S-72.2211 Mobile Communication Systems and Services

Exercise session 4   22.2.2008
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1. Hybrid ARQ

   Hybrid ARQ(HARQ) is an enhanced type of ARQ error control method which includes the error correction capability in addition of error detection for further improving the performance. Depending on what content is transmitted in retransmissions and how to deal with the retransmitted packets, different types of HARQ schemes are defined. In chase-combining HARQ, the packet with the same content is retransmitted and the replicated packets are chase combined in a way so that equivalently we obtain a packet with its SNR being the summation of the SNRs of the replicated packets. Assume we are using a modulation/coding scheme(MCS) with transmission rate of k and exponential packet error rate (PER) function as given below

   $$P_e(\gamma) = e^{(-\gamma/\gamma_c)} \tag{1}$$

   Where γ denotes received SNR of the packet and γ_c is a constant which is dependent on the MCS.

   Assume that our channel distribution happens to be the same as (1). Formulate the residual packet error probability, the expected number of transmission and the expected throughput of the system, with an instantaneously observed SNR value of γ, under the assumption that the maximum allowed transmission number is T and the retransmissions are performed during the channel coherence time. Calculate the value of the expected throughput after it is averaged over the channel distribution when $T = 1$ and $T = 2$

   Solution:

   To proceed, we clarify the definition a bit. The residual packet error probability is defined as the probability that a packet is still erroneous after T number of transmissions. The average number of transmission is defined as the number of transmission required on average for the packet to be successfully decoded in the receiver.
   Since the packet error probability of a t replicated chase-combined packet is expressed by $P_{t,e}(\gamma) = P_e(t\gamma)$, it follows that the residual packet error probability is expressed by

$$P_{res}(\gamma, T) = \prod_{t=1}^{T} P_{t,e}(\gamma) = \prod_{t=1}^{T} P_e(t\gamma) = \prod_{t=1}^{T} e^{-t\gamma/\gamma_c} = e^{-\frac{T(T+1)}{2}\frac{\gamma}{\gamma_c}}$$

As for the average number of transmissions, we express it by

$$\begin{aligned}
&Nn(\gamma, T) = \sum_{t=1}^{T} Prob(the\ packet\ needs\ i^{th}\ transmission) \\
&= 1 + \sum_{t=2}^{T} Prob(the\ packet\ needs\ i^{th}\ transmission) \\
&= 1 + \sum_{t=1}^{T-1} Prob(decoded\ failed\ for\ the\ former\ t\ transmissions) \\
&= 1 + \sum_{t=1}^{T-1} P_{res}(\gamma, t) = 1 + \sum_{t=1}^{T-1} \prod_{j=1}^{t} P_e(j\gamma) \\
&= 1 + \sum_{t=1}^{T-1} e^{-\frac{t(t+1)}{2}\frac{\gamma}{\gamma_c}}
\end{aligned}$$

By definition, the expected throughput is expressed by

$$G(\gamma, T) = k \frac{Prob(successful\ decodingof 1 packet)}{E < Number\ of\ Tx\ needed\ for\ successful\ decoding >}$$

$$= k \frac{1 - P_{res}(\gamma, T)}{Nn(\gamma, T)} = k \frac{1 - e^{-\frac{T(T+1)}{2}\frac{\gamma}{\gamma_c}}}{1 + \sum\limits_{t=1}^{T-1} e^{-\frac{t(t+1)}{2}\frac{\gamma}{\gamma_c}}}$$

The expected throughput averaged over the channel distribution is calculated as

$$\overline{G}(T) = \int_0^\infty G(\gamma, T) f(\gamma) d\gamma = \int_0^\infty k \frac{1 - e^{-\frac{T(T+1)}{2}\frac{\gamma}{\gamma_c}}}{1 + \sum\limits_{t=1}^{T-1} e^{-\frac{t(t+1)}{2}\frac{\gamma}{\gamma_c}}} e^{(-\gamma/\gamma_c)} d\gamma$$

For $T = 1$, $\overline{G}(1) = \frac{\gamma_c}{2}$
For $T = 2$, $\overline{G}(2) = \gamma_c(ln(4) - \frac{5}{6}) = 0.553\gamma_c$

2. Adaptive modulation and coding

   Adaptive modulation and coding (AMC) is a link adaptation method which makes use of the channel fluctuation. Allowed to switch among a set of modulation/coding schemes (MCS) with different transmission rates, the data rate of AMC system is enhanced in favorable channel conditions and is reduced when channel degrades. Assume a hexagonal cellular system with reuse factor of 7 and consider simply 1-slope path loss model. Besides, no power control is applied in the system. The MCS set to be used in the AMC system together with the corresponding data rates and the minimum required operation CIR is shown in the table below. What is the average date rate in the system? Path loss exponent of $\alpha = 3.5$ is assumed in calculation.

Hint: For hexagonal cells, the reuse distance (the ratio of the distance between the co-channel cells D to the cell radius R) should satisfy $D/R = \sqrt{(3k)}$ where $k = i^2 + ij + j^2$ and $i, j = 1, 2, 3 \cdots$.

	User rate (kbits/s)	Required CIR [dB]
MCS 1	8.8	9
MCS 2	11.2	11
MCS 3	14.8	12
MCS 4	17.6	16
MCS 5	22.4	17
MCS 6	29.6	19
MCS 7	44.8	23
MCS 8	54.4	28
MCS 9	59.2	40

Solution:

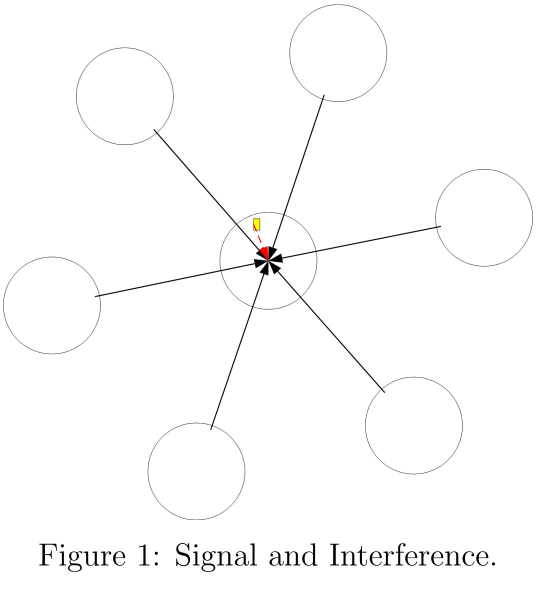

Figure 1: Signal and Interference.

Denote the distance between a user and the serving BS as r, we have the following inference according to our channel model: the longer the r is, the weaker the user's CIR is and thus the lower MCS should be used. For each of the MCS set, there exists one upper limit on r for the MCS to be used. Higher than that r, the signal would be too weak to achieve the required CIR. Assume that only the first tier interference are taken into account and consider the worst case that the interfering sources are located at the closest possible position to the center BS, it is straightforward to express the observed CIR value for a user who is r distance away from the BS as below

$$CIR_{MCS} \approx \frac{r_{MCS}^{-\alpha}}{6(\sqrt{(3k)}-1)^{-\alpha}} \Longrightarrow r_{MCS} = \frac{\sqrt{3k}-1}{\sqrt[\alpha]{6 \cdot CIR_{MCS}}}$$

The results of the upper limit of r for the MCS set, among others, are summarized in the following table. It is observed that there exists a certain range r for each MCS to give the highest transmission rate among the MCSs that fulfill the CIR requirement. It is always possible to use higher date rate than that can be provided by the lowest MCS since the upper limit of r for MCS 2 is already larger than the cell radius.

The average user date rate ($\bar{R}$) is calculated by summing over different user rates (k_i) weighted with the fractional area where the corresponding rate is used.

$$\bar{R} = \sum_{i=1}^{9} A_i \times k_i$$

The fractional area A_i for MCS i is calculated by

$$A_i \approx \frac{\pi(r_i^2 - r_{i+1}^2)}{\pi \cdot 1^2} \ , \ for\ i = 2 \sim 9\ and\ r_{10} = 0$$

	Upper limit in r for the MCS to be used	A_i
MCS 1	1.18	0
MCS 2	1.04	0.0494
MCS 3	0.975	0.3890
MCS 4	0.7494	0.0692
MCS 5	0.7017	0.1139
MCS 6	0.6152	0.1548
MCS 7	0.4729	0.1078
MCS 8	0.3403	0.0919
MCS 9	0.1545	0.0239

Evaluating for each coding class and weighting user date rate with the corresponding fractional area, we get the average date rate of 25.91 kbits/s.

3. AMC and CQI reporting error

   Assume an AMC system with 2 turbo-coded MCSs and their transmission rates are denoted k_1 and k_2 ($k_1 < k_2$), respectively. The BLock Error Rate (BLER) function of turbo code is approximated by a step function so that each of the MCSs is characterized by a minimum required SNR value, γ_1 and γ_2 ($\gamma_1 < \gamma_2$) respectively.

   a) Assume the Channel Quality Indicator (CQI) consisted of the received SNR information is perfectly feedback to the transmitter, how would you choose the switching point for the two MCSs to achieve the best achievable average transmission rate?

   b) If there exist Gaussian report error (channel report error in dB domain characterized by Gaussian distribution) in CQI and the variance of the Gaussian report error is σ^2, how do you choose the switching point to achieve the optimal throughput?

   A channel with uniform SNR distribution is assumed.

   Solution:

   a) With a step BLER function, each of the MCSs only works whenever its minimum required SNR is achieved by the instantaneous channel SNR and once it is achieved, it works perfectly without any possibility of decoding error. Apparently, both of the MCSs will be able to function properly if SNR is smaller than γ_1. Since MCS$_2$ only works when SNR is higher than γ_2, we will observed the best throughput if the switching point γ_s is set to γ_2.

   b) First we need to think what's the effect of CQI error to the system. If the CQI error is so that the transmitter makes the decision of choosing MCS$_1$ instead of MCS$_2$ (the real SNR is better than the reported SNR), what we lose is the opportunity for transmitting with higher rate. On the other hand, if the CQI error is so that the transmitter makes the decision of choosing MCS$_2$ instead of MCS$_1$ (the real SNR is worse than the reported SNR), we lose that transmission totally. The penalty for aggressive selection of MCS is big! If the switching point remains to be γ_2, and we use MCS$_2$ for Tx at that SNR, the Tx is successful with probability 1/2 and the expected throughput at γ_2 is $k_2/2$. Apparently, if $k_1 > k_2/2$, we should be more conservative of the switching point γ_s. If $k_1 < k_2/2$, we should be more aggressive of the switching point γ_s. Assume that $k_1 > k_2/2$, our strategy of selecting the optimal switching point is to be conservative somehow so that the expected throughput of using MCS$_2$ at γ_s is equal to the expected throughput of using MCS$_1$. In other words, we want to find γ_s according to the following criterion

$$\begin{aligned}&k_1 = k_2 \cdot Prob(Tx\ succeeds\ under\ CQI\ error)\\ &\Longrightarrow k_1 = k_2 \cdot Prob(\Delta\gamma > \gamma_2 - \gamma_s)\end{aligned}$$

This leads to the following expression.

$$k_1 = k_2 \int\limits_{\gamma_2-\gamma_s}^{\infty} f_\Delta(\Delta\gamma) d\Delta\gamma$$

Simplifying it, we have

$$\frac{k_1}{k_2} = Q(\frac{\gamma_2 - \gamma_s}{\sigma}) \Longrightarrow \gamma_s = \gamma_2 - \sigma \cdot Q^{-1}(\frac{k_1}{k_2})$$

4. Near-far effect and power control

   In a cellular system with cell radius 1 km and the maximum MS Tx power 20 dBm, consider the received power of two users. One is 50 m away from BS and the other one is at the cell edge.

   a) Assume a CDMA environment without power control. What is the difference of the received power between the two users? Calculate the power control dynamic range needed in this case.
   b) Assume a GSM environment with spectrum mask like the one in the lecture slides, what is the dynamic range of power control in this case?

   Consider the cases with path loss exponent of 2,3 and 4 respectively.

   Solution

   a) Denote the path loss exponent as α and consider only the single-slope channel model, the received power of the two users is:

$$\begin{aligned}&P_1 = \tfrac{P_{t,max}}{50^\alpha} = 20 - 10 \cdot \alpha \cdot log_{10}(50) \approx 20 - 17\alpha\ [dBm]\\ &P_2 = \tfrac{P_{t,max}}{1000^\alpha} = 20 - 10 \cdot \alpha \cdot log_{10}(1000) \approx 20 - 30\alpha\ [dBm]\\ &\Longrightarrow \Delta P = P_1 - P_2 = 13\alpha\ [dB]\end{aligned}$$

The purpose of power control is to compensate for the received power difference between the users. Therefore, the dynamic range of power control should be at least be equal to the received power difference. The results are summarized in the following table.

α	P_1 [dBm]	P_2 [dBm]	ΔP [dB]	PC [dB]
2	-14	-40	26	26
3	-31	-70	39	39
4	-48	-100	52	52

b) GSM spectral mask is shown in the following figure. The inter-channel interference must be at least 30 dB below the Tx power of the occupied channel as shown in the figure.

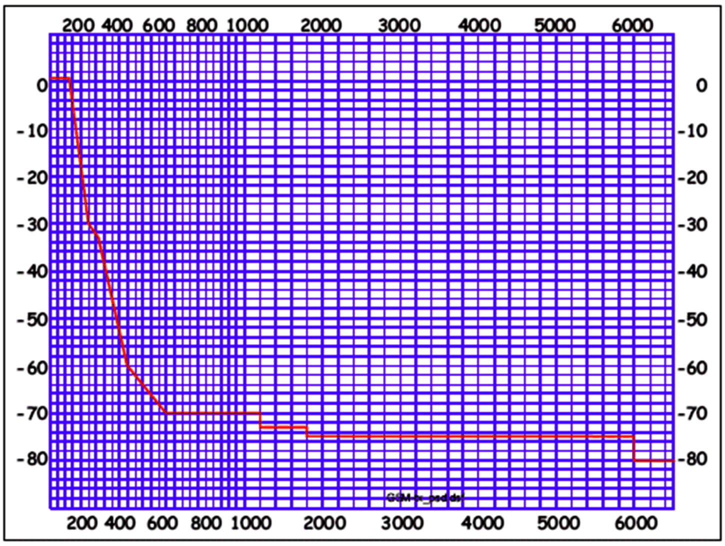

Figure 2: GSM Spectrum Mask.

In GSM system, the two users considered here would not occupy the same channel at the same time. However, there exists inter-channel interference and the worst case happens when these two users are using neighboring channels for transmission. Consider the channel (channel 2) which is occupied by user 2 located at the cell edge, the inter-channel interference caused by user 1 would be 30 dB below that of the received power in channel 1 from user 1. The calculation of the received power difference and the dynamic range for power control remains the same as the last subtask. The results are summarized in the following table.

α	P_1 [dBm]	P_2 [dBm]	ΔP [dB]	PC [dB]
2	-44	-40	-4	0
3	-61	-70	9	9
4	-78	-100	22	22