S-72.2211 Mobile Communication Systems and Services

Exercise session 5 29.2.2008
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1. Power Control

Power control (PC) is introduced in the down-link direction to reduce the average transmit power level in the cell. With single-slope channel model and perfect compensation of path loss by PC, the transmit power $P_{tx}(r)$ when MS is at distance r from the BS with a cell radius R and path loss exponent n is expressed by

$$P_{tx}(r) = P_{max}(\frac{r}{R})^n \tag{1}$$

Where P_{max} is the maximum transmit power used for the cell-edge users. Assume for users very close to the BS, we have a minimum transmit power $P_{min} \approx 0$. The mean transmit power averaged over the spatial distribution of MSs is expressed by

$$P_{txm} = \int_0^R \int_0^{2\pi} p(r,\phi) P_{tx}(r) d\phi dr \tag{2}$$

Where $p(r,\phi)$ is the MS distribution function. When MSs are more concentrated around the BS, a spherically symmetric probability density function used to describe the MSs' distribution is

$$p(r,\phi) = \frac{(k+1)r^k}{R^{k+1}} \frac{1}{2\pi},\ r \in [0,R],\ \phi \in [0,2\pi],\ k < 1. \tag{3}$$

The spatially uniform distribution of MSs is obtained from (3) when $k = 1$.

a) How large is the fractional area of the cell where 50% of the MSs nearest to the BS are located, when $k = 0$ and $k = 1$?

b) How much average power reduction is achieved if we have uniform user distribution when $n = 2, 3, 4, 5$?

c) Repeat the calculation for a non-uniform, spherically symmetrical distribution with $k = 0$

Solution

a) The probability of having 50% of the MSs within a circular area around the BS is calculated by utilizing (3)

$$P = 0.5 = \int_0^{R_{50}}\int_0^{2\pi} p(r,\phi)drd\phi = \int_0^{R_{50}}\int_0^{2\pi} \frac{(k+1)r^k}{R^{k+1}}\frac{1}{2\pi}d\phi dr$$

$$= \int_0^{R_{50}} \frac{(k+1)r^k}{R^{k+1}}dr \int_0^{2\pi}\frac{1}{2\pi}d\phi = \frac{r^{k+1}}{R^{k+1}}\Big|_0^{R_{50}} = (\frac{R_{50}}{R})^{k+1}$$

It is straightforward to see that

$$\frac{R_{50}}{R} = 0.5^{1/(k+1)} \rightarrow \frac{A_{50}}{A_{cell}} = (\frac{R_{50}}{R})^2 = 0.5^{2/(k+1)}$$

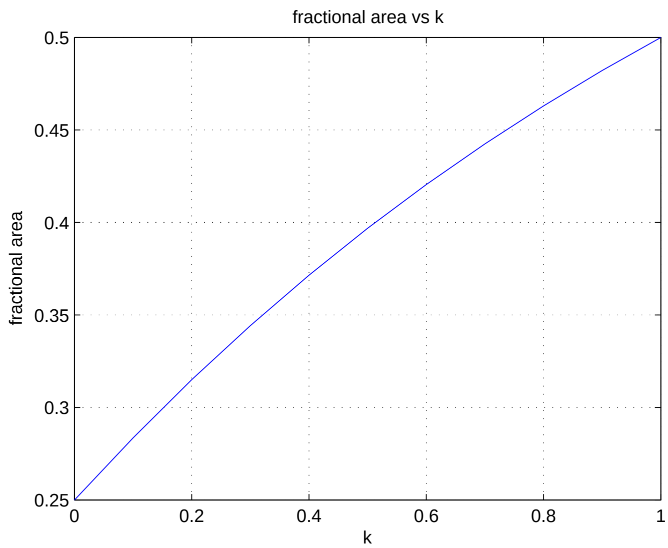

Figure 1: Fractional Area vs k.

b) The average transmit power is calculated by replacing (1) and (3) into (2)

$$P_{txm} = \int_0^{R}\int_0^{2\pi} p(r,\phi)P_{tx}(r)d\phi dr = \int_0^{R}\int_0^{2\pi} \frac{(k+1)r^k}{R^{k+1}}\frac{1}{2\pi}P_{max}\frac{r^n}{R^n}d\phi dr$$

$$= P_{max} \int_0^R \frac{(k+1)r^k}{R^(k+1)} \frac{r^n}{R^n} dr = (k+1) P_{max} \int_0^R \frac{r^{k+n}}{R^{k+1+n}} dr$$

$$= P_{max} \frac{k+1}{k+1+n} \frac{r^{k+1+n}}{R^{k+1+n}} |_0^R = P_{max} \frac{k+1}{k+1+n}$$

$$\rightarrow \Delta P = 10 \cdot \log_{10}(\frac{P_{max}}{P_{txm}}) = 10 \cdot \log_{10}(\frac{k+1+n}{k+1})$$

The results are summarized in the following table

	n=2	n=3	n=4	n=5
k=1	ΔP=3.01dB	ΔP=3.98dB	ΔP=4.78dB	ΔP=5.44dB
k=0	ΔP=4.78dB	ΔP=6.02dB	ΔP=6.99dB	ΔP=7.78dB

It is observed that the amount of power reduction is higher when the path loss exponent is higher and the users are more concentrated around the BS.

2. Power Control

Assume the similar PC scheme as the previous task, with a uniform user distribution. However, the minimum transmit power P_{min} is no longer 0 and the minimum transmit power P_{min} is used whenever the PC result requires a power level less than P_{min}.

a) At which distance, P_{min} is exactly achieved according to PC result?

b) Calculate the average transmit power over all possible user positions in the cell.

c) How much power reduction is reached by PC compared to the maximum transmit power P_{max}?

Give the results assuming that the power control range is 10, 20, 30, 40 dB and the path loss exponent is 2, 2.5, 3, 3.5, 4, 4.5, 5.

Solution

a) The PC scheme is now

$P_{tx}(r) = P_{max}(\frac{r}{R})^n$ when $r \geq r_{min}$
$P_{tx}(r) = P_{min}$ when $r < r_{min}$

Where r_{min} is the largest distance that P_{min} may still be used. Therefore,

$$P_{tx}(r_{min}) = P_{max}(\frac{r_{min}}{R})^n = P_{min}$$

$$\Longrightarrow r_{min} = R \cdot (\frac{P_{min}}{P_{max}})^{1/n}$$

Where P_{max}/P_{min} is recognized as the power control dynamic range. The values of r_{min} corresponds to different values of power control range and path loss exponent are given below.

PC range	n=2	n=2.5	n=3	n=3.5	n=4	n=4.5	n=5
10 dB	0.32	0.40	0.46	0.52	0.56	0.60	0.63
20 dB	0.10	0.16	0.22	0.27	0.32	0.36	0.40
30 dB	0.03	0.06	0.10	0.14	0.18	0.22	0.25
40 dB	0.01	0.03	0.05	0.07	0.10	0.13	0.16

b) The average transmit power is calculated according to (2) with new PC scheme.

$$P_{txm} = \int_0^R \int_0^{2\pi} P_{tx}(r) p(r,\phi) dr d\phi = \int_0^R \int_0^{2\pi} P_{tx}(r) \frac{2r}{2\pi R^2} dr d\phi = \frac{2}{R^2} \int_0^R P_{tx}(r) r dr$$
$$= \frac{2}{R^2} \left(\int_0^{r_{min}} P_{min} r dr + \int_{r_{min}}^R P_{max}(\frac{r}{R})^n r dr \right)$$
$$= \frac{2}{R^2} \left(P_{min} \int_0^{r_{min}} r dr + P_{max} \int_{r_{min}}^R (\frac{r}{R})^n r dr \right)$$
$$= P_{min} \frac{r_{min}^2}{R^2} + 2P_{max} \left(\frac{R^{n+2}}{(n+2)R^{n+2}} - \frac{r_{min}^{n+2}}{(n+2)R^{n+2}} \right)$$

Replace $r_{min} = R \cdot (\frac{P_{min}}{P_{max}})^{1/n}$ into the above expression, we have

$$P_{txm} = P_{min}(\frac{P_{min}}{P_{max}})^{2/n} + \frac{2P_{max}}{n+2}(1 - (\frac{P_{min}}{P_{max}})^{(2+n)/n})$$
$$= P_{max}(\frac{P_{min}}{P_{max}})^{(2+n)/n} + \frac{2P_{max}}{n+2}(1 - (\frac{P_{min}}{P_{max}})^{(2+n)/n})$$

c) The reduction in transmit power is obtained by

$$-10\cdot\log_{10}(\frac{P_{txm}}{P_{max}}) = -10\cdot\log_{10}\left((\frac{P_{min}}{P_{max}})^{(2+n)/n} + \frac{2}{n+2}\left(1-(\frac{P_{min}}{P_{max}})^{(2+n)/n}\right)\right)$$

Where the ratio P_{min}/P_{max} is recognized as the inverse of power control range. Replace the power control range and path loss exponent with their corresponding values, we have the following results.

PC range	n=2	n=2.5	n=3	n=3.5	n=4	n=4.5	n=5
10 dB	2.97	3.44	3.84	4.19	4.51	4.78	5.03
20 dB	3.01	3.52	3.98	4.39	4.76	5.11	5.42
30 dB	3.01	3.52	3.98	4.39	4.77	5.12	5.44
40 dB	3.01	3.52	3.98	4.39	4.77	5.12	5.44

It is noted that we don't need a large power control range to harvest most of the achievable power control gain in our model.

3. WCDMA Capacity

In a DS-CDMA system the up-link fractional load is defined as

$$\eta = (1+f)\sum_{i=1}^{N}\frac{\rho_i\gamma_i}{G_i} \tag{4}$$

and the interference margin is

$$IM = 10\cdot\log_{10}(\frac{1}{1-\eta}) \tag{5}$$

Where ρ is the channel activity factor, γ is the target SINR after despreading, G is the spreading factor and f is the ratio of other cell interference to own cell interference.

a) How many speech users ($\rho_i = 0.4,\ \gamma_i = 8\ dB,\ G_i = 256$) can be served when $f = 0.75$, and the fractional load target is 0.7?

b) If a new user ($\rho_{N+1} = 1,\ \gamma_{N+1} = 4\ dB,\ G_{N+1} = 32$) is admitted in the cell, how many dB must the interference margin be increased from the value in subtask a) to maintain all the connections?

Solution

a) The number of users N can be calculated by equating (4) to the required value 0.7.

$$\eta = 0.7 = (1+f)\sum_{i=1}^{N}\frac{\rho_i\gamma_i}{G_i} = (1+0.75)N\frac{0.4\cdot 10^{8/10}}{256}$$

$$N = \frac{256\cdot 0.7}{1.75\cdot 0.4\cdot 10^{0.8}} = 40.57 \rightarrow N = 40$$

b) When a new user is admitted, the fractional load is calculated by (4) first and then the interference margin by (5).

$$\eta = (1+f)\sum_{i=1}^{N+1}\frac{\rho_i\gamma_i}{G_i} = (1+0.75)\left(40\frac{0.4\cdot 10^{8/10}}{256} + \frac{1\cdot 10^{0.4}}{32}\right)$$
$$= 1.75\cdot(0.3943+0.0785) = 0.8274$$

$$\Delta IM = IM_{new} - IM_{old} = 10\cdot\log_{10}(\tfrac{1}{1-\eta_{new}}) - 10\cdot\log_{10}(\tfrac{1}{1-\eta_{old}})$$
$$= 10\cdot\log_{10}(\tfrac{1-\eta_{old}}{1-\eta_{new}}) = 10\cdot\log_{10}(\tfrac{1-1.75\cdot 0.3943}{1-0.8274}) = 2.54\ dB$$

4. WCDMA Capacity

In the WCDMA system the chip rate is 3.84 Mchips/s.

a) Assume a single cell system with the user bit rate after channel coding 15 kbits/s and the E_b/I_0 requirement for proper reception 5 dB, where E_b is the bit energy and I_0 is the experienced interference. With user activity factor 0.4 and AWGN noise not considered, how many users in a cell, theoretically, can be simultaneously served in the up-link direction?

b) How many users with 10 times higher received power can exist in a cell before the total number of users is halved in a single cell system?

c) Repeat the calculation for a multicell system when the other to own cell interference ratio is 0.6.

Solution

a) When a single service with constant rate is used, the capacity in number of users is obtained from the following SIR expression.

$$\frac{GP}{\rho(1+f)(N-1)P} = \gamma_0 \rightarrow N = \frac{1}{\rho \cdot (1+f)} \cdot \frac{G}{\gamma_0} + 1$$

Since the term of processing gain divided with the SIR target (G/γ_0) can be calculated from the chip rate, the bit rate and the E_b/I_0 target, the capacity is obtained by inserting the given parameters. In the single cell system

$$N = \frac{1}{0.4 \cdot (1+0)} \cdot \frac{3.84 \cdot 10^6/(15 \cdot 10^3)}{10^{5/10}} + 1 = 203.4 \rightarrow N = 203$$

For a multiple cell system, the capacity is obtained as

$$N = \frac{1}{0.4 \cdot (1+0.6)} \cdot \frac{3.84 \cdot 10^6/15 \cdot 10^3}{10^{5/10}} + 1 = 127.5 \rightarrow N = 127$$

b) Assume M out of N users with 10 times higher received power exist in the system, the SIR expression is given by

$$\gamma_0 = \frac{GP}{\rho(1+f)[(N-M-1)+10M]P} \rightarrow M = \frac{1}{9}[\frac{G}{\gamma_0\rho(1+f)} - N + 1]$$

Now, we want to know the value of M given that the total number of users in the cell is halved. In a single cell network, it is calculated as

$$M = \frac{1}{9}[\frac{256}{10^{0.5} \cdot 0.4(1+0)} - \frac{203}{2} + 1] = 11.4 \rightarrow M = 11$$

In a multiple cell system we take into account the other cell interference and it is calculated as

$$M = \frac{1}{9}[\frac{256}{10^{0.5} \cdot 0.4(1+0.6)} - \frac{127}{2} + 1] = 7.11 \rightarrow M = 7$$

By comparing the results from the single cell and the multiple cell systems we can see the impact of the other cell interference to the system capacity.