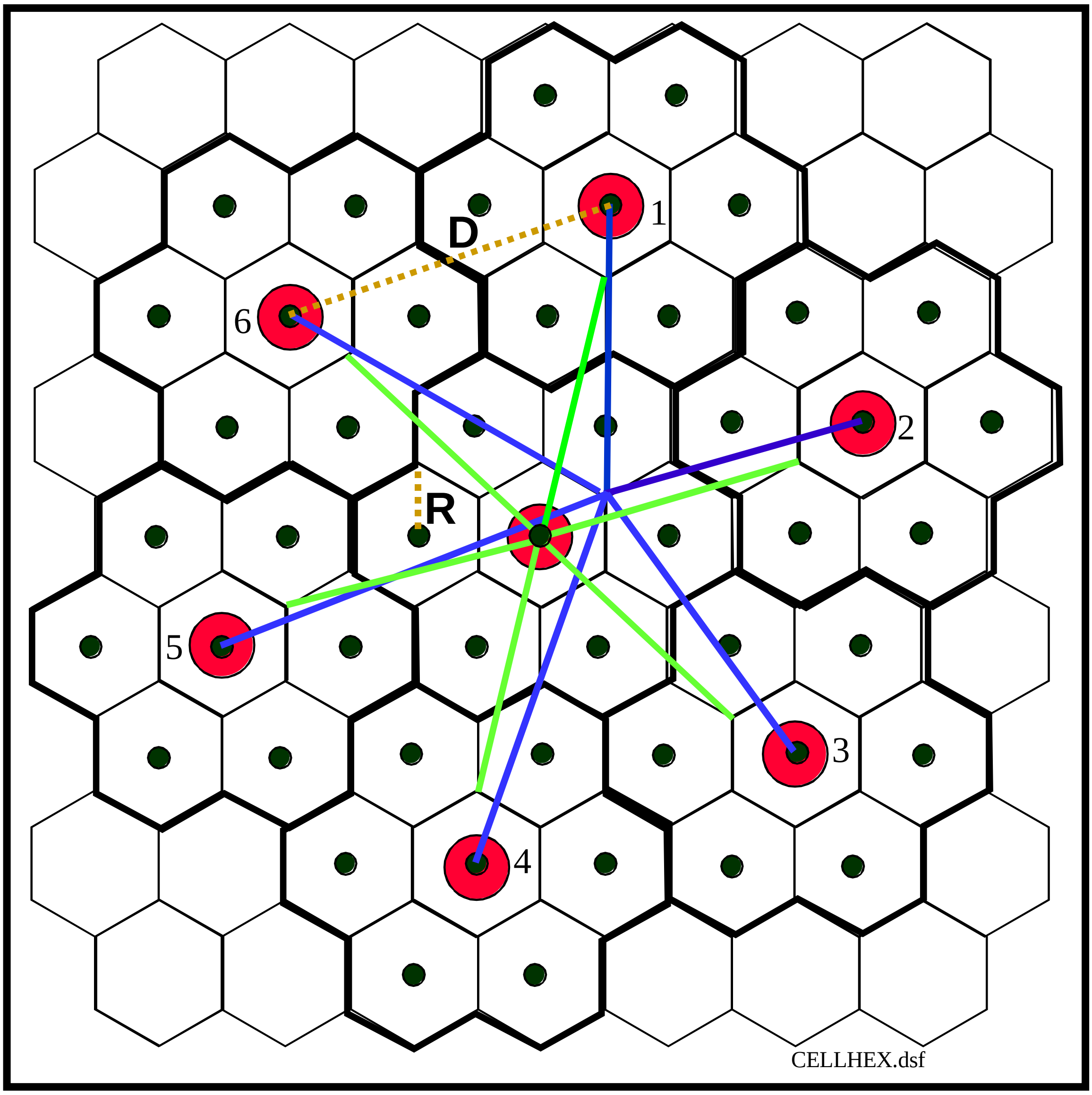
S-72.3220 RADIO COMMUNICATION SYSTEMS
EXERCISES.2007

1. Determine an approximative value for the spectrum efficiency in GSM under the following assumptions:
   - voice service with 2 % blocking probability is provided
   - the co-channel protection ratio requirement is 9 dB
   - a hexagonal cell layout with 1 km cell radius is applied
   - the average path loss exponent is $n = 4$
   - The mobile GSM1800 operator has a 2×7 MHz bandwidth



SOLUTION
In this case it feels natural to use Erlang/MHz/km^2 as the spectrum efficiency unit

We start by determining the minimum distance for spectrum reuse. In a cellular network this will result in the minimum reuse factor. We are here only considering the down-link case.

The figure shows the frequency reuse in a ideally hexagonal cell structure when the frequency reuses factor is 7. This implies that all cells in the set of 7 cells all use different frequencies. The distance between base stations using the same set of carriers is *D* and the cell radius is *R*.

A more accurate analysis would require a probability of exceeding the protection ratio in the cell taking into account ahadowing etc. Still it would be valid only for this in practice unrealilstic cell structure.

Here we take a much simpler approach by considering the worst cell location and upper bounding the interference. This is a often used approach in the literature.

Assuming that all base stations have the same transmit power, the carrier to interference ratio (CIR) in a mobile station in a cell corner is approximated by

$$CIR_1 \approx \frac{P/cR^n}{P/c\,(D-R)^n} = \frac{(D-R)^n}{R^n} = \left(\frac{D}{R}-1\right)^n$$

Taking into account the other interfering base station in the first tier around the investigated cell, and upper bounding the total CIR by setting their distance to the interfered mobile station to be the same as the distance to the nearest interfering BS we get

$$CIR_{tot} \approx \frac{P/cR^n}{6P/c\,(D-R)^n} = \frac{(D-R)^n}{6R^n} = \frac{1}{6}\left(\frac{D}{R}-1\right)^n$$

The impact of BSs more fare away is neglected. To some extent their interference is considered by the upper bounding done here.

Now the required normalized minimum reuse distance can be solved:

$$\frac{D}{R} = 1 + \left(6\cdot CIR_{tot}\right)^{1/n} = 1 + \left(6\cdot 10^{0.9}\right)^{1/4} = 3.627$$

The numerical value is obtained with the given parameters.

Next we will determine the minimum size of the reuse pattern. In a hexagonal cell structure the connection between the normalized reuse distance and the reuse pattern size M is given by

$$\frac{D}{R} = \sqrt{3M}$$

where the feasible reuse factors are obtained from the expression

$$M = (i + j)^2 - ij, \quad i,j = 0,1,2,3,...$$

$\rightarrow$ feasible M-values: 1,3,4,7,9,12,13,16,19,21,25,27,...

For the omitted M-values the resulting CIR-values in the used approach would be lower or equal to the next lower feasible value.

$$M = \frac{1}{3}\left(\frac{D}{R}\right)^2 = \frac{1}{3}\ 3.627^{\ 2} = 4.385 \rightarrow M_{\min} = 7$$

The Mmin-value is used to determine the number of carrier in each BS and finally the number of voice channels.

The frequency band available to the operator for down-link use is 7 MHz, and the carrier spacing is 0.2 MHz in GSM. Thus the number of carriers in a BS is

$$N_{carrier\_BS} = \frac{N_{tot}}{M_{\min}} = \frac{B/\Delta f}{M_{\min}} = \frac{7/0.2}{7} = 5$$

Each carrier is divided into 8 time-slots in GSM, which theoretically offers 40 traffic channels. However, perhaps 2 time-slots are reserved for signalling, which leaves us with 38 voice channels.

To obtain the cell voice traffic in Erlang, we have to use the Erlang-B model, which states that the blocking probability as a function of the traffic T and the number of voice channels N is

$$B = \frac{T^N / N!}{\sum_{n=0}^{N} T^n / n!}$$

To solve T when B and N are known is a numerical task, and we will here use tables from the literature.

For the actual parameter values B = 0.02 and N = 38 the corresponding offered traffic is 29.17 Erlang and the served traffic which is using the air interface is

$$T_{served} = (1 - B) \cdot T_{offered} = 0.98 \cdot 29.17 = 28.59 \text{ Erlang}$$

Now it is possible to calculate the spectrum efficiency.

$$\eta = \frac{T_{served,NW}}{B_{NW} A_{NW}} = \frac{N_{cell} T_{served,cell}}{B_{NW} N_{cell} A_{cell}} = \frac{T_{served,cell}}{B_{NW} A_{cell}}$$

$$= \frac{T_{served,cell}}{B_{NW}\left(6 \cdot \frac{1}{2} R \cdot \frac{R\sqrt{3}}{2}\right)} = \frac{T_{served,cell}}{B_{NW} \left(1.5\sqrt{3}R^2\right)}$$

$$= \frac{28.59}{2 \cdot 7 \cdot \sqrt{6.75} \cdot 1^2} \frac{\text{Erlang}}{\text{MHz} \cdot \text{km}^2} = \frac{28.59}{36.373} \frac{\text{Erlang}}{\text{MHz} \cdot \text{km}^2}$$

$$= 0.786 \frac{\text{Erlang}}{\text{MHz} \cdot \text{km}^2}$$

It should be noted that power control, discontinuous transmission, and sectorization can reduce the interference in the GSM-network and thus increase the spectrum efficiency.

The same effect is achieved by reducing cell size.