S-72.3210 Channel modeling for radio communication systems
Additional demo problems

A1. In a fixed radio link design the antenna height is selected to guarantee that the 1st Fresnel zone is free of obstacles during standard propagation conditions ($\partial N/\partial h = -40$ NU/km) for the situation depicted in Fig.1.

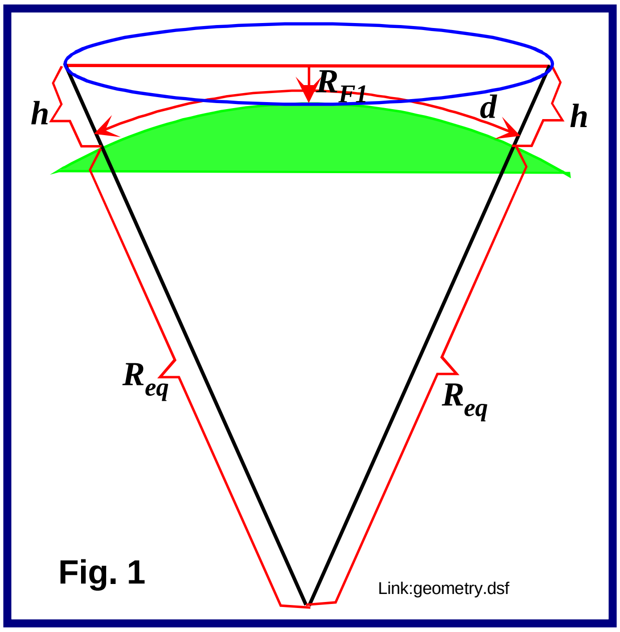


Fig. 1

a. Derive by geometrical considerations a symbolic expression for the required antenna height h above the reference ground.

b. Calculate the approximate numerical antenna height value h on the frequency f = 3 GHz for the path lengths i) d = 50 km, ii) d = 100 km, and iii) d = 150 km. You may use the expression cos(2x) = 1 – 2sin^2(x). The geometrical Earth radius is assumed to be 6370 km.

SOLUTION

a) By considering a right-angled triangle we get

$$R_{eq} + h = \frac{R_{eq} + R_{F1}}{\cos\left(\dfrac{d}{2R_{eq}}\right)} = \frac{R_{eq} + R_{F1}}{1 - 2\sin^2\left(\dfrac{d}{4R_{eq}}\right)}$$

$$\rightarrow h = \frac{R_{eq} + R_{F1}}{1 - 2\sin^2\left(\dfrac{d}{4R_{eq}}\right)} - R_{eq} = \frac{2R_{eq}\sin^2\left(\dfrac{d}{4R_{eq}}\right) + R_{F1}}{1 - 2\sin^2\left(\dfrac{d}{4R_{eq}}\right)}$$

$$\square \; \frac{d^2}{8R_{eq}} + R_{F1} = \frac{d^2}{8R_{eq}} + \sqrt{\frac{\lambda d}{4}}$$

b) The wavelength at 3 GHz is λ = 0.1 m

The equivalent Earth radius is

$$R_{eq} \approx \frac{R_o}{1 + R_o \frac{\partial n}{\partial h}} = \frac{6370}{1 - 6370 \cdot 0.000040} = 8548 \text{ km}$$

i) d = 50 km

$$h \approx \frac{d^2}{8R_{eq}} + \sqrt{\frac{\lambda d}{4}} = \frac{50^2}{8 \cdot 8548} + \sqrt{\frac{0.05 \cdot 50000}{4}}$$
$$= 0.03656 \text{ km} + 25.00 \text{ m} = 61.56 \text{ m}$$

ii) d = 100 km

$$h \approx \frac{d^2}{8R_{eq}} + \sqrt{\frac{\lambda d}{4}} = \frac{100^2}{8 \cdot 8548} + \sqrt{\frac{0.05 \cdot 100000}{4}}$$
$$= 0.14623 \text{ km} + 35.36 \text{ m} = 181.59 \text{ m}$$

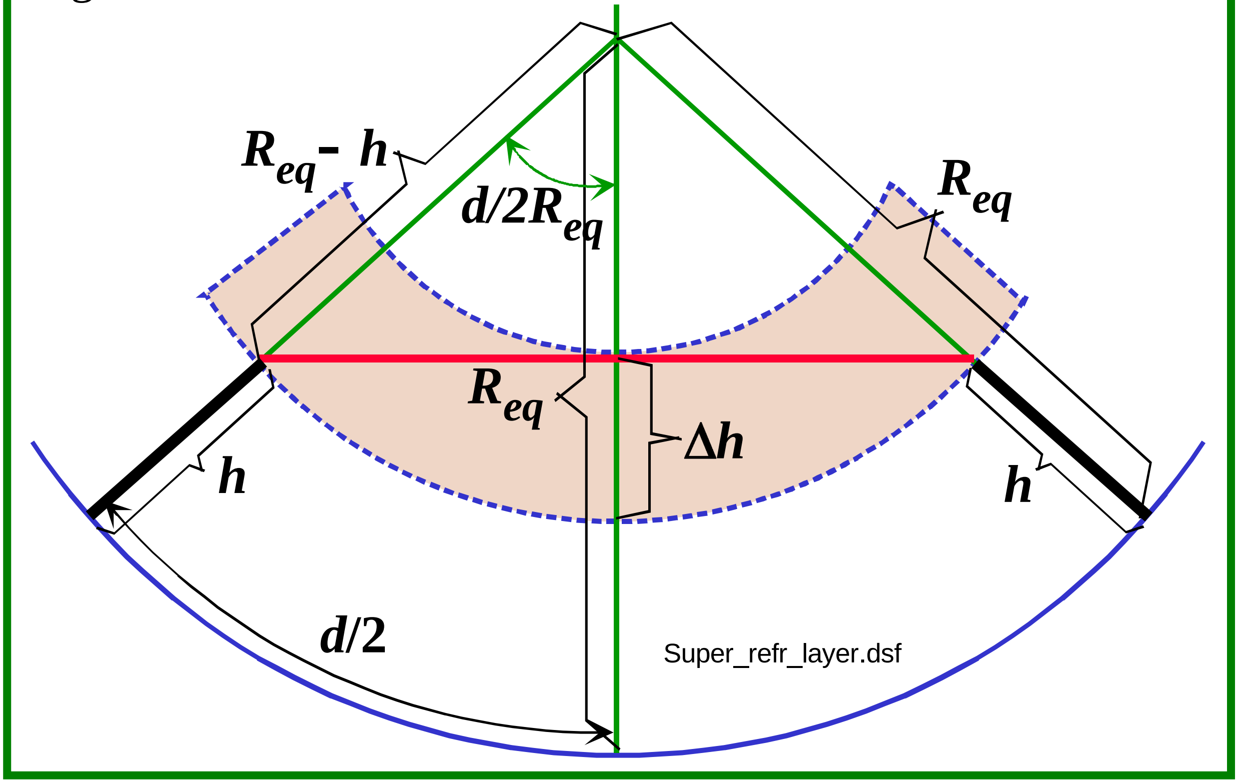
iii) d = 150 km

$$h \approx \frac{d^2}{8R_{eq}} + \sqrt{\frac{\lambda d}{4}} = \frac{150^2}{8 \cdot 8548} + \sqrt{\frac{0.05 \cdot 150000}{4}}$$
$$= 0.32902 \text{ km} + 43.30 \text{ m} = 372.32 \text{ m}$$

A2. Determine from the geometry given in Fig. 1 an expression of the minimum vertical thickness Δh of a super-refractive layer so that the entire ray path is within the layer. Use suitable approximations and calculate the layer thickness on a 40 km path for the refractivity gradients i) –300 N-units/km, ii) –400 N-units/km, and iii) –500 N-units/km. Calculate also the corresponding vertical changes of the refractivity through the layer.

SOLUTION

Fig. 1



From the figure one can see that

$$R_{eq} - h - \Delta h = \ R_{eq} - h \ \cos\left(\frac{d}{2R_{eq}}\right)$$

$$\rightarrow \Delta h = R_{eq} - h - \ R_{eq} - h \ \cos\left(\frac{d}{2R_{eq}}\right) = \ R_{eq} - h \ \left(1 - \cos\left(\frac{d}{2R_{eq}}\right)\right)$$

As the angle $d/2R_{eq}$ is small, the cosine-function can be replaced by the two first terms of its Taylor-series, and as $h \ll R_{eq}$ the expression can be further simplified:

$$\rightarrow \Delta h \approx \; R_{eq} - h \; \left(1 - \left(1 - \frac{1}{2}\left(\frac{d}{2R_{eq}}\right)^2\right)\right) = \; R_{eq} - h \; \frac{d^2}{8R_{eq}^2} \approx \frac{d^2}{8R_{eq}}$$

The equivalent Earth radius is approximately depending on the vertical gradient of the refractive index in the following way:

$$R_{eq} \approx \frac{R_o}{1 + R_o \dfrac{\partial n}{\partial h}}$$

$$\frac{\partial N}{\partial h} = -300 \frac{\text{NU}}{\text{km}} \rightarrow R_{eq} = \frac{6370}{1 - 0.0003 \cdot 6370} = \; - \; 6992 \text{ km}$$

$$\rightarrow \Delta h = \frac{40^2}{8 \cdot 6992} = 0.0314 \text{ km},$$

$$\rightarrow \Delta N = \frac{\partial N}{\partial h} \cdot \Delta h = -300 \cdot 0.0314 = -9.42 \text{ N-units}$$

$$\frac{\partial N}{\partial h} = -400 \frac{\text{NU}}{\text{km}} \rightarrow R_{eq} = \frac{6370}{1 - 0.0004 \cdot 6370} = \; - \; 4115 \text{ km}$$

$$\rightarrow \Delta h = \frac{40^2}{8 \cdot 4115} = 0.0486 \text{ km} \rightarrow \Delta N = -400 \cdot 0.0486 = -19.44 \text{ N-units}$$

$$\frac{\partial N}{\partial h} = -500 \frac{\text{NU}}{\text{km}} \rightarrow R_{eq} = \frac{6370}{1 - 0.0005 \cdot 6370} = \; - \; 2915 \text{ km}$$

$$\rightarrow \Delta h = \frac{40^2}{8 \cdot 2915} = 0.0686 \text{ km} \rightarrow \Delta N = -500 \cdot 0.0686 = -34.30 \text{ N-units}$$

Note! If the 1st Fresnel zone should be inside the layer, the thickness would be several tens of meters larger.

A3. How large is the diffraction loss (dB) caused by a knife-edge obstacle, when it covers exactly the full diameter of the first Fresnel zone

SOLUTION

When the 1st Fresnel zone is completely occupied by a knife-edge obstacle the Fresnel diffraction parameter value is

$$\nu = h\sqrt{\frac{2}{\lambda}\left(\frac{l_1 + l_2}{l_1 l_2}\right)} = \frac{h}{R_{F1}} \cdot \sqrt{2} = \sqrt{2}$$

As $\nu > -0.7$ Eq. 2 – 56 can be used

$$\Delta L_{knife-edge} = 6.9 + 20\log\left(\sqrt{\left(\nu - 0.1\right)^2 + 1} + \nu - 0.1\right)$$

$$= 6.9 + 20\log\left(\underbrace{\sqrt{\left(\sqrt{2} - 0.1\right)^2 + 1}}_{1.5414} + \underbrace{\sqrt{2} - 0.1}_{1.3142}\right) \quad ????$$

$$= 6.9 + 20\log\left(2.9656\right) = 7.37 \text{ dB}$$

A4 Fig. 1 shows the geometry of a multiple screen diffraction path where the screen height above the flat ground is h_s and the distance between the N screens is b. The transmitter antenna with the height h_{tx} above ground is on the distance $b/2$ from the first screen and the receiver antenna with the height h_{rx} above ground is on the distance $b/2$ from the Nth and last screen. Without loss of generalithy it is assumed that the transmitter antenna is higher than the receiver antenna.

a) Derive using Deygot's method the excess loss in symbolic form for the given path.

b) Calculate a numerical value for the excess loss in dB relative to free space loss when d = 2 km, b = 100 m, h_s = 25 m, h_{tx} = 15 m, h_{rx} = 1.5 m, and the frequency f = 2000 MHz.

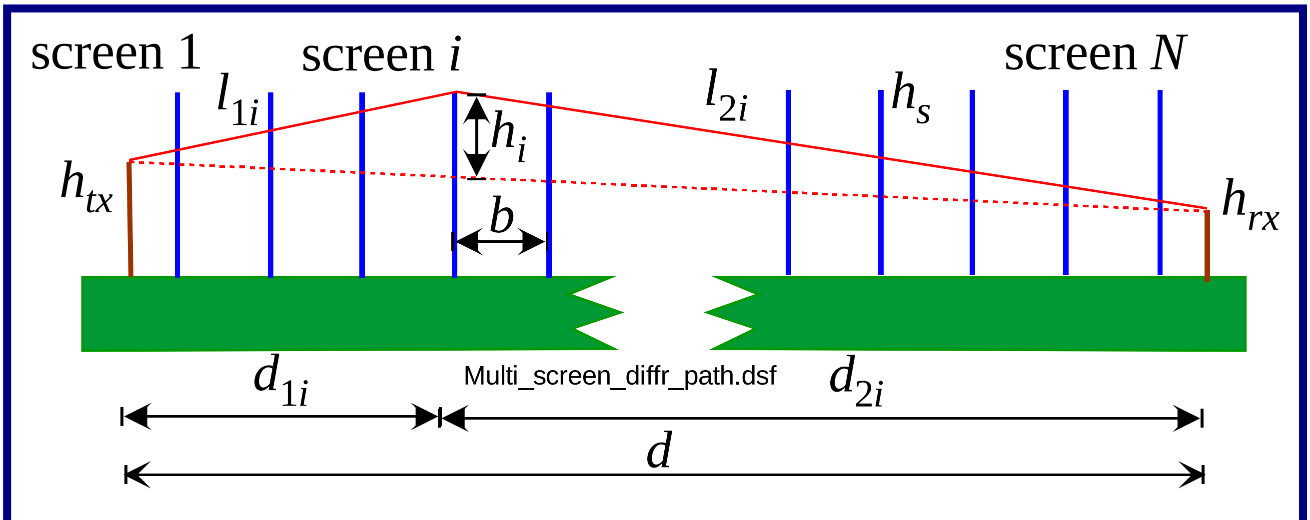


Fig. 1

SOLUTION

a) From the path geometry it can be seen that the path length along ground is $d = Nb$.

In Deygot's method one has first to calculate the Fresnel parameter ni for each screen as it were a single obstacle.

$$\nu_i = h_i \sqrt{\frac{2}{\lambda} \cdot \left(\frac{1}{l_{1i}} + \frac{1}{l_{2i}} \right)}$$

From Fig. 2 one can write the height of the screen above the theoretical line-of-sight path to be

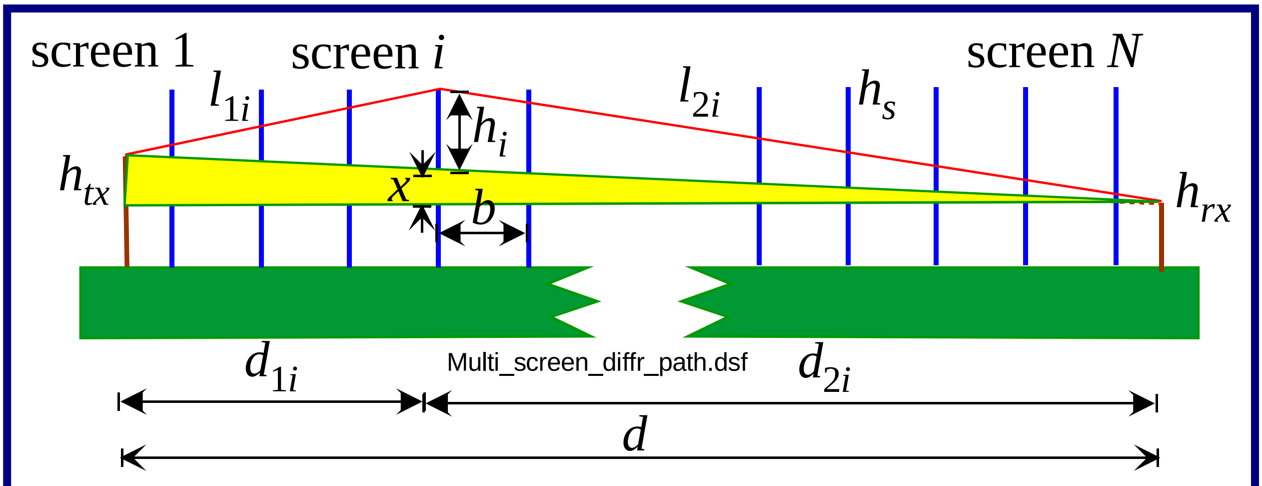


Fig. 2

$$h_i = h_s - h_{rx} - x$$

By considering similar triangles one gets

$$x = (h_{tx} - h_{rx}) \cdot \frac{d_{2i}}{d}$$

From Fig. 1 it can be deduced that

$$d_{1i} = (i - 0.5) \cdot b = (i - 0.5) \cdot \frac{d}{N}$$

$$d_{2i} = d - d_{1i} = d - (i - 0.5) \cdot \frac{d}{N} = (N - i + 0.5) \cdot \frac{d}{N}$$

The path lengths from the transmitter and receiver to Screen *i* are obtained from the right-angled triangles in Fig. 3

$$l_{1i} = \sqrt{d_{1i}^2 + (h_s - h_{tx})^2} = \sqrt{(i - 0.5)^2 \cdot \frac{d^2}{N^2} + (h_s - h_{tx})^2}$$

$$l_{2i} = \sqrt{d_{2i}^2 + (h_s - h_{rx})^2} = \sqrt{(N - i + 0.5)^2 \cdot \frac{d^2}{N^2} + (h_s - h_{rx})^2}$$

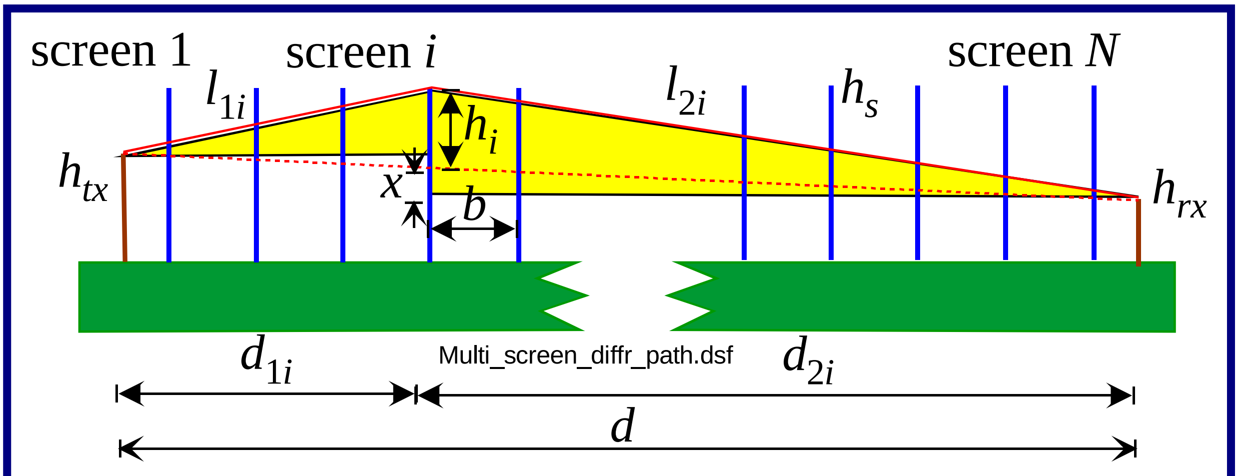


Fig. 3

The Fresnel-parameters of the individual screens can now be written as

$$\nu_i = h_i \sqrt{\frac{2}{\lambda}\cdot\left(\frac{1}{l_{1i}}+\frac{1}{l_{2i}}\right)} = \left(h_s - h_{rx} - (h_{tx} - h_{rx})\cdot\frac{N-i+0.5}{N}\right)\cdot$$

$$\sqrt{\frac{2}{\lambda}\cdot\left(\frac{1}{\sqrt{(i-0.5)^2\cdot\frac{d^2}{N^2}+(h_s-h_{tx})^2}}+\frac{1}{\sqrt{(N-i+0.5)^2\cdot\frac{d^2}{N^2}+(h_s-h_{rx})^2}}\right)}$$

From Figs. 1…3 it is clear that h_i increases as the distance to the lower antenna decreases. As the expression under the square root in the Fresnel-paramter is a convex function of i with a minimum near to the path midpoint it follows that the maximum Fresnel-parameter occurs for that screen which is nearest to the lower antenna, in this case the receiver antenna or

$$\nu_{\max} = \nu_N == \left(h_s - h_{rx} - (h_{tx} - h_{rx})\cdot\frac{0.5}{N}\right)\cdot$$

$$\sqrt{\frac{2}{\lambda}\cdot\left(\frac{1}{\sqrt{(N-0.5)^2\cdot\frac{d^2}{N^2}+(h_s-h_{tx})^2}}+\frac{1}{\sqrt{0.25\cdot\frac{d^2}{N^2}+(h_s-h_{rx})^2}}\right)}$$

According to Deygot’s method the excess heights for the remaining individual diffraction paths from the higher antenna to the top of the screen (nearest to the lower antenna) and the screen with the maximum Fresnel-parameter should be determined,

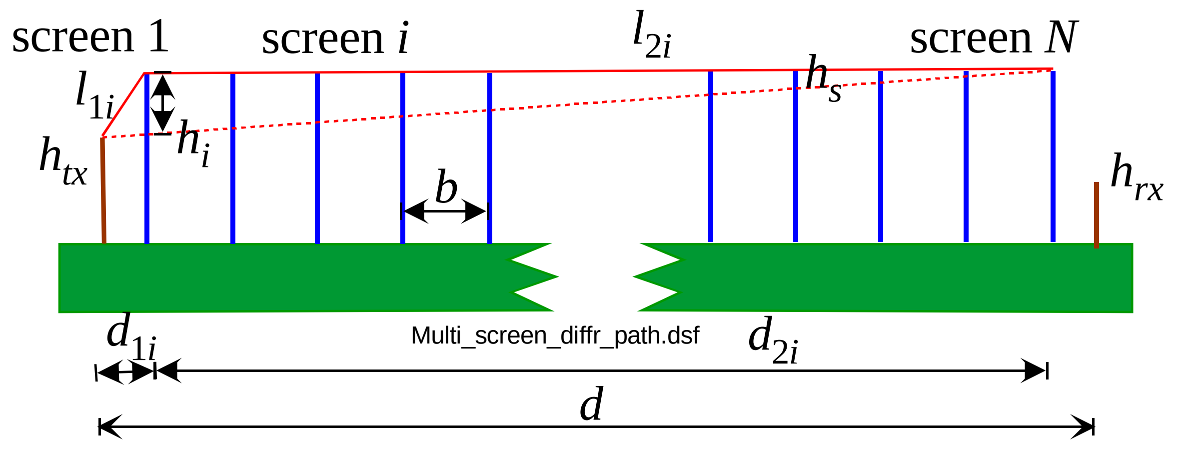


Fig. 4

Wtth the same reasoning as above the result is that the maximum Fresnel-parameter occurs for the screen nearest to the higher antenna, in this case Screen 1, Now

$$l_{11}=\sqrt{\frac{b^2}{4}+\left(h_s-h_{tx}\right)^2}=\sqrt{\frac{d^2}{4N^2}+\left(h_s-h_{tx}\right)^2}$$

$$l_{21}=\left(N-1\right)b=\frac{N-1}{N}d$$

$$h_1=\left(h_s-h_{tx}\right)\cdot\frac{l_{21}}{d-0.5b}=\left(h_s-h_{tx}\right)\cdot\frac{\frac{N-1}{N}d}{d-0.5\frac{d}{N}}=\left(h_s-h_{tx}\right)\cdot\frac{\frac{N-1}{N}}{1-0.5\frac{1}{N}}$$

$$=\left(h_s-h_{tx}\right)\cdot\frac{N-1}{N-0.5}$$

$$\nu_{\max}=\nu_1'=\left(h_s-h_{tx}\right)\cdot\frac{N-1}{N-0.5}\cdot\sqrt{\frac{2}{\lambda}\cdot\left(\frac{1}{\sqrt{\frac{d^2}{4N^2}+\left(h_s-h_{tx}\right)^2}}+\frac{1}{\frac{N-1}{N}d}\right)}$$

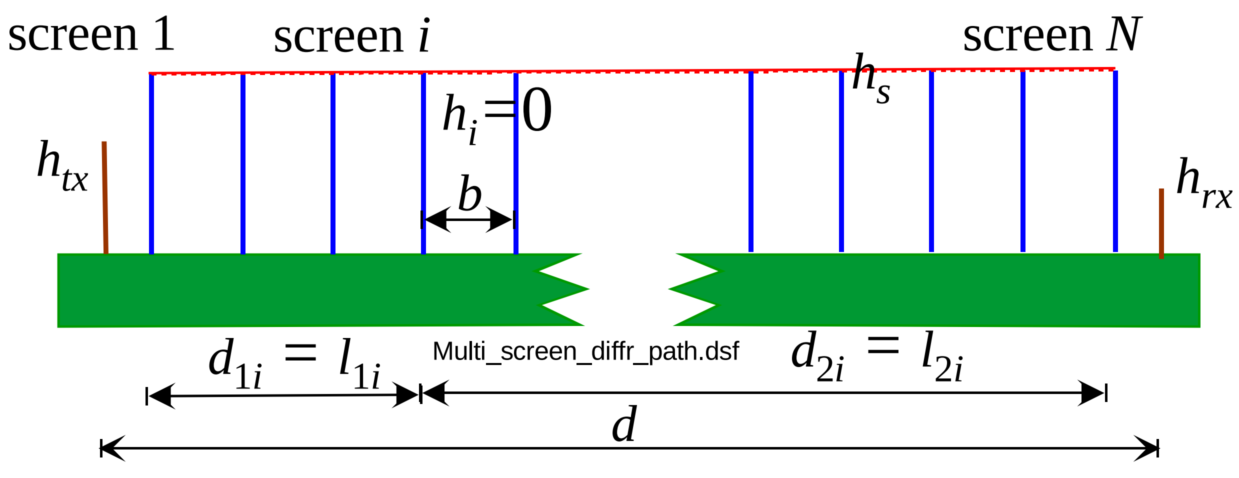


Fig. 5

For the remaining subsets of screens in Deygot's ,method the excess height is zero as Fig. 5 shows. This also implies that all following Fresnel-parameters take tha value zero. The total excess diffraction loss is

$$\Delta L_{msd} = L(\nu_1) + L(\nu_N) + (N-2)L(0)$$

$$= L\left((h_s - h_{tx}) \cdot \frac{N-1}{N-0.5} \cdot \sqrt{\frac{2}{\lambda} \cdot \left(\frac{1}{\sqrt{\frac{d^2}{4N^2} + (h_s - h_{tx})^2}} + \frac{1}{\frac{N-1}{N} d} \right)} \right)$$

$$+ L\left(\left(h_s - h_{tx} + (h_{tx} - h_{rx}) \cdot \frac{N-0.5}{N} \right) \cdot \sqrt{\frac{2}{\lambda} \cdot \left(\frac{1}{\sqrt{(N-0.5)^2 \cdot \frac{d^2}{N^2} + (h_s - h_{tx})^2}} + \frac{1}{\sqrt{0.25 \cdot \frac{d^2}{N^2} + (h_s - h_{rx})^2}} \right)} \right)$$

$$+ (N-2)L(0)$$

where $$L\ \ \nu\ \ = 6.9 + 20\log\left(\sqrt{\ \nu - 0.1\ ^{2} + 1} + \nu - 0.1\right)$$

b)

$$\nu_1' = (h_s - h_{tx}) \cdot \frac{N-1}{N-0.5} \cdot \sqrt{\frac{2}{\lambda} \cdot \left(\frac{1}{\sqrt{\frac{d^2}{4N^2} + (h_s - h_{tx})^2}} + \frac{1}{\frac{N-1}{N} d} \right)}$$

$$= (25-15) \cdot \frac{20-1}{20-0.5} \cdot \sqrt{\frac{2}{0.15} \cdot \left(\frac{1}{\sqrt{\frac{2000^2}{4 \cdot 20^2} + (25-15)^2}} + \frac{1}{\frac{20-1}{20} \cdot 2000} \right)}$$

$$= 10 \cdot \frac{19}{19.5} \cdot \sqrt{\frac{2}{0.15} \cdot \left(\frac{1}{\sqrt{2500+100}} + \frac{1}{\frac{19}{20} \cdot 2000} \right)}$$

$$= 9.744 \cdot \sqrt{\frac{2}{0.15} \cdot \left(\frac{1}{50.990} + \frac{1}{1900} \right)} = 9.744 \cdot \sqrt{0.26851} = 5.049$$

$$\nu_N = \left(h_s - h_{rx} - (h_{tx} - h_{rx}) \cdot \frac{0.5}{N} \right) \cdot$$

$$\sqrt{\frac{2}{\lambda} \cdot \left(\frac{1}{\sqrt{(N-0.5)^2 \cdot \frac{d^2}{N^2} + (h_s - h_{tx})^2}} + \frac{1}{\sqrt{0.25 \cdot \frac{d^2}{N^2} + (h_s - h_{rx})^2}} \right)}$$

$$= \left(25 - 1.5 + (15 - 1.5) \cdot \frac{0.5}{20} \right) \cdot$$

$$\sqrt{\frac{2}{0.15} \cdot \left(\frac{1}{\sqrt{(20-0.5)^2 \cdot \frac{2000^2}{20^2} + (25-15)^2}} + \frac{1}{\sqrt{0.25 \cdot \frac{2000^2}{20^2} + (25-1.5)^2}} \right)}$$

$$= \left(23.5 - 13.5 \cdot \frac{0.5}{20}\right) \cdot \sqrt{\frac{2}{0.15} \cdot \left(\frac{1}{\sqrt{19.5^2 \cdot 100^2 + 10^2}} + \frac{1}{\sqrt{0.25 \cdot 100^2 + 23.5^2}}\right)}$$

$$= 23.1625 \cdot \sqrt{\frac{2}{0.15} \cdot \left(\frac{1}{\sqrt{3802500 + 100}} + \frac{1}{\sqrt{2500 + 552.25}}\right)}$$

$$= 23.1625 \cdot \sqrt{\frac{2}{0.15} \cdot \left(\frac{1}{1950.026} + \frac{1}{55.247}\right)} = 23.1625 \cdot \sqrt{0.24818} = 11.539$$

The excess loss due to multiple screen diffraction is

$$\Delta L_{msd} = L\;\; \nu_1 \;\; + L\;\; \nu_{20} \;\; + 18 \cdot L\;\; 0$$

$$= 6.9 + 20\log\left(\sqrt{\;5.049 - 0.1\;^2 + 1} + 5.049 - 0.1\right)$$

$$+ 6.9 + 20\log\left(\sqrt{\;11.539 - 0.1\;^2 + 1} + 11.539 - 0.1\right)$$

$$+ 18 \cdot \left(6.9 + 20\log\left(\sqrt{\;0 - 0.1\;^2 + 1} + 0 - 0.1\right)\right)$$

$$= 6.9 + 20\log\;\; 9.998$$

$$+ 6.9 + 20\log\;\; 22.922$$

$$+ 18 \cdot\;\; 6.9 + 20\log\;\; 0.9050$$

$$= 26.90 + 34.10 + 18 \cdot 6.03 = 169.54 \text{ dB}$$

A5 This task deals with the calculation of excess loss due to diffraction over a knife edge obstacle without and with a reflected ray at 2 GHz. The paths geometries are shown in Fig. 2.

a) Determine the excess diffraction loss due to a single knife edge obstacle at the distance *d* = 1000 m from the transmitter. The distance of the receiver from the obstacle is *b* = 2 m. The transmitter antenna and obstacle height above the flat ground is 20 m and 1.5 m respectively.

b) In this case there is a reflecting wall at the distance *w* = 20 m behind the obstacle. How large is the excess loss of the diffracted and reflected ray at the receiver if the reflection loss is 3 dB?

c) Between which values will the excess loss vary of the sum pf the directly diffracted ray and the reflected ray depending on the phase difference between the two rays?

Notice that the approximate heights of the obstacle for the two rays above corresponding lines of sight are given in Fig, 2.

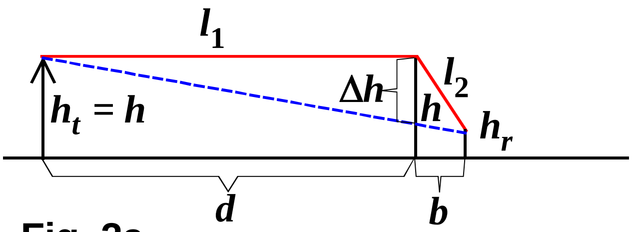


Fig. 2a

$$\Delta h = \frac{d}{d+b} \cdot (h_t - h_r)$$

$$l_2 = \sqrt{b^2 + (h_t - h_r)^2}$$

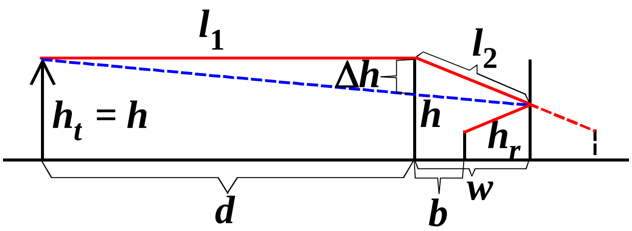


Fig. 2b

diff_reflx_model.dsf

$$\Delta h = \frac{d}{d+w} \cdot \frac{w}{2w-b} \cdot (h_t - h_r)$$

$$l_2 = \sqrt{w^2 + \left(\frac{w}{2w-b} \cdot (h_t - h_r)\right)^2}$$

A6. The delay power spectrum of a radio channel is $P(\lambda) = \frac{P_o}{\tau}\exp\left(-\frac{\lambda}{\tau}\right)\mathrm{u}(\lambda)$, where $\mathrm{u}(\lambda)$ is the unit step function.

a) Determine the delay spread and coherence bandwidth of the channel as these are defined as the interval over which respective absolute-valued spectrum falls 20 dB under its maximum value.

$$F\left\{\exp\left(-\frac{\lambda}{\tau}\right)\mathrm{u}(\lambda)\right\} = \frac{\tau}{1+j2\pi f\tau}$$

b) If the coherence bandwidth is defined as 1/delay spread in task a, how many dB has the absolute value of the frequency correlation function fallen below its maximum value at this frequency difference?

SOLUTION

a)

$$\frac{P(\lambda)}{P_{\max}} = \frac{P_o}{\tau}\exp\left(-\frac{\lambda}{\tau}\right)\Big/\frac{P_o}{\tau} = 0.01 \rightarrow \exp\left(\frac{\lambda}{\tau}\right) = 100$$

$$\rightarrow \lambda = \ln(100)\cdot\tau = 4.605\tau$$

$$F\left\{\frac{P_o}{\tau}\exp\left(-\frac{\lambda}{\tau}\right)\mathrm{u}(\lambda)\right\} = \frac{P_o}{1+j2\pi f\tau} \rightarrow |P(f)| = \frac{P_o}{\sqrt{1+(2\pi f\tau)^2}}$$

$$\frac{P(B_{coh})}{P_{\max}} = \frac{P_o}{\sqrt{1+(2\pi B_{coh}\tau)^2}}\Big/P_o = 0.01 \rightarrow 1+(2\pi B_{coh}\tau)^2 = 10000$$

$$\rightarrow B_{coh} = \frac{\sqrt{9999}}{2\pi\tau} = \frac{15.91}{\tau}$$

b)

$$\frac{P(B_{coh})}{P_{\max}} = \frac{P_o}{\sqrt{1+\left(2\pi\frac{1}{4.605\tau}\tau\right)^2}}\Bigg/P_o = \frac{1}{\sqrt{1+\left(\frac{2\pi}{4.605}\right)^2}}$$

$$= 0.591 \rightarrow -2.28\text{ dB}$$

A7 What is the maximum bandwidth of an ideal bandpass signal that it could be considered to be a narrow-band signal in the equalizer test channel model in GSM. This model has 6 taps of equal average strength at 0, 3.2, 6.4, 9.6, 12.8, and 16.0 μs. The required signal to error signal ratio is 20 dB.

SOLUTION

$$\lambda_{rms}^2 = \frac{1\cdot 0^2 + 1\cdot 3.2^2 + 1\cdot 6.4^2 + 1\cdot 9.6^2 + 1\cdot 12.8^2 + 1\cdot 16^2}{1+1+1+1+1+1}$$
$$= \frac{1}{6}\left(0 + 10.24 + 40.96 + 92.16 + 163.84 + 256\right) = 93.87\ \mu\text{s}^2$$

$$\gamma = \frac{1}{4\pi^2 B_{rms}^2 \tau_{rms}^2} = \frac{3}{4\pi^2 B^2 \tau_{rms}^2}$$
$$\rightarrow B = \frac{\sqrt{3}}{2\sqrt{\gamma}\pi\tau_{rms}} = \frac{\sqrt{3}}{2\cdot 10\cdot \pi\sqrt{93.87}}\ \text{MHz} = 2.845\ \text{kHz}$$

A8. Calculate the average path loss on a 2 km path on 2.1 GHz with worst street orientation using the COST237 Walfisch Ikegami model with the following parameters: average roof height 20 m, base station antenna height 35 m, street width 20 m, average building spacing 100m, and mobile station antenna height 1.5m.

SOLUTION

$$L_o = 32.5 + 20\log d\ \ km\ + 20\log f\ \ MHz$$
$$= 32.5 + 20\log\ 2\ + 20\log\ 2100\ = 32.5 + \ 6.02 + 66.44 = 104.96\text{ dB}$$

$$L_{rts} = -16.9 - 10\log w\ \ m\ + 10\log f\ \ MHz$$
$$+ 20\log\ h_{roof}\ \ m\ - h_{rx}\ \ m\ \ + L_{ori}$$
$$= -16.9 - 10\log\ 20\ + 10\log\ 2100\ + 20\log\ 20 - 1.5\ + 4$$
$$= -16.9 - 13.01 + 33.22 + 25.34 + 4 = 32.65\text{ dB}$$

$$L_{msd} = L_{bsh} + k_a + k_d\log d\ \ km\ + k_f\log f\ \ MHz\ - 9\log b\ \ m$$

$$L_{bsh} = -18\log\ 1 + h_{tx}\ \ m\ - h_{roof}\ \ m\ \ ,\quad h_{tx} > h_{roof}$$
$$= -18\log\ 1 + 35 - 20\ = -21.67\text{ dB}$$

$$k_a = 54\text{ dB},\quad h_{tx} > h_{roof}$$

$$k_d = 18\text{ dB},\quad h_{tx} > h_{roof}$$

$$k_f = \begin{cases} -4 + 0.7\left(\dfrac{2100}{925} - 1\right) = -3.11\text{ dB}, & \text{small cities} \\ -4 + 1.5\left(\dfrac{2100}{925} - 1\right) = -2.09\text{ dB}, & \text{large cities} \end{cases}$$

$$L_{msd} = L_{bsh} + k_a + k_d\log d\ \ km\ + k_f\log f\ \ MHz\ - 9\log b\ \ m$$
$$= -21.67 + 54 + 18\log\ 2\ - \begin{Bmatrix} 3.11 \\ 2.09 \end{Bmatrix}\log\ 2100\ - 9\log\ 100$$
$$= -21.67 + 54 + 5.42 - \begin{Bmatrix} 10.33 \\ 6.94 \end{Bmatrix} - 18 = \begin{Bmatrix} 9.42\text{ dB} \\ 12.81\text{ dB} \end{Bmatrix}$$

$$L = L_o + L_{rts} + L_{msd} = 104.96 + 32.65 + \begin{Bmatrix} 9.42 \\ 12.81 \end{Bmatrix} = \begin{Bmatrix} 147.0\text{ dB} \\ 150.4\text{ dB} \end{Bmatrix}$$

A9. Coverage planning in a cellular network shows that 50% of the locations have coverage at a 5 km distance from the base station. The path loss exponent is 3.5. How large is the cell radius if the coverage requirement at the cell border is i) 90%, ii) 95%, and iii) 99%? The standard deviation of the shadow facing is 8 dB.

SOLUTION

The additional loss from shadow fading is given by

$$P\ \ L > L_i + \Delta L\ \ = \int_{\Delta L/\sigma_L}^{\infty} \frac{\exp\left(-\frac{x^2}{2}\right)}{\sqrt{2\pi}} dx = Q\left(\frac{\Delta L}{\sigma_L}\right)$$

$$\Delta L = \sigma_L \text{ INVQ } \ P\ \ L > L_i + \Delta L\ \ \ = \sigma_L \text{ INVQ } \ 1 - P_{coverage}$$

$$P_{coverage} = 0.9 \rightarrow \Delta L = 8 \cdot \text{INVQ}\ \ 0.1\ \ = 8 \cdot 1.282 = 10.26 \text{ dB}$$

$$P_{coverage} = 0.95 \rightarrow \Delta L = 8 \cdot \text{INVQ}\ \ 0.05\ \ = 8 \cdot 1.645 = 13.16 \text{ dB}$$

$$P_{coverage} = 0.99 \rightarrow \Delta L = 8 \cdot \text{INVQ}\ \ 0.01\ \ = 8 \cdot 2.326 = 18.61 \text{ dB}$$

When the other link arameters stay constant the increased path loss can only be compensated by a shorter cell range. With a single slope path loss model this implies that

$$L_1 = L_o + \Delta 10 n \lg\ \ d_1/d_o\ \ = L_o + \Delta L + 10 n \lg\ \ d_2/d_o$$

$$\rightarrow 10 n \lg\ \ d_1/d_o\ \ = \Delta L + 10 n \lg\ \ d_1/d_o$$

$$\rightarrow \Delta L = 10 n\ \ \lg\ \ d_1/d_o\ \ - \lg\ \ d_2/d_o\ \ = 10 n \lg\ \ d_1/d_2$$

$$\rightarrow d_2 = d_1 10^{-0.1 \Delta L/n}$$

$$P_{\text{coverage}} = 0.9 \rightarrow \Delta L = 10.26 \text{ dB} \infty d_2 = 5 \cdot 10^{-0.1 \cdot 10.26/3.5} = 2.546 \text{ km}$$

$$P_{\text{coverage}} = 0.95 \rightarrow \Delta L = 13.16 \text{ dB} \infty d_2 = 5 \cdot 10^{-0.1 \cdot 13.16/3.5} = 2.104 \text{ km}$$

$$P_{\text{coverage}} = 0.99 \rightarrow \Delta L = 18.61 \text{ dB} \rightarrow d_2 = 5 \cdot 10^{-0.1 \cdot 18.61/3.5} = 1470 \text{ km}$$

A10. A good DVB-T receiver needs a field strength of 20 dBμV/m to deliver a satisfying picture. The transmitter network is planned for 80 km range with 90 % coverage with the receiver antenna height corresponding to average terrain cover height. How large transmit power (EIRP) is needed to fulfill the requirement at 600 MHz according to the ITU-R Recommendation P1546? The effective transmitter antenna height is 300 m..

SOLUTION

From the actual ITU-R Recommendation P1546 graph the field strength level at 80 km distance is 30 dBμV/m with a transmitter power of 1 kW EIRP and 50 % location probability. This is 10 dB higher than the required level.

The shadow fading margin (*SFM*) for 90 % location probability is

$$SFM = E_q - E_{50} = \mathrm{INVQ}\left(\frac{q}{100}\right) \cdot \sigma_L \quad f$$

q is the probability that the field strength level exceeds a given value, in this case 100 – 90 = 10 %

The standard deviation of shadow fading for signals having a bandwidth larger than 1 MHz is according the Rec. P1546 5.5 dB.

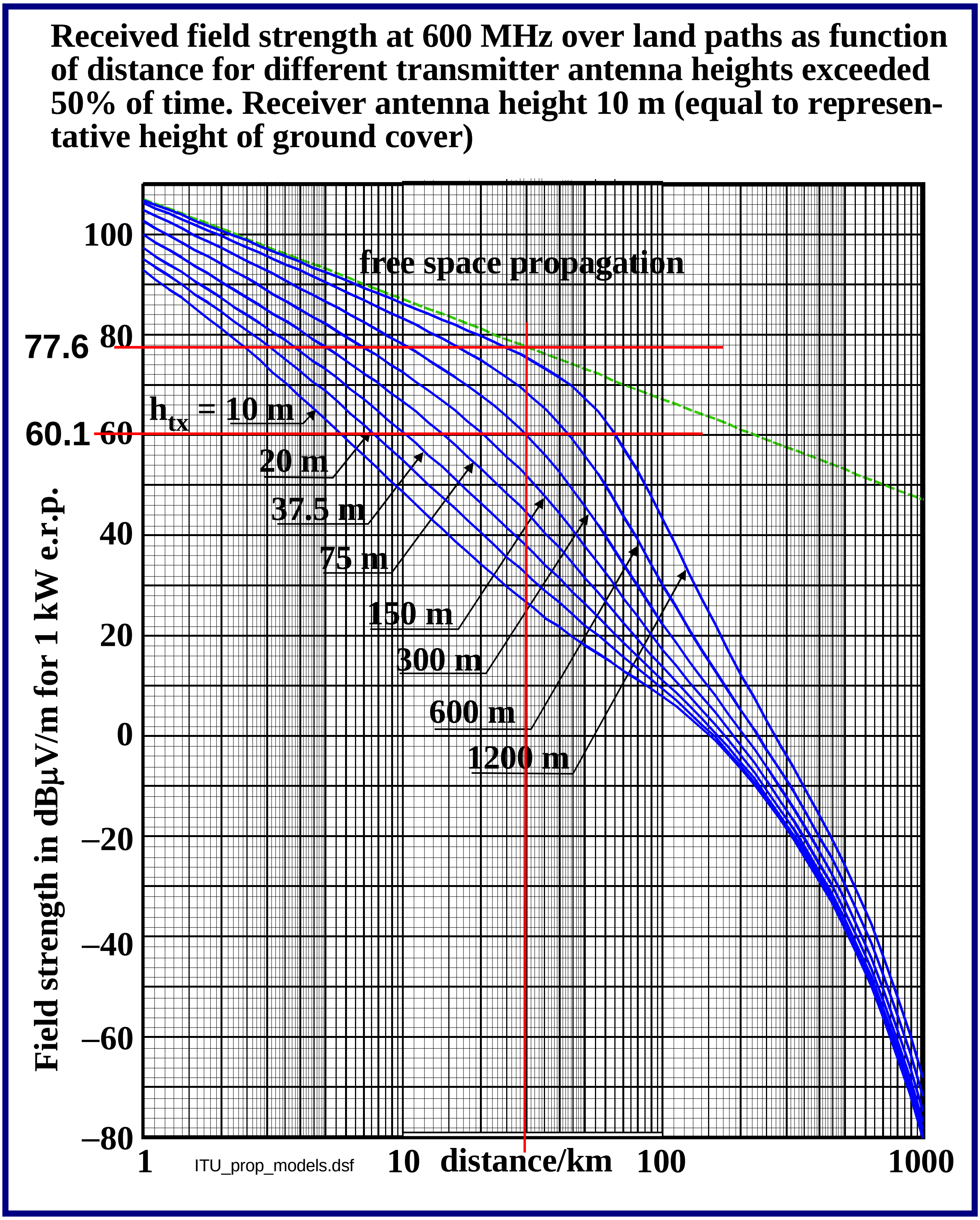
Thus $$SFM = \mathrm{INVQ}\left(\frac{10}{100}\right) \cdot 5.5 = 1.282 \cdot 5.5 = 7.05 \text{ dB}$$

The needed transmitter EIRP level is

$$P_{tx} = 10\lg(1 \text{ kW}) + E_{required} - E_{1 \text{ kW}} + SFM$$
$$= 30 \text{ dBW} - 10 + 7.05 = 27.05 \text{ dBW} \leftrightarrow 0.507 \text{ kW EIRP}$$

A11. Which receiver antenna height will give a field strength equal to the free space value on an over-land 30 km path at 600 MHz according to the ITU-R Rec. P1546. The transmitter antenna height is 300 m and the path is in rural environment.

SOLUTION



From the graph can be seen that the free space field is 77.8 dBμV/m, and the field with 10 m receiver antenna height is 60.1 dBμV/m.

Thus a receiver antenna height giving 77.8 – 60.1 = 17.7 dB higher field strength is the largest appropriate value as that gives the free space field which cannot be exceeded.

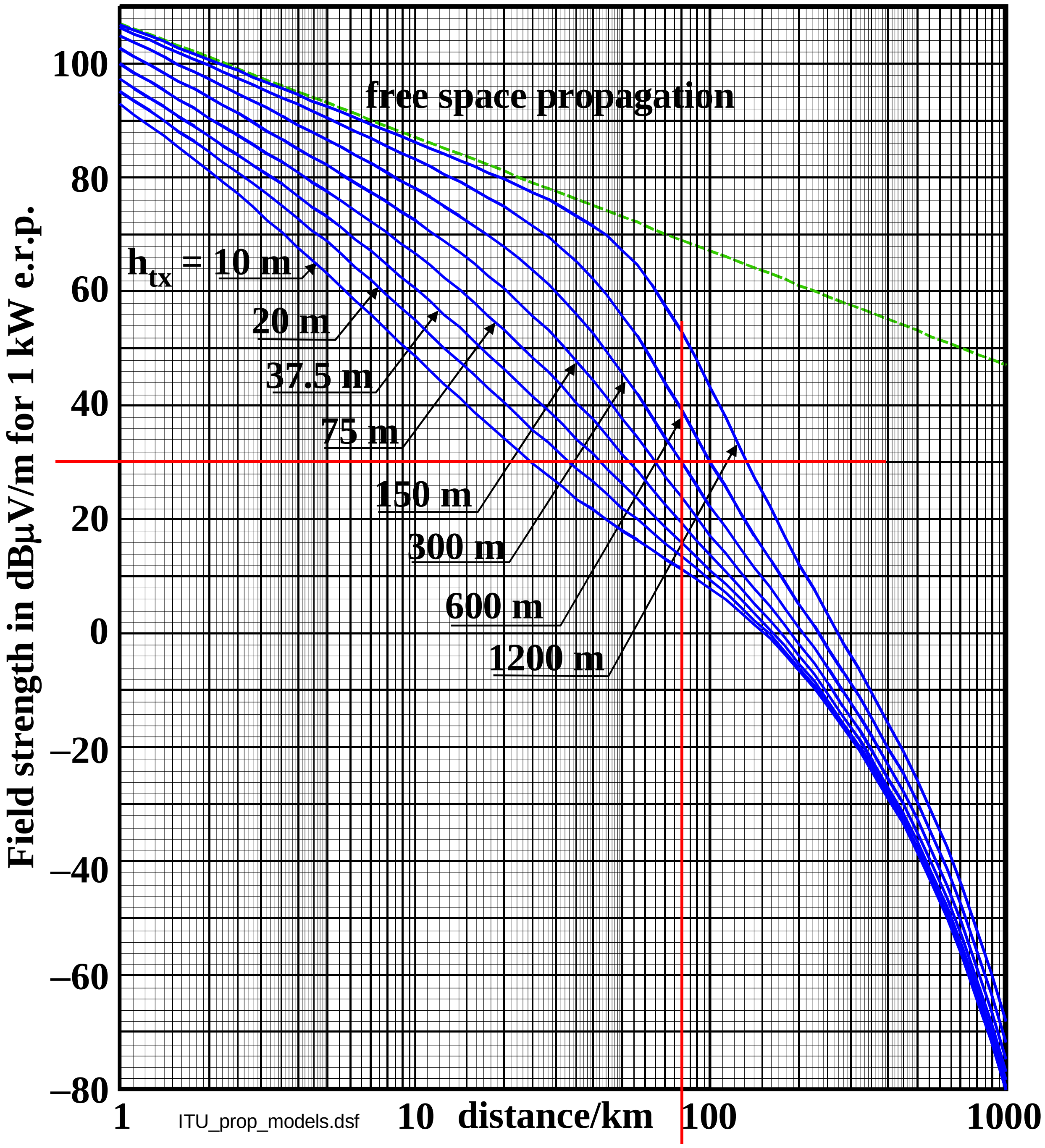
The increase of field strength vs. receiver antenna height is given by

$$\Delta E = \left(3.2 + 6.2\log(f)\right)\log\left(h_{rx}/h_{cm}\right)$$

$$\rightarrow h_{rx} = h_{cm}10^{\frac{\Delta E}{3.2+6.2\log(f)}} = 10\cdot 10^{\frac{17.7}{3.2+6.2\log(600)}}$$

$$= 10\cdot 10^{\frac{17.7}{20.42}} = 73.55\ \text{m}$$

Received field strength at 600 MHz over land paths as function of distance for different transmitter antenna heights exceeded 50% of time. Receiver antenna height 10 m (equal to representative height of ground cover)



A12 In long-haul fixed radio tropospheric multipath propagation is the main cause for signal fading and enhancement. The ITU-R Recommendation P530 gives expressions for the fading and enhancement distributions.

The radio link parameters are the following: f = 7.5 GHz, d = 45 km, dN_1 = –350 NU/km, and $h_L = h_{tx} = h_{rx} = 80$ m

a) Which fade depth is exceeded 0.005% of time?
b) Which enhancement is exceeded 0.005% of time?

SOLUTION

a) The percentage of time a certain fade depth A is exceeded is given by the expression whem the link path topography is unknown

$$p_w = Kd^{3.0}\ 1+\left|\varepsilon_p\right|^{-1.2} \cdot 10^{0.033f-0.001h_L-A/10}$$

where $K = 10^{-6.2-0.0029dN_1}$

Insertion of given values gives

$$K = 10^{-6.2-0.0029\cdot\ -350} = 10^{-5.185}$$

$$p_w = 10^{-5.185}\cdot 45.0^{3.0}\cdot\ 1+\left|0\right|^{-1.2}\cdot 10^{0.033\cdot 7.5-0.001\cdot 80-A/10}$$
$$= 10^{-5.185}\cdot 10^{4.9596}\cdot 10^{0.2475-0.08}\cdot 10^{-A/10}$$
$$= 10^{-0.0579}\cdot 10^{-A/10}$$

Now the fade depth can be solved

NB. The above expressions give the fade probability value, not the probability percentage.

$$p_w = 10^{-0.0579-A/10} \rightarrow -0.0579 - A/10 = \lg\ p_w$$
$$\rightarrow A = 10\cdot\ -0.0579 - \lg\ p_w\ = 10\cdot\ -0.0579 - \lg\ 0.00005$$
$$= 10\cdot\ -0.0579 -\ -4.3001\ = 10\cdot\ -0.0579 + 4.3001\ = 42.43 \text{ dB}$$

As A > 25 dB the probability expression and thus the result is valid.

b) The enhancement probability is given by the expression

$$p_{ew} = P\left(\Delta L < -E\right) = 0.01 \cdot 10^{\left(-1.7+0.2A_{0.01}-E\right)/3.5}$$

First the value of $A_{0.01}$ must be obtained (0.01 means p_w = 0.01 %),

$$\rightarrow A = 10 \cdot \left(-0.0579 - \lg\left(p_w\right)\right) = 10 \cdot \left(-0.0579 - \lg\left(0.0001\right)\right)$$

$$= 10 \cdot \left(-0.0579 - \left(-4\right)\right) = 10 \cdot \left(-0.0579 + 4\right) = 40.06 \text{ dB}$$

Now the actual enhancement value can be solved

$$p_{ew} = 0.01 \cdot 10^{\left(-1.7+0.2\cdot 40.06-E\right)/3.5} = 10^{-2+\left(6.312-E\right)/3.5}$$

$$= 10^{\left(-0.688-E\right)/3.5} \rightarrow \frac{-0.688-E}{3.5} = \lg\left(p_{ew}\right)$$

$$\rightarrow E = -0.688 - 3.5 \cdot \lg\left(p_{ew}\right) = -0.688 - 3.5 \cdot \lg\left(0.00005\right) = 14.37 \text{ dB}$$

As E > 10 dB the enhancement probability expression and thus the result are valid, A13. The typical urban channel model for GSM1800 contains six flat Rayleigh-fading taps having a Doppler-spectrum according to Clarke's model. A case with a mobile station moving 125 km/h is performed.

a) How many times in a second does a tap signal pass below –10 dB and pass above +10 dB relative to the r.m.s. value?

b) How long is the average duration of a signal fade below –10 dB and the average duration of a signal enhancement above +10 dB relative to the r.m.s. value?

SOLUTION

a) The level crossing rate for a Rayleigh-fading narrow-band signal in a channel with Clarke's Doppler-shift model is

$$N_A = \sqrt{2\pi}\, f_{D\max}\, \rho \exp\left(-\rho^2\right)$$

The maximum Doppler-shift is

$$f_{D\max} = \frac{v}{c} \cdot f = \frac{125/3.6}{3 \cdot 10^8} \cdot 1.8 \cdot 10^9 = 208.33 \text{ Hz}$$

The absolute value of the faded signal level is

$$\rho^2 = 10^{-10/10} = 0.1$$

The absolute value of the enhanced signal level is

$$\rho^2 = 10^{10/10} = 10$$

Then the level crossing rates upwards or downwards through the –10 dB level is

$$N_{A=-10\text{ dB}} = \sqrt{2 \cdot \pi} \cdot 208.33 \cdot \sqrt{0.1} \cdot \exp\left(-0.1\right) = 149.42 \text{ s}^{-1}$$

Then the level crossing rates upwards or downwards through the +10 dB level is

$$N_{A=+10\text{ dB}} = \sqrt{2 \cdot \pi} \cdot 208.33 \cdot \sqrt{10} \cdot \exp\left(-10\right) = 0.07497 \text{ s}^{-1} = 4.498 \text{ min}^{-1}$$

b) The average duration of a fade deeper than a given value is

$$T_A = \frac{\exp\left(\rho^2\right) - 1}{f_{D\max}\rho\sqrt{2\pi}} = \frac{\exp\left(0.1\right) - 1}{208.33 \cdot \sqrt{0.1} \cdot \sqrt{2\pi}} = 6.369 \cdot 10^{-4}\ \text{s} = 636.9\ \mu\text{s}$$

The duration of an enhancement is defined as

$$T_A = \frac{\mathrm{P}\left(a > A\right)}{N_A} = \frac{\exp\left(-\rho^2\right)}{f_{D\max}\rho\sqrt{2\pi}\exp\left(-\rho^2\right)} = \frac{1}{f_{D\max}\rho\sqrt{2\pi}}$$

Insertion of the actual values gives

$$T_A = \frac{1}{208.33 \cdot 10 \cdot \sqrt{2\pi}} = 1.9150 \cdot 10^{-4}\ \text{s} = 191.5\ \mu\text{s}$$