6	–13	–3	1.0		
MCS5	22.4	–7.5	2.5	6.5		
MCS6	29.6	–10	0	4.0		
MCS7	44.8	–15	–5	–1.0		
MCS8	54.2	–20	–10	–6.0		
MCS9	59.2	–24	–14	–10.0		

Coding class	User net rate kbit/s	r/R-ratio, TU50, ΔS = 14 dB				
		$n = 2$	$n = 3$	$n = 3.5$	$n = 4$	$n = 5$
MCS1	8.8			>1.000		
MCS2	11.2			>1.000		
MCS3	14.8			>1.000		
MCS4	17.6			>1.000		
MCS5	22.4			>1.000		
MCS6	29.6			>1.000		
MCS7	44.8			0.936		
MCS8	54.2			0.674		
MCS9	59.2			0.518		

When the cell is dimensioned for a –90 dBm sensitivity, the average user rate is:

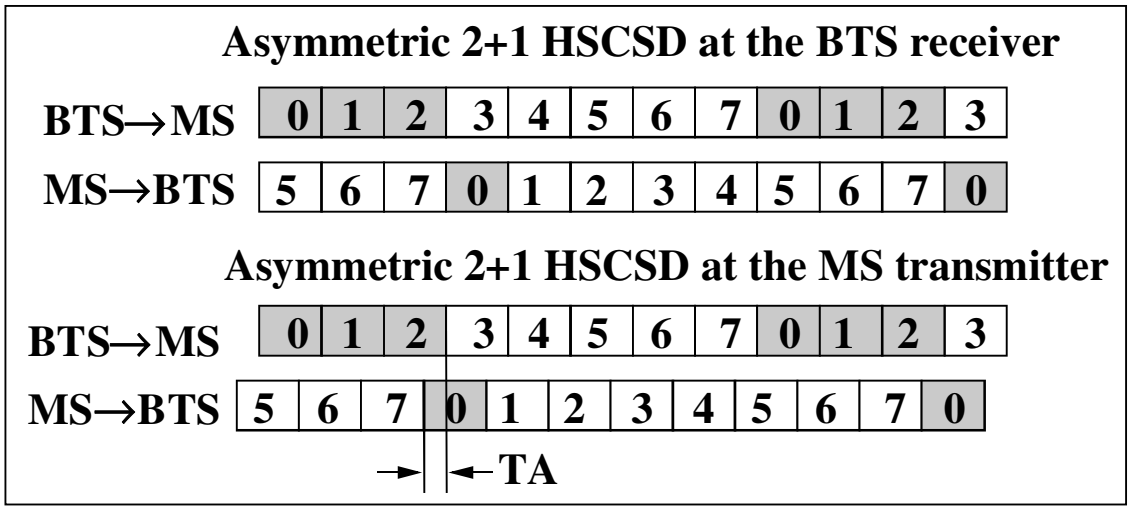
$$n = 3.5 \rightarrow R_{bm} = 52.753 \text{ kbit/s}$$

Home work 8.

a) How long time will the transmitted and received symbols overlap in the 3+1 multi-slot EDGE mobile station, when it transmits and receives normal bursts with maximum timing advance?

b) Determine the theoretical degradation of receiver sensitivity at symbol error probability 0.01 of the uncoded symbols in i) the static AWGN-channel, and ii) the single-tap Rayleigh-fading channel, when link adaption switches from MCS1 to MCS5?

SOLUTION



a) 2p

In this part I will accept more than one correct answer because in the material there is more than one rate given for EDGE without fully describing in which scenario these rates refer to.

b) 4p

i) in the AWGN-channel without fading the theoretical bit error probabilities are:

$$\text{GMSK, BT} = 0.3 \rightarrow P_b = \text{Q}\left(\sqrt{\frac{1.78P_{rx}}{N_o R_s}}\right)$$

$$\rightarrow P_{rxMCS1} = 3.05 N_o R_b$$

$$\text{8PSK } P_{b,8PSK} = \frac{2}{3}\mathrm{Q}\left(\sqrt{\frac{0.879 \cdot P_{rx}}{N_o R_b}}\right)$$

$$\rightarrow P_{rx,MCS5} \approx 5.36 N_o R_b$$

Thus in the AWGN-channel the degradation is

$$\Delta SNR = 2.45 \text{ dB}$$

ii) in the flat fading Rayleigh-channel the average bit error probability is

$$P_b = \frac{1}{2}\left[1 - \sqrt{\frac{k\gamma_o}{2 + k\gamma_o}}\right]$$

In GMSK $k = 1.78$ which gives

$$\rightarrow \gamma_o = 27.25$$
$$\rightarrow P_{rx,MCS1} = 27.25 N_o R_b$$

In 8PSK $k = 0.879$ and the coefficient before the Q-function = 2/3, which gives

$$\rightarrow \gamma_o = 36.22$$
$$\rightarrow P_{rx,MCS1} = 36.22 N_o R_b$$

Thus in the Rayleigh-fading AWGN-channel the degradation is

$$\Delta SNR = 1.24 \text{ dB}$$

Home work 9.

The maximum processing gain in the WCDMA-system is 512.

a) How many simultaneous users with the activity factor 0.4 can coexist in theory (pole capacity) in a single-cell network, if a sufficient performance requires a signal to interference ratio of 3 dB. The up-link is investigated and all users are received with the same power at the base station (ideal power control).

b) The power control is malfunctioning for one user, which is transmitting with constant power corresponding to that required at the cell border. How near to the base station (measured in cell radius) this user can come without reducing the total number more than half of the number in sub-task b? The path loss exponent is 3.5, and slow and fast fading are not considered.

SOLUTION

a) 3p

$$M = 642{,}52 \rightarrow 642$$

b) 3p

$$\frac{r}{R_{cell}} = 0.192$$

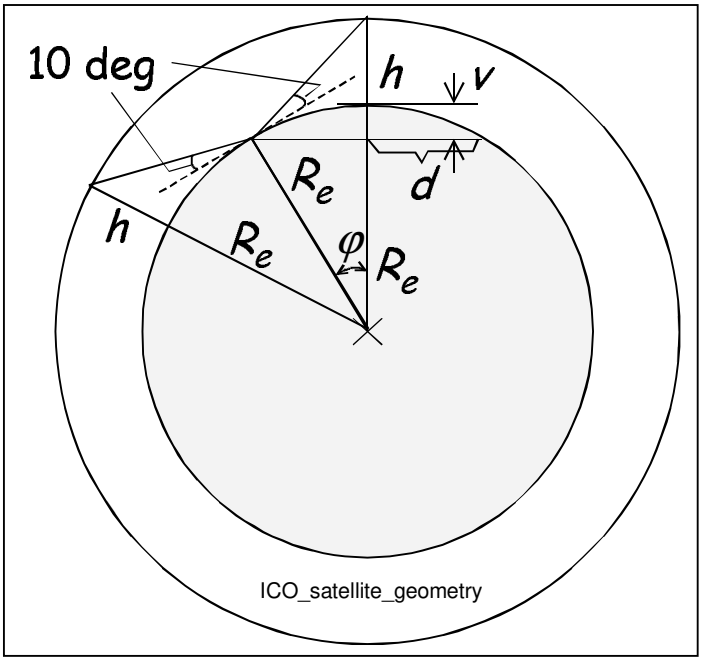
Comment: Wrong parameter value was used in the previous result.

Home work 10.

A mobile satellite system uses satellites in circular orbits at 1000 km altitude.

a) How large is the coverage area of a satellite, if this is defined as the area from which the satellite elevation (angle above the tangential plane in the observation point) is more than 10 degrees at a given time instant?

b) During how long time will the satellite be observed with more than 10 degree elevation, if it passes through zenith in the observation point?



Area of a spherical segment:

$$A = 2\pi R_e v$$

Full orbit time:

$$T = \frac{2\pi}{\sqrt{\mu}} r^{1.5},$$

$$\mu = 3.986 \cdot 10^{14} \text{ m}^3/\text{s}^2$$

SOLUTION

a) 3p

$$A = 2\pi R_e v = 17998744 \text{ km}^2$$

b) 3p

$$T_{obs} = \frac{\varphi}{360} T = 378.8 \text{ s}$$

Twice this value will also be accepted. These results have been calculated using Earth radius 6370km. This is the average radius. Pole or zenith radii are accepted also.