S-72.3210 Channel modeling for radio communication systems
Exam 31.10.2006

1. In a fixed radio link exists a refraction situation depicted in Fig.1 with two layers with the boundary between them at the antennas. The ray in the lower layer propagates along a straight line and in the upper super-refractive layer along circles with a radius less than $R_o + h$.

a. Define the conditions the vertical refractivity in the two layers must fulfill in this propagation situation.

b. For which path length *d* the received rays have opposite phases at the receiver antenna, when the carrier frequency is 6 GHz and the upper ray just runs at the border between the layers where tha refractivity is the same as in the lower layer.
Hint. Derive the actual path length difference using the true ray path geometry and approximate the result by using the Taylor serues $\sin(x) \approx x - x^3/3! + \ldots$ and $h \ll R_o$.

SOLUTION

a) In the lower layer

$$\left(\frac{\partial n}{\partial h}\right)_1 = -\frac{1}{R_1} = -\frac{1}{\infty} = 0$$

In the upper layer

$$\left(\frac{\partial n}{\partial h}\right)_2 = -\frac{1}{R_1} \leq -\frac{1}{R_o + h} \approx -\frac{1}{R_o} = -\frac{1}{6370}$$

$$\left(\frac{\partial N}{\partial h}\right)_2 \leq -\frac{1000000}{6370} = -157.0 \text{ NU/km}$$

b) As the refractivity is constant along both ray paths, it is sufficient to determine the geometric path length difference and put it equal to half a wavelength (0.05 m)

$$d_1 = 2\ R_o + h\ \sin\left(\frac{d}{2R_o}\right), \qquad d_2 = \ R_o + h\ \frac{d}{2R_o}$$

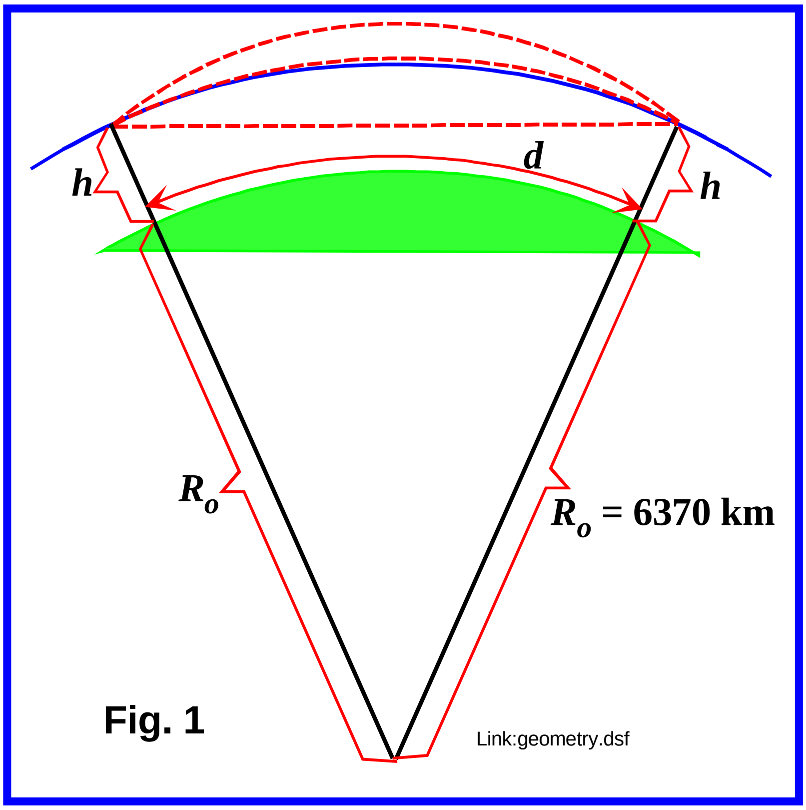


Fig. 1

$$d_2 - d_1 = R_o + h \;\frac{d}{R_o} - 2\; R_o + h \;\sin\left(\frac{d}{2R_o}\right)$$

$$= R_o + h \left[\frac{d}{R_o} - 2\left(\frac{d}{2R_o} - \frac{1}{6}\left(\frac{d}{2R_o}\right)^3 + ..\right)\right]$$

$$\approx R_o + h \;\frac{1}{3}\left(\frac{d}{2R_o}\right)^3 \approx \frac{1}{24}\cdot\frac{d^3}{R_o^2} = \frac{\lambda}{2}$$

$$\rightarrow d = 12R_o^2\lambda^{\;1/3} = 12\cdot 6370^2\cdot 0.000025^{\;1/3} = 23.004\text{ km}$$

2. The excess loss due to diffraction over a knife edge obstacle being 10 m above the line-of-sight on a 10 km path at 3 GHz is considered.

a) Show that the minimum excess loss is obtained when the obstacle is at the middle of the path.

b) How many dB does this excess diffraction loss change if the obstacle is located 1 km from the receiver instead of being at the middle of the path?

SOLUTION

a) The excess loss is an increasing function of the parameter ν

$$\nu = h\sqrt{\frac{2}{\lambda}\cdot\frac{d}{d_1 d_2}} = h\sqrt{\frac{2}{\lambda}\cdot\frac{d}{\varepsilon d\,(1-\varepsilon)\,d}} = h\sqrt{\frac{2}{\lambda}\cdot\frac{1}{\varepsilon\,(1-\varepsilon)\,d}}$$

ν obtains its minimum value when $f(\varepsilon) = \varepsilon(1-\varepsilon) = \varepsilon - \varepsilon^2$ obtains its maximum value. This takes place when $\frac{df(\varepsilon)}{d\varepsilon} = 1 - 2\varepsilon = 0 \rightarrow \varepsilon = 0.5$

As $\frac{d^2 f(\varepsilon)}{d\varepsilon^2} = -2$, the extreme value is a maximum, ν gets its minimum value as does the excess diffraction loss when the obstacle is in the middle of the path.

b) When $\varepsilon = 0.5$,

$$\nu = h\sqrt{\frac{2}{\lambda}\cdot\frac{1}{0.5\cdot 0.5\cdot d}} = 10\sqrt{\frac{2}{0.1}\cdot\frac{1}{0.5\cdot 0.5\cdot 10000}} = 0.894$$

$$\rightarrow L_1 = 6.9 + 20\log\left(\sqrt{(0.894-0.1)^2+1} + (0.894-0.1)\right)$$

$$= 6.9 + 20\log\left(\sqrt{0.794^2+1} + 0.794\right)$$

$$= 6.9 + 20\log(2.071) = 13.22 \text{ dB}$$

When $\varepsilon = 0.1$,

$$\nu = h\sqrt{\frac{2}{\lambda}\cdot\frac{1}{0.1\cdot 0.9\cdot d}} = 10\sqrt{\frac{2}{0.1}\cdot\frac{1}{0.1\cdot 0.9\cdot 10000}} = 1.491$$

$$\to L_2 = 6.9 + 20\log\left(\sqrt{(1.491 - 0.1)^2 + 1} + (1.491 - 0.1)\right)$$

$$= 6.9 + 20\log\left(\sqrt{1.391^2 + 1} + 1.391\right)$$

$$= 6.9 + 20\log(3.104) = 16.74 \text{ dB}$$

$$L_2 - L_1 = 16.74 - 13.22 = 3.52 \text{ dB}$$

3. The delay power spectrum of a radio channel is $P(\lambda) = \frac{P_o}{\tau}\exp\left(-\frac{\lambda}{\tau}\right)u(\lambda)$, where $u(\lambda)$ is the unit step function.

a) Determine the delay spread and coherence bandwidth of the channel as these are defined as the interval over which respective absolute-valued spectrum falls 20 dB under its maximum value.

$$F\left\{\exp\left(-\frac{\lambda}{\tau}\right)u(\lambda)\right\} = \frac{\tau}{1 + j2\pi f\tau}$$

b) If the coherence bandwidth is defined as 1/delay spread in task a, how many dB has the absolute value of the frequency correlation function fallen below its maximum value at this frequency difference?

SOLUTION

a)

$$\frac{P(\lambda)}{P_{\max}} = \frac{P_o}{\tau}\exp\left(-\frac{\lambda}{\tau}\right)\Big/\frac{P_o}{\tau} = 0.01 \to \exp\left(\frac{\lambda}{\tau}\right) = 100$$

$$\to \lambda = \ln(100)\cdot\tau = 4.605\tau$$

$$F\left\{\frac{P_o}{\tau}\exp\left(-\frac{\lambda}{\tau}\right)u(\lambda)\right\} = \frac{P_o}{1 + j2\pi f\tau} \to \left|P(f)\right| = \frac{P_o}{\sqrt{1 + (2\pi f\tau)^2}}$$

$$\frac{P(B_{coh})}{P_{\max}} = \frac{P_o}{\sqrt{1 + (2\pi B_{coh}\tau)^2}}\Big/P_o = 0.01 \to 1 + (2\pi B_{coh}\tau)^2 = 10000$$

$$\to B_{coh} = \frac{\sqrt{9999}}{2\pi\tau} = \frac{15.91}{\tau}$$

b)

$$\frac{P(B_{coh})}{P_{\max}} = \frac{P_o}{\sqrt{1 + \left(2\pi\frac{1}{4.605\tau}\tau\right)^2}}\Bigg/P_o = \frac{1}{\sqrt{1 + \left(\frac{2\pi}{4.605}\right)^2}}$$

$$= 0.591 \to -2.28\text{ dB}$$

4. What is the maximum bandwidth of an ideal bandpass signal that it could be considered to be a narrow-band signal in the equalizer test channel model in GSM. This model has 6 taps of equal average strength at 0, 3.2, 6.4, 9.6, 12.8, and 16.0 μs. The required signal to error signal ratio is 20 dB.

SOLUTION

$$\lambda_{rms}^2 = \frac{1\cdot 0^2 + 1\cdot 3.2^2 + 1\cdot 6.4^2 + 1\cdot 9.6^2 + 1\cdot 12.8^2 + 1\cdot 16^2}{1+1+1+1+1+1}$$

$$= \frac{1}{6}\left(0 + 10.24 + 40.96 + 92.16 + 163.84 + 256\right) = 93.87\ \mu\text{s}^2$$

$$\gamma = \frac{1}{4\pi^2 B_{rms}^2 \tau_{rms}^2} = \frac{3}{4\pi^2 B^2 \tau_{rms}^2}$$

$$\rightarrow B = \frac{\sqrt{3}}{2\sqrt{\gamma}\pi\tau_{rms}} = \frac{\sqrt{3}}{2\cdot 10\cdot \pi\sqrt{93.87}}\ \text{MHz} = 2.845\ \text{kHz}$$

5. Calculate the average path loss on a 2 km path on 2.1 GHz with worsts street orientation using the COST237 Walfisch Ikegami model with the following parameters: average roof height 20 m, base station antenna height 35 m, street width 20 m, average building spacing 100m, and mobile station antenna height 1.5m.

SOLUTION

$$L_o = 32.5 + 20\log d\ \ km\ \ + 20\log f\ \ MHz$$
$$= 32.5 + 20\log\ 2\ \ + 20\log\ 2100\ \ = 32.5 + \ 6.02 + 66.44 = 104.96 \text{ dB}$$

$$L_{rts} = -16.9 - 10\log w\ \ m\ \ + 10\log f\ \ MHz$$
$$+ 20\log\ h_{roof}\ \ m\ - h_{rx}\ \ m\ \ + L_{ori}$$
$$= -16.9 - 10\log\ 20\ \ + 10\log\ 2100\ \ + 20\log\ 20 - 1.5\ \ + 4$$
$$= -16.9 - 13.01 + 33.22 + 25.34 + 4 = 32.65 \text{ dB}$$

$$L_{msd} = L_{bsh} + k_a + k_d \log d\ \ km\ \ + k_f \log f\ \ MHz\ \ - 9\log b\ \ m$$

$$L_{bsh} = -18\log\ 1 + h_{tx}\ \ m\ - h_{roof}\ \ m\ \ , \quad h_{tx} > h_{roof}$$
$$= -18\log\ 1 + 35 - 20\ \ = -21.67 \text{ dB}$$

$$k_a = 54 \text{ dB}, \quad h_{tx} > h_{roof}$$

$$k_d = 18 \text{ dB}, \quad h_{tx} > h_{roof}$$

$$k_f = \begin{cases} -4 + 0.7\left(\dfrac{2100}{925} - 1\right) = -3.11 \text{ dB}, & \text{small cities} \\ -4 + 1.5\left(\dfrac{2100}{925} - 1\right) = -2.09 \text{ dB}, & \text{large cities} \end{cases}$$

$$L_{msd} = L_{bsh} + k_a + k_d \log d \;\; km \;\; + k_f \log f \;\; MHz \;\; - 9 \log b \;\; m$$

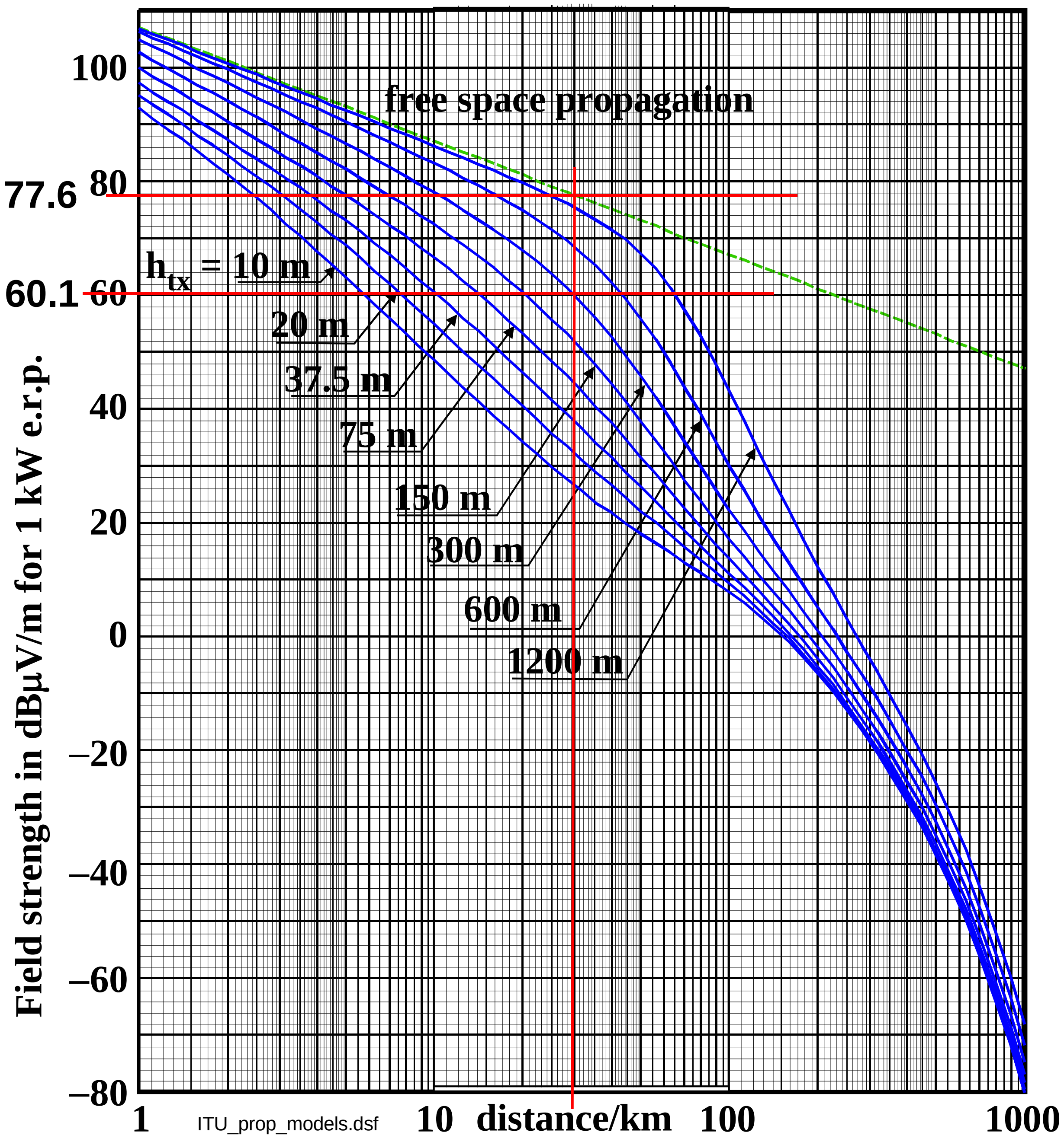
$$= -21.67 + 54 + 18 \log \;\; 2 \;\; - \begin{Bmatrix} 3.11 \\ 2.09 \end{Bmatrix} \log \;\; 2100 \;\; - 9 \log \;\; 100$$

$$= -21.67 + 54 + 5.42 - \begin{Bmatrix} 10.33 \\ 6.94 \end{Bmatrix} - 18 = \begin{Bmatrix} 9.42 \text{ dB} \\ 12.81 \text{ dB} \end{Bmatrix}$$

$$L = L_o + L_{rts} + L_{msd} = 104.96 + 32.65 + \begin{Bmatrix} 9.42 \\ 12.81 \end{Bmatrix} = \begin{Bmatrix} 147.0 \text{ dB} \\ 150.4 \text{ dB} \end{Bmatrix}$$

6. Which receiver antenna height will give a field strength equal to the free space value on an over-land 30 km path at 600 MHz according to the ITU-R Rec. P1546. The transmitter antenna height is 300 m and the path is in rural environment.

Received field strength at 600 MHz over land paths as function of distance for different transmitter antenna heights exceeded 50% of time. Receiver antenna height 10 m (equal to representative height of ground cover)



From the graph can be seen that the free space field is 77.8 dBμV/m, and the field with 10 m receiver antenna height is 60.1 dBμV/m.

Thus a receiver antenna height giving 77.8 – 60.1 = 17.7 dB higher field strength is the largest appropriate value as that gives the free space field which cannot be exceeded.

The increase of field strength vs. receiver antenna height is given by

$$\Delta E = \; 3.2 + 6.2\log(f) \;\; \log \;\; h_{rx}/h_{cm}$$

$$\rightarrow h_{rx} = h_{cm} 10^{\frac{\Delta E}{3.2+6.2\log(f)}} = 10 \cdot 10^{\frac{17.7}{3.2+6.2\log(600)}}$$

$$= 10 \cdot 10^{\frac{17.7}{20.42}} = 73.55 \text{ m}$$