S-72.3210
Channel modeling for radio communication systems
Exercise 1

1. A 10 kW MF-transmitter at 1 MHz uses an antenna with 5 dBi antenna gain. The receiver antenna gain is 0 dBi, and the total noise level in the receiver input is -74 dBm. To achieve the QoS-requirements a 20 dB SNR is needed. Based on the ITU-curves for ground wave propagation, estimate the maximum range over land (σ =3 mS/m, ε = 40).

SOLUTION

The path loss is obtained in the following way:

$$P_{rx} \geq S = P_n + SNR = P_{tx} + G_{tx} - L_p + G_{rx}$$
$$\rightarrow L_p \leq P_{tx} + G_{tx} + G_{rx} - P_n - SNR$$
$$= 70 \text{ dBm} + 5 + 0 - (-74 \text{ dBm}) - 20 = 129 \text{ dB}$$

On the other hand the relationship between path loss and received field is given by:

$$L_p = 142.0 + 20\lg f_{MHz} - E_{dB\mu V/m,1\,\text{kW}}$$

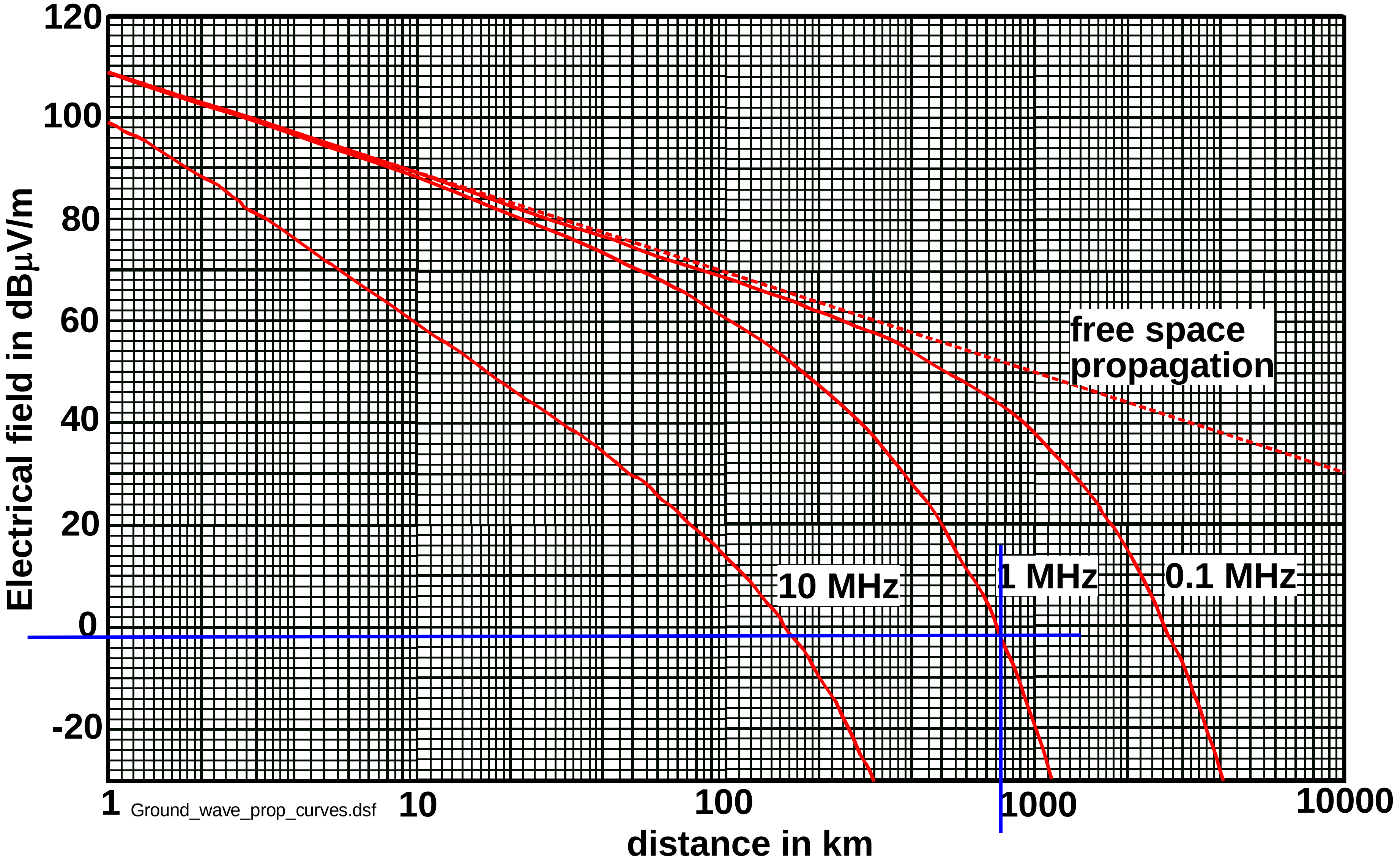
The above expression is valid only for an EIRP of 1 kW. Equaling the two expressions of path loss gives the necessary field strength with that transmit power:

$$129 = 142.0 + 20\lg f_{MHz} - E_{dB\mu V/m},1\text{ kW}$$
$$\rightarrow E_{dB\mu V/m,1\,\text{kW}} = 142.0 + 20\lg f_{MHz} - 129 = 142.0 + 0 - 129$$
$$= 13\,\text{dB}\mu\text{V/m}$$

As the field strength curves in the ITU-recommendation are drawn assuming 1 kW transmitted EIRP and the actual EIRP is 10 + 5 = 15 dB higher, the field strength level to look for the maximum distance in the ITU field strength figures is 13 – 15 = – 2 dBμV/m.

Fig. 4 in ITU Rec. P.368-8 gives the range estimate 770 km.

Ground-wave propagation over land, σ = 3 mS/m, ε = 40



2. What would be the range of the radio system in Problem 1 over sea with average salinity (σ = 5 S/m, ε = 70), when the transmitter is on land 200 km from the sea shore (mixed path)?

SOLUTION

From the mixed path expressions for three different ground sections it is easy to obtain the expressions for two different ground sections.

$$E_R = E_1 \; d_1 \; - E_2 \; d_1 \; + E_2 \; d_1 + d_2$$
$$E_T = E_2 \; d_2 \; - E_1 \; d_2 \; + E_1 \; d_1 + d_2$$
$$E = 0.5 \; E_R + E_T$$

In the actual situation we have

$$E_R = E_1 \; 200 \; - E_2 \; 200 \; + E_2 \; d \; \approx 48 - 60 + E_2 \; d \; = E_2 \; d \; - 12$$
$$E_T = E_2 \; d - 200 \; - E_1 \; d - 200 \; + E_1 \; d$$
$$E = 0.5 \; E_1 \; d \; - E_1 \; d - 200 \; + E_2 \; d \; + E_2 \; d - 200 \; - 12 \; = -2 \text{ dB}\mu\text{V/m}$$

From the ground wave propagation curves we can read the following field values in dBμV/m

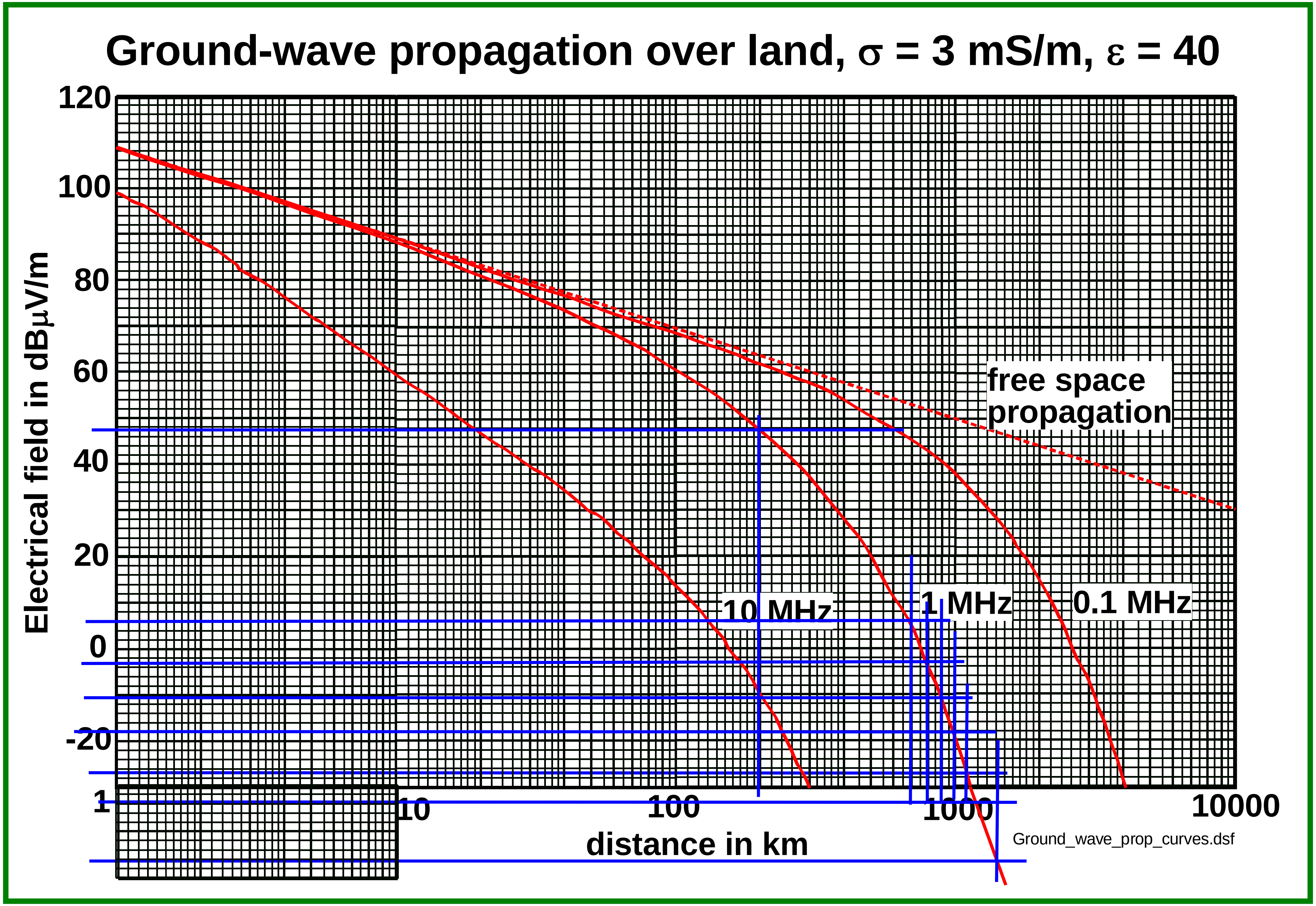
d	200	800	900	1000	1100	1200	1400
E1(d)	48	-3	-11	-18	-27	-34	-46
E2(d)	60	30	26	22	16	12	4

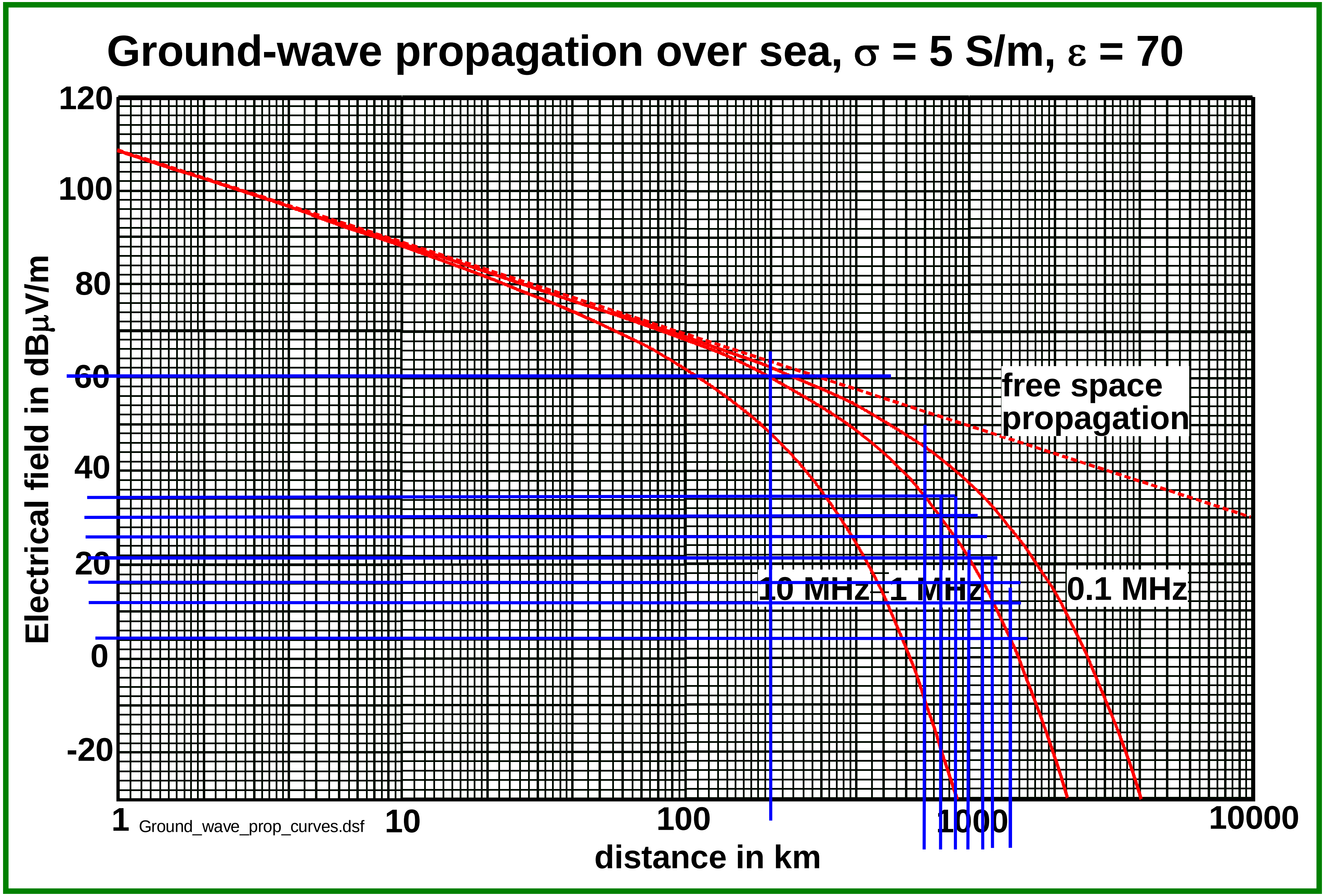
d = 1000 km $\rightarrow$
$$E = 0.5 \; E_1 \; 1000 \; - E_1 \; 800 \; + E_2 \; 1000 \; + E_2 \; 800 \; - 12$$
$$= 0.5 \; -18 + 3 + 22 + 30 - 12 \; = 12.5 \text{ dB}\mu\text{V/m}$$

d = 1200 km $\rightarrow$
$$E = 0.5 \; E_1 \; 1200 \; - E_1 \; 1000 \; + E_2 \; 1200 \; + E_2 \; 1000 \; - 12$$
$$= 0.5 \; -34 + 18 + 12 + 22 - 12 \; = 3.0 \text{ dB}\mu\text{V/m}$$

d = 1400 km $\rightarrow$
$$E = 0.5 \; E_1 \; 1400 \; - E_1 \; 1200 \; + E_2 \; 1400 \; + E_2 \; 1200 \; - 12$$
$$= 0.5 \; -46 + 34 + 4 + 12 - 12 \; = -8 \text{ dB}\mu\text{V/m}$$

Linear interpolation gives a range of about 1300 km.


Ground-wave propagation over land, σ = 3 mS/m, ε = 40
Electrical field in dBμV/m
120
100
80
60
40
20
0
-20
1
10
100
1000
10000
distance in km
free space propagation
10 MHz
1 MHz
0.1 MHz
Ground_wave_prop_curves.dsf


Ground-wave propagation over sea, σ = 5 S/m, ε = 70
Electrical field in dBμV/m
120
100
80
60
40
20
0
-20
1
Ground_wave_prop_curves.dsf
10
100
1000
10000
distance in km
free space
propagation
10 MHz
1 MHz
0.1 MHz

18. The bad urban channel model in the TETRA defined in the TETRA specification comprises 2 independent Rayleigh-fading taps with 5 μs delay difference and the average power level of the delayed tap is 3 dB smaller than the average power level of the direct tap. In TETRA the modulation method is π/4-shifted DQPSK, The used pulse spectrum shape with the roll-off parameter $\alpha = 0.35$. The symbol rate over the air interface is $R_S = 18$ kBd.

a) Derive the autocorrelation function of the time variant transfer function $R_H(\Delta t, \Delta f)$.

b) Derive the autocorrelation function of the time variant impulse response $R_h(\Delta t, \Delta\lambda)$ and determine the r.m.s. delay spread.

c) Check using the method outlined in the lecture material whether the transmitted signal can be regarded as narrow-band in this channel when the bit error probability target is 0.001.

d) To which transfer function correlation value does the maximum r.m.s. bandwidth for flat fading in sub-task c) correspond?

SOLUTION

a) As in Problem 14 the autocorrelation function of the channel transfer function is

$$P_H(\Delta f, \Delta t) = R_{h_1}(\Delta t) + R_{h_2}(\Delta t) e^{-j2\pi\Delta f \tau}$$

b) The inverse Fourier-transform gives the autocorrelation function of the channel impulse response

$$P_h(\lambda, \Delta t) = R_{h_1}(\Delta t)\delta(\lambda) + R_{h_2}(\Delta t)\delta(\lambda - \tau)$$

The r.m.s. delay-spread is calculated using $P_h(\lambda, 0)$:

$$P_h(\lambda, 0) = R_{h1}(0)\delta(\lambda) + R_{h2}(\Delta\tau)\delta(\lambda - \tau) = P_1\delta(\lambda) + P_2\delta(\lambda - \tau)$$

$$\lambda_{rms} = \sqrt{\frac{\int \lambda^2 P_S(\lambda, 0)\, d\lambda}{\int P_S(\lambda, 0)\, d\lambda}} = \sqrt{\frac{\tau_1^2 P_1 + \tau_2^2 P_2}{P_1 + P_2}}$$

$$= \sqrt{\frac{0^2 \cdot 1 + 5^2 \cdot 10^{-0.1\cdot 3}}{1 + 10^{-0.1\cdot 3}}} = \sqrt{\frac{0 + 12.53}{1 + 0.501}} = 2.89\ \mu\text{s}$$

c) The necessary snr-value for obtaining the average bit error probability 0.001 in an AWGN-channel is solved from the bit error probability of QPSK:

$$P_b = Q\left(\sqrt{3.09}\right) = 0.001 \rightarrow \gamma = INVQ\left(10^{-3}\right) = 3.09^2 = 9.55$$

The r.m.s. signal bandwidth for flat fading can be calculated from the expression derived in the lecture material

$$\gamma = \frac{1}{4\pi^2 B_{rms}^2 \tau_{rms}^2}$$

from which we get

$$. \; B_{rms} = \sqrt{\frac{1}{4\pi^2 \gamma \tau_{rms}^2}} = \frac{1}{2\pi\tau_{rms}\sqrt{\gamma}} = \frac{1}{2\cdot\pi\cdot 2.89\cdot 3.09}\ \text{MHz} = 17.82\ \text{kHz}$$

The r.m.s. bandwidth of a digital signal using a raised cosine pulse spectrum is obtained in following way.

Signal spectrum:

$$X(f) = \begin{cases} \dfrac{1}{R_s}, & |f| < (1-\alpha)\dfrac{R_s}{2} \\ \dfrac{1}{2R_s}\left[1+\cos\left(\dfrac{\pi}{\alpha}\left(\dfrac{|f|}{R_s} - \dfrac{1}{2}(1-\alpha)\right)\right)\right], & (1-\alpha)\dfrac{R_s}{2} < |f| < (1+\alpha)\dfrac{R_s}{2} \\ 0, & |f| > (1+\alpha)\dfrac{R_s}{2} \end{cases}$$

Definition of the r.m.s. bandwidth

$$B_{rms}^2 = \frac{\int_{-\infty}^{\infty} f^2 |X(f)|^2 df}{\int_{-\infty}^{\infty} |X(f)|^2 df}$$

With the given signal spectrum

$$B_{rms}^2 = \frac{\int_0^{(1-\alpha)\frac{R_S}{2}} f^2 df + \int_{(1-\alpha)\frac{R_S}{2}}^{(1+\alpha)\frac{R_S}{2}} f^2 \frac{1}{4}\left[1+\cos\left(\frac{\pi}{\alpha}\left(\frac{|f|}{R_S}-\frac{1}{2}(1-\alpha)\right)\right)\right]^2 df}{\int_0^{(1-\alpha)\frac{R_S}{2}} df + \int_{(1-\alpha)\frac{R_S}{2}}^{(1+\alpha)\frac{R_S}{2}} \frac{1}{4}\left[1+\cos\left(\frac{\pi}{\alpha}\left(\frac{|f|}{R_S}-\frac{1}{2}(1-\alpha)\right)\right)\right]^2 df}$$

Inserting in the second integral terms

$$x = \frac{\pi}{\alpha}\left(\frac{f}{R_S}-\frac{1}{2}(1-\alpha)\right) \rightarrow (1-\alpha)\frac{R_S}{2} \Rightarrow 0,\ (1+\alpha)\frac{R_S}{2} \Rightarrow \pi$$

$$\rightarrow f = \frac{\alpha R_S}{\pi}x + \frac{R_S}{2}(1-\alpha) = \frac{\alpha R_S}{\pi}\left(x+\frac{\pi R_S}{2\alpha R_S}(1-\alpha)\right) = A(x+B)$$

$$\rightarrow df = Adx$$

$$A = \frac{\alpha R_S}{\pi},\ B = \frac{\pi}{2\alpha}(1-\alpha) = \frac{R_S(1-\alpha)}{2A}$$

Then

$$B_{rms}^2 = \frac{(1-\alpha)^3\frac{R_S^3}{24} + \frac{A^3}{4}\int_0^{\pi}(x+B)^2\left[1+\cos(x)\right]^2 dx}{(1-\alpha)\frac{R_S}{2} + \frac{A}{4}\int_0^{\pi}\left[1+\cos(x)\right]^2 dx}$$

$$= \frac{(1-\alpha)^3\frac{R_S^3}{24} + \frac{A^3}{4}\int_0^{\pi}(x+B)^2\left[2\cos^2\left(\frac{x}{2}\right)\right]^2 dx}{(1-\alpha)\frac{R_S}{2} + \frac{A}{4}\int_0^{\pi}\left[2\cos^2\left(\frac{x}{2}\right)\right]^2 dx}$$

$$= \frac{(1-\alpha)^3\frac{R_S^3}{24} + A^3\int_0^{\pi}(x+B)^2\cos^4\left(\frac{x}{2}\right)dx}{(1-\alpha)\frac{R_S}{2} + A\int_0^{\pi}\cos^4\left(\frac{x}{2}\right)dx}$$

Changing the variable $y = \frac{x}{2} \rightarrow dx = 2dy$

$$B_{rms}^2 = \frac{1-\alpha \;\; {}^3 \frac{R_s^3}{24} + 2A^3 \int_0^{\pi/2} \;\; 2y + B \;\; {}^2 \cos^4 \;\; y \;\; dy}{1-\alpha \;\; \frac{R_s}{2} + 2A \int_0^{\pi/2} \cos^4 \;\; y \;\; dy}$$

$$= \frac{1-\alpha \;\; {}^3 \frac{R_s^3}{24} + 2A^3 \int_0^{\pi/2} \;\; 4y^2 + 4By + B^2 \;\; \cos^4 \;\; y \;\; dy}{1-\alpha \;\; \frac{R_s}{2} + 2A \int_0^{\pi/2} \cos^4 \;\; y \;\; dy}$$

$$\int_0^{\pi/2} 4y^2 \cos^4 \;\; y \;\; dy = \frac{\pi^3}{16} - \frac{15}{32}\pi$$

$$\int_0^{\pi/2} 4By \cos^4 \;\; y \;\; dy = \frac{3}{16}\pi^2 B - B$$

$$\int_0^{\pi/2} B^2 \cos^4 \;\; y \;\; dy = \frac{3}{16}\pi B^2$$

$$B_{rms}^2 = \frac{(1-\alpha)^3 \frac{R_s^3}{24} + 2A^3\left(\frac{\pi^3}{16} - \frac{15}{32}\pi + \frac{3}{16}\pi^2 B - B + \frac{3}{16}\pi B^2\right)}{(1-\alpha)\frac{R_s}{2} + 2A\frac{3}{16}\pi}$$

$$= \frac{(1-\alpha)^3 \frac{R_s^3}{24} + 2\frac{\alpha^3 R_s^3}{\pi^3}\left(\begin{array}{l}\frac{\pi^3}{16} - \frac{15}{32}\pi + \frac{3}{16}\pi^2 \frac{\pi}{2\alpha}(1-\alpha) \\ \qquad - \frac{\pi}{2\alpha}(1-\alpha) + \frac{3}{16}\pi \frac{\pi^2}{4\alpha^2}(1-\alpha)^2\end{array}\right)}{(1-\alpha)\frac{R_s}{2} + 2\frac{\alpha R_s}{\pi}\cdot\frac{3}{16}\pi}$$

$$= \frac{R_s^2}{12}\cdot\frac{(1-\alpha)^3 + 48\frac{\alpha^3}{\pi^3}\left(\begin{array}{l}\frac{\pi^3}{16} - \frac{15}{32}\pi + \frac{3\pi^3}{32\alpha}(1-\alpha) \\ \qquad - \frac{\pi}{2\alpha}(1-\alpha) + \frac{3\pi^3}{64\alpha^2}(1-\alpha)^2\end{array}\right)}{(1-\alpha) + \frac{3\alpha}{4}}$$

$$= \frac{R_s^2}{12}\cdot\frac{(1-\alpha)^3 + \left(\begin{array}{l}3\alpha^3 - \frac{45\alpha^3}{2\pi^2} + \frac{9}{2}\alpha^2(1-\alpha) \\ \qquad - \frac{24}{\pi^2}\alpha^2(1-\alpha) + \frac{9}{4}\alpha(1-\alpha)^2\end{array}\right)}{\left(1 - \frac{\alpha}{4}\right)}$$

Insertion of $\alpha = 0.35$ into the last expression gives

$$B_{rms,TETRA} = \sqrt{0.271}\cdot R_s = 0.521\cdot 18 = 9.37 \text{ kHz}$$

This is clearly less than the r.m.s. bandwidth the channel allows, so on this error probability the TETRA-signal can be regarded as a narrow-band signal.

d) The correlation is based on the absolute value of the autocorrelation function of the channel transfer function

$$\left|P_H(\Delta f,0)\right| = \left|P_1 + P_2 e^{-j2\pi\Delta f\tau}\right| = \sqrt{P_1^2 + P_2^2 + 2P_1P_2\cos\left(2\pi\Delta f\tau\right)}$$

and the correlation calculation follows

$$\rho = \frac{\left|P_H\left(\Delta f,0\right)\right|}{\left|P_H(0,0)\right|} = \frac{\sqrt{P_1^2 + P_2^2 + 2P_1P_2\cos\left(2\pi\Delta f\tau\right)}}{P_1 + P_2}$$

$$= \frac{\sqrt{1+10^{-0.6} + 2\cdot 1\cdot 10^{-0.3}\cos\left(2\pi\cdot 0.01782\cdot 5\right)}}{1+10^{-0.3}}$$

$$= \frac{\sqrt{1+0.251+1.002\cos\left(0.5598\right)}}{1+0.501} = 0.965$$

A flat fading condition requires a rather high correlation.