S-72.2211 Mobile Communication Systems and Services

Principles of DS-CDMA

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Spread Spectrum Techniques

- Spreading the transmission bandwidth to be $>>$ information symbol frequency
  - introducing redundancy by occupying more frequency
- Frequency Hopped Spread Spectrum (FH-SS)
- Direct Sequence Spread Spectrum
  - spreading by multiplying linear modulation symbols with a spreading code
  - different spreading codes can be used to multiplex channels and/or for multiple access
    - Direct Sequence Code Division Multiple Access (DS-CDMA)

Spreading: Chips and Symbols

- A chip is the shortest modulated signal in a DS-CDMA system
  - Chip rate   signal bandwidth
- A symbol is spread over multiple chips
- The Spreading Factor (SF) tells how many chips are used to transmit one symbol

$$\text{spreading factor} = SF = \frac{R_c}{R_s} = \frac{\text{chip rate}}{\text{symbol rate}}$$

  - SF is a spectrum spreading factor: bit rate $\Rightarrow$ chip rate
- The Spreading Code is a sequence of SF chips
  - Usually the chips are +-1
  - Spreading code can be understood as $SF \times 1$ vector $\mathbf{c}$
  - normalization: $\mathbf{c}^{\mathrm{T}}\mathbf{c} = SF$
  - Example: SF=8 spreading code $\mathbf{c} = \begin{bmatrix} 1 & 1 & -1 & 1 & -1 & -1 & 1 & -1 \end{bmatrix}^{\mathrm{T}}$
- Spreading
  - Transmitted symbol x (any linear modulation)
  - Transmitted chip sequence $x\mathbf{c} = \begin{bmatrix} x & x & -x & x & -x & -x & x & -x \end{bmatrix}^{\mathrm{T}}$

Despreading

- At receiver, despreading is performed
- Example: 1-tap channel h,constant during transmission
- equivalent baseband signal model, one sample per chip
  - received signals during the SF chips of transmitting the spread symbol:

$$\mathbf{y} = hx\mathbf{c} + \mathbf{n} = \begin{bmatrix} hx & hx & -hx & hx & -hx & -hx & hx & -hx \end{bmatrix}^{\mathrm{T}} + \mathbf{n}$$

    where $\mathbf{n}$ is noise and interference
  - despreading by multiplying with the (transposed) spreading code:

$$\mathbf{z} = \mathbf{c}^{\mathrm{T}}\mathbf{y} = hx\mathbf{c}^{\mathrm{T}}\mathbf{c} + \mathbf{c}^{\mathrm{T}}\mathbf{n} = SF\, hx + \mathbf{c}^{\mathrm{T}}\mathbf{n}$$

- the chips carrying information about symbol x are coherently combined
- noise and interference is despread and non-coherently combined (as long as interference is not transmitted with the same spreading code)

Interference from Other Code Channel

- assume transmission on another code channel (e.g. another user) with same timing and same SF
  - symbol, code, channel of user of interest: $x_1,\ h_1,\ \mathbf{c}_1$
  - symbol, code, channel of interfering user: $x_2,\ h_2,\ \mathbf{c}_2$
- received signals during the SF chips

$$\mathbf{y} = h_1 x_1 \mathbf{c}_1 + h_2 x_2 \mathbf{c}_2 + \mathbf{n}$$

  where $\mathbf{n}$ is noise (and other interference)

- despreading:

$$\mathbf{z}_1 = \mathbf{c}_1^{\mathrm{T}} \mathbf{y} = SF\ h_1 x_1 + h_2 x_2 \mathbf{c}_1^{\mathrm{T}} \mathbf{c}_2 + \mathbf{c}_1^{\mathrm{T}} \mathbf{n}$$

- interference caused by transmission using $\mathbf{c}_2$ on $\mathbf{c}_1$ determined by *cross-correlation* $\mathbf{c}_1^{\mathrm{T}}\mathbf{c}_2$ of $\mathbf{c}_1$ and $\mathbf{c}_2$

Interference form Other Channel Tap

- Example: 2-tap channel $\left[\ h_1\ \ h_2\ \right]$
- received signals during the SF chips of transmitting the spread symbol:

$$\begin{aligned}\mathbf{y} = \ & h_1 x \ \left[1\ 1\ -1\ 1\ -1\ -1\ 1\ -1\right]^{\mathrm{T}} + \\ & h_2 x \ \left[0\ 1\ 1\ -1\ 1\ -1\ -1\ 1\right]^{\mathrm{T}} + \\ & h_2 x_0 \ \left[-1\ 0\ 0\ 0\ 0\ 0\ 0\ 0\right]^{\mathrm{T}} + \mathbf{n} \\ = \ & h_1 x \ \ \mathbf{c}^{\mathrm{T}} + h_2\ x \mathbf{c}_{\text{shift } 1}^{\mathrm{T}} + \left[\ h_2(x_0 - x)c_1\ \ 0\ 0\ 0\ 0\ 0\ 0\ 0\right]^{\mathrm{T}} + \mathbf{n}\end{aligned}$$

  - x_0 is the previous transmitted symbol, c_1 is the first chip in $\mathbf{c}$
  - $\mathbf{c}_{\text{shift } 1}$ is the permuted version of $\mathbf{c}$ where last chip is first
- despreading with timing of first channel tap:

$$\mathbf{z} = \mathbf{c}^{\mathrm{T}} \mathbf{y} \approx SF\ h_1 x + h_2 x\ \mathbf{c}^{\mathrm{T}} \mathbf{c}_{\text{shift } 1} + \mathbf{c}^{\mathrm{T}} \mathbf{n}$$

- Inter-Path Interference is characterized by *auto-correlation* $\mathbf{c}^{\mathrm{T}}\mathbf{c}_{\text{shift } n}$ of code with shifted versions of itself

Orthogonal and Non-orthogonal Codes I

- A family of spreading codes is a set $\{\mathbf{c}_j\}$
  - different spreading codes define different code multiplexed channels
- A family of spreading codes is orthogonal if

$$\mathbf{c}_i^{\mathrm{T}}\mathbf{c}_j = SF\delta_{ij}$$

  - Here δ_{ij} is the Kronecker delta, $\delta_{ij} = \begin{cases} 0, & i \neq j \\ 1, & i = j \end{cases}$
- at most SF orthogonal codes with length SF
  - example for $SF = 4$:

$$\mathbf{c}_1 = \begin{bmatrix} 1 \\ 1 \\ 1 \\ 1 \end{bmatrix} \quad \mathbf{c}_2 = \begin{bmatrix} 1 \\ -1 \\ 1 \\ -1 \end{bmatrix} \quad \mathbf{c}_3 = \begin{bmatrix} 1 \\ 1 \\ -1 \\ -1 \end{bmatrix} \quad \mathbf{c}_4 = \begin{bmatrix} 1 \\ -1 \\ -1 \\ 1 \end{bmatrix}$$

  - these are mutually orthogonal:

$$\mathbf{c}_1^{\mathrm{T}}\mathbf{c}2 = \mathbf{c}_1^{\mathrm{T}}\mathbf{c}3 = \mathbf{c}_1^{\mathrm{T}}\mathbf{c}4 = \mathbf{c}_2^{\mathrm{T}}\mathbf{c}3 = \mathbf{c}_2^{\mathrm{T}}\mathbf{c}4 = \mathbf{c}_3^{\mathrm{T}}\mathbf{c}4 = 0$$

  - and normalized:

$$\mathbf{c}_1^{\mathrm{T}}\mathbf{c}1 = \mathbf{c}_2^{\mathrm{T}}\mathbf{c}2 = \mathbf{c}_3^{\mathrm{T}}\mathbf{c}3 = \mathbf{c}_4^{\mathrm{T}}\mathbf{c}4 = SF = 4$$

Orthogonal and Non-orthogonal Codes II

- Orthogonality is lost if codes are received with different timing
- Autocorrelation properties of orthogonal codes are poor
  - for example the autocorrelation of $\mathbf{c}_1 = [1\ 1\ 1\ 1]$ with any shifted version of itself is $SF = 4$
  - this leads to poor resistance against inter-path interference
- For *random spreading codes*, cross & and autocorrelation are

$$E\left\{\mathbf{c}_i^{\mathrm{T}}\mathbf{c}_j\right\} = \sqrt{SF} \ \text{ for } i \neq j \qquad E\left\{\mathbf{c}^{\mathrm{T}}\mathbf{c}_{\ \mathrm{shift}n}\right\} = \sqrt{SF} \ \text{ for } n \neq 0$$

  - expected cross-correlation of two random codes is $\sqrt{SF}$
  - expected auto-correlation of random code with shifted versions of itself is $\sqrt{SF}$

Multicode CDMA vs. TDMA

- With orthogonal codes, the channel can be divided into SF orthogonal code channels, each with symbol rate 1/SF of the chip rate
- Just as with TDMA, one can divide the chip rate into SF orthogonal time domain channels with rate 1/SF
  - in TDMA, symbols/channels are multiplexed in the time domain
  - in orthogonal CDMA, symbols/channels are multiplexed in the code domain $\Rightarrow$ Multicode transmission
- in orthogonal CDMA with given SF, at most SF orthogonal channels can be designed

Multicode CDMA vs. TDMA Example I

- 4 symbols on 4 signaling channels, possibly with different wireless channels h_1, h_2, h_3, h_4:
- Time Division Multiplexed channels: $\mathbf{y} = \begin{bmatrix} h_1x_1 \\ h_2x_2 \\ h_3x_3 \\ h_4x_4 \end{bmatrix} + \mathbf{n}$
- 4 Code Division Multiplexed channels with $SF = 4$:
  - define spreading matrix using SF orthogonal codes: $\mathbf{C} = \begin{bmatrix} \mathbf{c}_1 & \mathbf{c}_2 & \mathbf{c}_3 & \mathbf{c}_4 \end{bmatrix}$
  - for orthogonal spreading, $\mathbf{C}$ is orthogonal: $\mathbf{C}^{\mathrm{T}}\mathbf{C} = \mathbf{C}\mathbf{C}^{\mathrm{T}} = SF\,\mathbf{I}$
  - Rx signal is sum of Rx signals of the CDM channels:

$$\begin{aligned}\mathbf{y} &= \tfrac{1}{2}(h_1x_1\mathbf{c}_1 + h_2x_2\mathbf{c}_2 + h_3x_3\mathbf{c}_3 + h_4x_4\mathbf{c}_4) + \mathbf{n} \\ &= \tfrac{1}{2}\begin{bmatrix} \mathbf{c}_1 & \mathbf{c}_2 & \mathbf{c}_3 & \mathbf{c}_4 \end{bmatrix}\begin{bmatrix} h_1x_1 \\ h_2x_2 \\ h_3x_3 \\ h_4x_4 \end{bmatrix} + \mathbf{n} = \tfrac{1}{\sqrt{SF}}\mathbf{C}\begin{bmatrix} h_1x_1 \\ h_2x_2 \\ h_3x_3 \\ h_4x_4 \end{bmatrix} + \mathbf{n}\end{aligned}$$

  - normalization by $\frac{1}{2}$ to have same Tx power in TDM and CDM

Multicode CDMA vs. TDMA Example II

- despreading of all channels: 4 despreading outputs z_i
  - include scaling with $1/\sqrt{SF} = 1/2$ into despreading
    - scales received signals and interference + noise similarly

$$\mathbf{z} = \begin{bmatrix} z_1 \\ z_2 \\ z_3 \\ z_4 \end{bmatrix} = \frac{1}{\sqrt{SF}}\mathbf{C}^{\mathrm{T}}\mathbf{y} = \frac{1}{\sqrt{SF}}\mathbf{C}^{\mathrm{T}}\mathbf{C}\begin{bmatrix} h_1x_1 \\ h_2x_2 \\ h_3x_3 \\ h_4x_4 \end{bmatrix} + \frac{1}{\sqrt{SF}}\mathbf{C}^{\mathrm{T}}\mathbf{n}$$

  - with orthogonal spreading $\mathbf{z} = \begin{bmatrix} h_1x_1 \\ h_2x_2 \\ h_3x_3 \\ h_4x_4 \end{bmatrix} + \tilde{\mathbf{n}}$
  - noise hase not been coloured in despreading:

$$\mathrm{E}\left\{\tilde{\mathbf{n}}\tilde{\mathbf{n}}^{\mathrm{H}}\right\} = \frac{1}{SF}\mathrm{E}\left\{\mathbf{C}^{\mathrm{T}}\mathbf{n}\mathbf{n}^{\mathrm{H}}\mathbf{C}\right\} = \frac{1}{SF}\mathbf{C}^{\mathrm{T}}\underbrace{\mathrm{E}\left\{\mathbf{n}\mathbf{n}^{\mathrm{H}}\right\}}_{=N_0\mathbf{I}}\mathbf{C} = N_0\mathbf{I}$$

- In 1-tap channel TDM & multicode orthogonal CDM equivalent
  - difference in Peak-to-average power Ratio (PAR), higher in CDM

Multicode CDMA vs. TDMA: Conclusion

- Apart for PAR, TDM and orthogonal CDM differ in wideband multitap channels
  - in TDM Inter-Symbol Interference (ISI) caused
  - in CDM, inter-chip interference results in Inter-Path Interference (IPI)
- To mitigate ISI in TDM, multipath equalizers required
  - polynomic complexity in delay spread
- in CDM, IPI is mitigated by RAKE receiver
  - linear complexity in delay spread
  - with increasing IPI, the number of multicodes in CDM can be diminished, to keep received signal at target level
- With fixed bandwidth & chip rate, CDMA with large SF offers less complex receiver processing against multipath interference than TDMA and a more graceful diminishing of transmission rate with increasing interference

Spreading in Cellular Systems

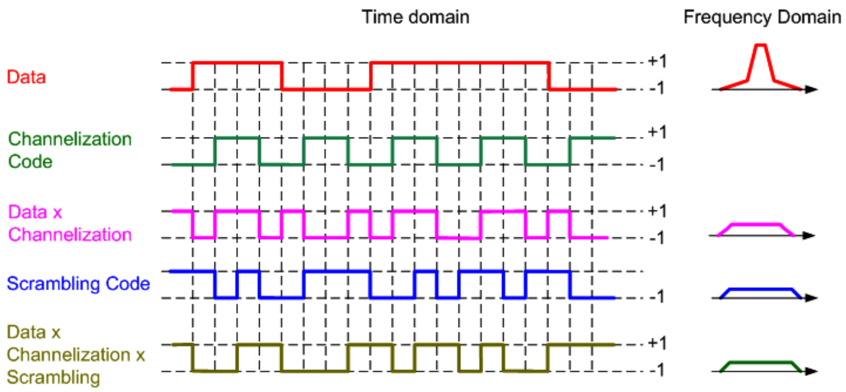
- Downlink is intra-cell synchronous by definition
  - Transmissions to all users have the same timing
  - Orthogonal spreading may be used
- Including possible multicode transmissions to a user
- To synchronize intra-cell UL, accurate Timing Advance is required
  - Less than a fraction of the chip rate
  - If UL is intra-cell synchronized, orthogonal spreading may be used
  - If UL is not synchronous, orthogonal spreading for different users cannot be used
- Pseudo random spreading is used to randomize interference
- In WCDMA, chip rate 4 Mcps, UL synchonization not considered
  - required TA accuracy would be $< 10^{-7}$ s
  - Orthogonal spreading can be used for multicode transmission from a user
- In CDMA systems, resources used in different cells are not orthogonal
  - Reuse 1
  - Pseudo-random spreading (“scrambling”) to mitigate inter-cell interference

Spreading Partitioning

- functions of spreading in Cellular system
  1. to provide immunity against multipath interference
     - good autocorrelation properties
  2. to whiten the inter-cell interference (randomization)
  3. to provide orthogonal CDM for synchronized intra-cell channels
- these somewhat conflicting targets are achieved by partitioning spreading code into two parts
  - scrambling code
    - a long sequence of pseudo-random +-1:s generated by mathematically defined random sign generator, known at BS and MS
    - fulfils targets 1 and 2
  - channelization code
    - performs the spreading from symbol rate to chip rate
    - length SF
    - family of SF orthogonal codes, provides multiple orthogonal channels, if needed
    - fulfils target 3
- a symbol is spread by multiplying with a channelization code of length SF and a changing set of SF consecutive chips of the scrambling code

Spreading and Scrambling, Example

- $SF = 4$
- channelization code $\mathbf{c} = \begin{bmatrix} -1 & 1 & 1 & -1 \end{bmatrix}$
- pseudo-random scrambling code
- data symbols are BPSK, one bit per symbol, values ± 1
- spectrum is spread from bit rate to chip rate = 4 x bit rate



Processing Gain I

- multi-user CDMA (UL), N users
- the received signal of user k after despreading

$$z_k = \mathbf{c}_k^{\mathrm{T}}\mathbf{y} = SF\ h_k x_k + \sum_{\substack{i=1 \\ i \neq k}}^{N} h_i x_i \mathbf{c}_k^{\mathrm{T}} \mathbf{c}_i + \mathbf{c}_k^{\mathrm{T}}\mathbf{n}$$

$$\approx \underbrace{SF\ h_k x_k}_{\text{signal}} + \underbrace{\sqrt{SF} \sum_{\substack{i=1 \\ i \neq k}}^{N} h_i x_i}_{\text{Multiple Access Interference}} + \underbrace{\mathbf{c}_k^{\mathrm{T}}\mathbf{n}}_{\text{noise}}$$

- the last approximate equality uses expected cross-correlation of pseudo-random spreading
- expected noise power is $\mathrm{E}\left\{\mathbf{c}_k^{\mathrm{T}}\mathbf{n}\mathbf{n}^{\mathrm{H}}\mathbf{c}_k\right\}\ R_c = \mathbf{c}_k^{\mathrm{T}}\mathbf{c}_k\ N_0\ R_c = SF\ N_0\ R_c$
  - the energy of the SF noise samples combined in despreading, times chip bandwidth (chip rate)

Processing Gain II

- expected signal power is $SF^2|h|^2|x|^2R_c = SF^2P_k$
  - P_k is the received signal power for user k, the channel gain times the transmitted symbol power (symbol energy/chip duration)
- expected interference power is

$$SF\ \mathrm{E}\left\{\sum_{\substack{i=1\\i\neq k}}^{N} h_i x_i \sum_{\substack{j=1\\j\neq k}}^{N} h_j^* x_j^*\right\} R_c = SF\sum_{\substack{i=1\\i\neq k}}^{N}|h_i|^2|x_i|^2R_c = SF\sum_{\substack{i=1\\i\neq k}}^{N}P_i$$

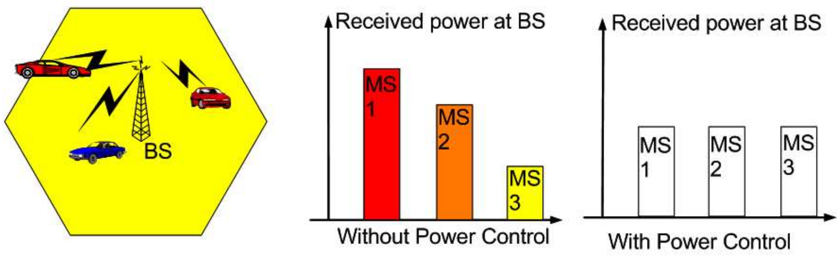
- the post-despreading SINR of user k is

$$\gamma_k = \frac{SF\ P_k}{\sum_{\substack{i=1\\i\neq k}}^{N}P_i + N_0R_c} \equiv G\ \frac{P_k}{\sum_{\substack{i=1\\i\neq k}}^{N}P_i + N_0R_c}$$

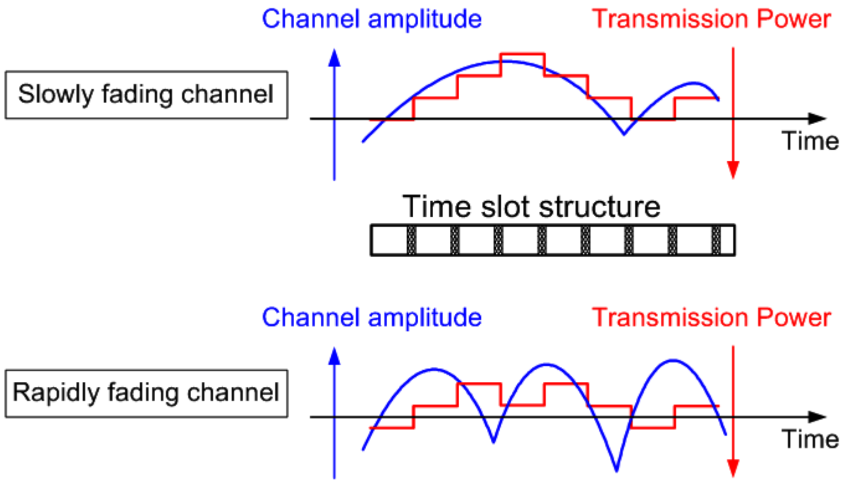
- the spreading factor provides the processing gain $G = SF$ against noise and interference
  - processing gain is 3dB per doubling of SF
  - here Multiple Access Interference (MAI) treated
  - similarly, spreading provides processing gain against Inter-Path Interference and Inter-Cell Interference

Near–Far Effect

- recall SINR of user k: $\gamma_k = \frac{G\ P_k}{\sum_{\substack{i=1\\i\neq k}}^{N}P_i+N_0R_c}$
- With the same Tx power, received power differences of UL users due to path loss may be up to 90 dB, due to fast fading up to 50dB.
- To overcome this, a user in a disadvantaged position would need a processing gain of the same magnitude, i.e. $G\ 10^9$
- This near-far effect causes a significant reduction of the capacity
- To cope with the near-far effect, effective uplink power control is required in DS-CDMA systems



Power Control in CDMA I



- Power control commands are given with fixed periods
- the target is to track fast fading (caused by mobilty of user)
  - the better channel, the less power transmitted

Power Control in CDMA II

Purpose of PC:

- Uplink
  - removes excessive intra-cell (and inter-cell) interference caused by transmissions of users close to BS
  - makes Rx power of users nearly equal to serve whole coverage area properly
- Downlink
  - removes excessive inter-cell interference caused by transmissions to users close to BS
  - keeps the received signal at minimum required level properly
- Both links
  - mitigate fast fading
    - creates reliable channels for circuit switched traffic

Advantages of DS-CDMA

- All resources can be used in all cells
  - wideband channel and large SF produce sufficient SINR for cell edge users $\Rightarrow$ high system capacity
- Protection against multipath interference
  - based on good autocorrelation properties of the spreading codes
  - if spreading factor is large
- Multipath diversity can be utilised by the RAKE-receiver
- It is easy to multiplex different channels in the code domain
  - control and transport channels, different users
- Silence periods in the transmitted signal do not consume resources
- Narrowband interference rejection
  - A narrowband signal will be spread in the correlation receiver
- Privacy and low probability of interception
  - signal can be detected only if spreading code is known
- In a hostile environment good anti-jamming properties

Disatvantages of DS-CDMA

- In UL, chip synchronisation between the users is overwhelming $\Rightarrow$ Multiple Access Interference between users
- In DL, Inter-Path Interference reduces orthogonality of users
- Accurate power control needed to avoid near-far problems which put distant users in an unfavourable situation
- CDMA is fundamentally an access scheme for low rates and many users
  - RAKE works well in severe multipath channel only if significant fraction of the possible orthogonal codes are not used (DL)
  - despreading in UL works well only for large SF
  - When striving for high data rates with high SINR requirements, IPI and MAI dominate performance
  - More complex receivers (chip equalizers) are needed to mitigate IPI and MAI
  - Simplicity of DS-CDMA is lost

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CDMA Capacity

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CDMA Uplink Capacity

- divide interfering users to own cell and other cell interference
  - N users in own cell, M interfering cells, N_i users in cell i
  - activity factors ρ_j and $\rho_{i,l}$ of own cell and other cell users
- multipath interference can be modeled as interfering user
- post-despreading SINR of user k is

$$\gamma_k = \frac{G\,P_k}{I_{\text{own}} + I_{\text{other}} + P_k/\gamma_{\text{RF}}}$$

  - average own and other cell interference powers, and SNR (before despreading) are

$$I_{\text{own}} = \sum_{j=1,\ j\neq k}^{N} \rho_j P_j \qquad I_{\text{other}} = \sum_{i=1}^{M}\sum_{l=1}^{N_i} \rho_{i,l} P_j \qquad \gamma_{\text{RF}} = \frac{P_k}{N_0 R_c}$$

- other cell interference is fraction of own cell interference:

$$I_{\text{other}} \approx f\ I_{\text{own}}$$

  - f depends on path loss and distribution of users, typically $f \approx 0.6$
- the received powers are subject to power control

UL Capacity, Similar Users

- Consider the case where all users receive the same service
  - target SINR γ_{target}
  - power control target $P_j = P$
  - same activity factor $\rho_j = \rho$

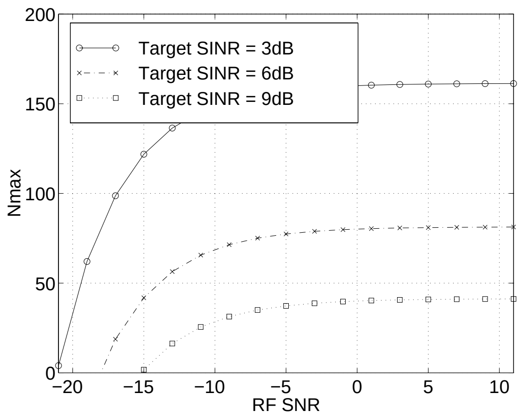
$$\gamma_k = \frac{G}{(1+f)(N-1)\rho + 1/\gamma_{\text{RF}}} \geq \gamma_{\text{target}}$$

- this constrains the number of users that can receive this service:

$$N \leq N_{\text{max}} = \frac{1}{\rho(1+f)} \left(\frac{G}{\gamma_{\text{target}}} - \frac{1}{\gamma_{\text{RF}}} \right) + 1$$

  - N_{max} is directly proportional to G
  - inversly proportional to γ_{target}

UL Capacity, Example



- parameters: $G = 256, \quad \rho = 0.5, \quad f = 0.6$
- capacity saturates when γ_{RF} grows: interference limitation
- N_{max} is significantly less than the spreading factor G

UL Capacity, Non-similar Users

- different users have different SINR targets and processing gains
  - target SINR for user j is γ_j, spreading factor is G_j
  - assume that the MAI affects all users in the same way: power control condition is
$$\frac{\gamma_j}{\gamma_k} = \frac{G_j P_j}{G_k P_k}$$
    - this means that the feedback coupling in PC needs not to be taken into account (how the user's PC affect other user's PC, and again the users PC)
    - valid if a user consumes a small fraction of the resources in the cell
    - the PC of one user does not affect the interference level in the cell
- interference can be expressed in terms of SINR target using the power control condition:

$$\gamma_k = \frac{G\ P_k}{(1+f)\sum_{j\neq k} \rho_j P_j + \frac{P_k}{\gamma_{\mathrm{RF}}}} = \frac{1}{(1+f)\sum_{j\neq k} \frac{\rho_j \gamma_j}{G_j \gamma_k} + \frac{P_k}{\gamma_{\mathrm{RF}}}}$$

Noise Rise

- the SINR can be calculated from the previous equation:

$$\gamma_k = \left(1 - (1+f)\sum_{j\neq k} \frac{\rho_j \gamma_j}{G_j}\right) G_k \gamma_{\mathrm{RF}}$$

- Define $\gamma_0 = G_k \gamma_{\mathrm{RF}}$, post-despreading SINR in absence of MAI
- the noise rise is

$$\frac{\gamma_0}{\gamma_k} = \frac{I_{\mathrm{own}} + I_{\mathrm{other}} + N_0 R_c}{N_0 R_c} = \left(1 - (1+f)\sum_{j\neq k} \frac{\rho_j \gamma_j}{G_j}\right)^{-1}$$

  - the increase in disturbances over thermal noise due to MAI
  - Note: if $(1+f)\sum_{j\neq k} \frac{\rho_j \gamma_j}{G_j} = 1$, noise rise is infinite
    - target SINR cannot be met at any SNR

Fractional Load

- the more users the higher load in the system
- the higher SINR targets, the higher load
- the more activitym the higher load
- the fractional load is

$$\eta = (1+f)\sum_j \frac{\rho_j \gamma_j}{G_j}$$

  - note: all users taken into account (also user k)
  - noise rise equation was derived based on the power control condition that assumed that a user takes negligible resources
    $\Rightarrow$ no essential difference betwwen fractional load and sum in noise rise
- relation of fractional and noise rise:

$$\frac{\gamma_0}{\gamma_k} \approx \frac{1}{1-\eta}$$

- when number of users and their SINR requirements grow so that $\eta \rightarrow 1$, the required SNR (and the Tx power) grows to infinity
  - infinitely interference limited network

Pole Capacity

- fractional load 1 determines the pole capacity of the CDMA system

$$\eta = (1+f)\sum_j \frac{\rho_j \gamma_j}{G_j} = 1$$

  - the number of users and SINR requirements that can be served if all users have ∞ Tx power
  - with finite Tx powers, capacity is less than pole capacity
- Example: similar users
- pole capacity equation:

$$\frac{1}{1+f} N_{\max} \frac{\rho\gamma}{G} = 1 \quad \Rightarrow \quad N_{\max} = \frac{G}{(1+f)\rho\gamma}$$

  - takle $\rho = 0.5, \quad G = 256, \quad \gamma = [3,6,9]\mathrm{dB} = [2,4,8]$
  - $f = 0.6 = 3/5$

$$N_{\max} = \frac{5}{8} \cdot 256 \cdot \left[1, \frac{1}{2}, \frac{1}{4}\right] = [160, 80, 40]$$

  - compare plot for similar users above

DL Fractional Load

- In DL CDMA, users in a cell typically use orthogonal spreading codes
- Multiple Access Interference arises from inter-path interference
  - partly destroys orthogonality of spreading codes
- can be modelled by an orthogonality factor α:

$$\eta = (1 - \alpha + f) \sum_j \frac{\rho_j \gamma_j}{G_j}$$

- in frequency flat (single path) channel, $\alpha = 1$
  - no in-cell interference
- in frequency selective fading, $\alpha < 1$

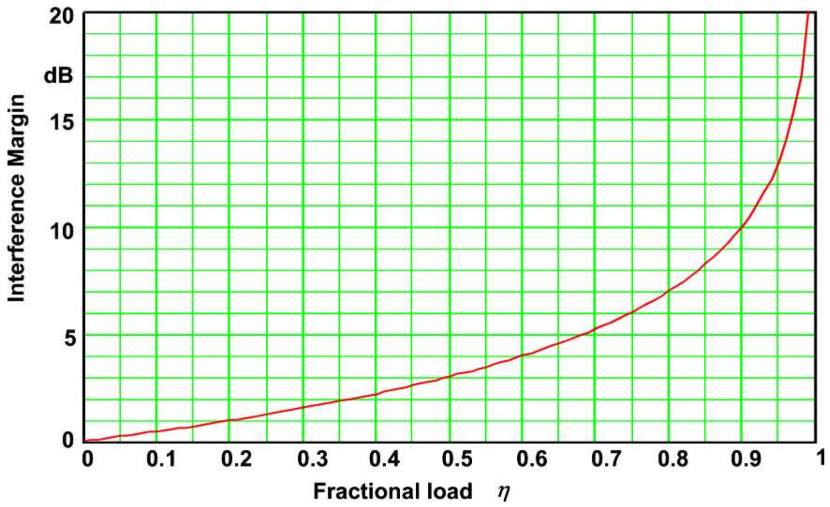
Interference Margin I

- According to noise rise equation, post-despreading SNR γ_0 is larger than SINR γ_k
- an Interference Margin (IM) needs to be added to the link budget

$$IM = 10 \log \left(\frac{1}{1 - \eta} \right)$$

  - takes care of the multiple access interference
  - when load is approaching the pole capacity, the IM becomes infinite
- interference margin, fractional load, noise rise:
  - different ways to view the same phenomenon: degree of interfernce limitation of CDMA

Interference Margin II



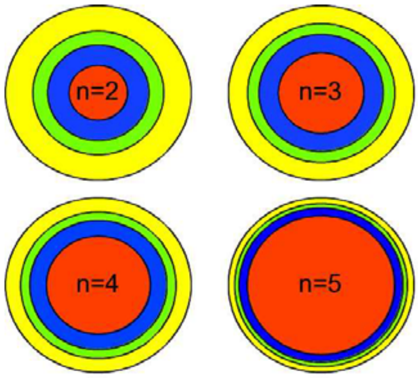
Load vs. Coverage I

- Consider the path loss model $L_p = L_0 + 10n\log(r)$ in [dB]
  - L_0 is the path loss at 1 km distance
  - n is the path loss exponent
  - r is the distance in km
- Link budget: $P_{\text{Tx}} - S = L_1 + L_p + IM$
  - P_{Tx} is the transmitter power level [dBm]
  - S is the receiver sensitivity [dBm]
  - IM is the interference margin due to traffic load
  - L_1 is the sum of system gains and losses except for L_p and IM
- determine coverage area $A_0 = \pi r_0^2$ for zero fractional load and coverage area $A = \pi r^2$ for non-zero fractional load
- find ratio from

$$10n\log r + IM = 10n\log r_0 \quad \Rightarrow \quad \frac{A}{A_0} = 10^{-IM/5n} = (1-\eta)^{2/n}$$

Load vs. Coverage II

η	n 2	3	4	5
0	1	1	1	1
0.5	0.5	0.63	0.71	0.87
0.7	0.3	0.45	0.55	0.79
0.9	0.1	0.22	0.32	0.63



- Cell breathing: Coverage area decreases with increasing traffic load
  - price from reuse 1: increasing interference at cell edge
  - breathing stronger for low path loss exponents
  - cell size must be planned according to maximum load