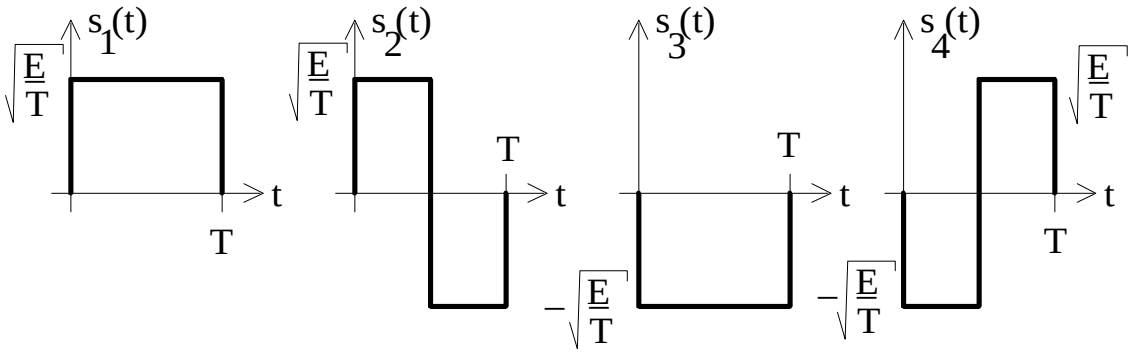
S-72.3230 Radio transmission and network access
Exercise 1 - 2

P1 In a four-symbol digital system with equally probable symbols the pulses in the figure are used in transmission over a AWGN-channel.



a) Convert the pulse waveforms into signal vectors using rectangular pulses as basis functions.
b) Sketch the optimum receiver using matched filters. Draw the filter impulse responses.
c) Using the vector representation draw the signal constellation in the r_1-r_2-coordinate system and derive the SEP-expression.
d) Derive the exact BEP-expression when Gray-coding is used

P2 In baseband binary transmission the symbol values -1 and +1 have equal probability. The decision circuit input noise is Laplace-distributed (symmetrically exponentially distributed):

$$p(n) = \frac{1}{\sigma\sqrt{2}} \exp\left(\frac{|n|\sqrt{2}}{\sigma} \right)$$

where σ is the r.m.s. noise amplitude.

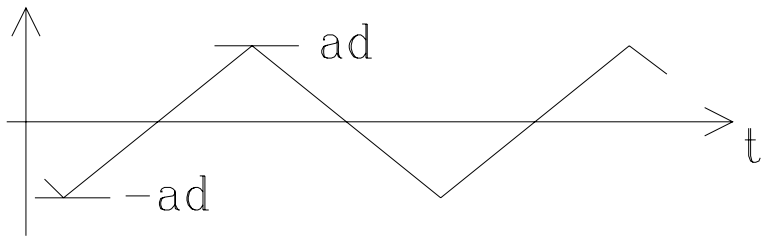
a) Derive the BEP-expression when the signal samples are $\pm s$.
b) Calculate how many dB better the signal to noise ratio must be in the Laplace-channel than in the Gaussian channel on the BEP-levels 10^{-3}, 10^{-6} and 10^{-9}.

Note! $Q(3.09) = 10^{-3}$, $Q(4.75) = 10^{-6}$, $Q(6.00) = 10^{-9}$

P3 In a bipolar binary system rectangular pulse (duration T) sequences are transmitted through an AWGN-channel with the pulse rate 1/T, and the receiver uses a matched filter. There is a timing error ΔT in decision sampling causing ISI. How large may $\Delta T/T$ be, that the degradation should not exceed 1.0 dB on the BEP-level 10^{-3}?

P4 Investigate how many dB a ML-reference receiver is degraded compared to a MAP-reference receiver as a function of the occurrence probability of a binary 0 in the AWGN-channel. Give numerical results for the bit error probabilities 10^{-3} and 10^{-6}.

P5 In a bipolar binary optimum PAM-system the decision threshold wanders as function of time as given in the figure below.



The constellation points are +d and –d.

a) Derive the expression of the average BEP.

b) Calculate the largest allowable value of the parameter a, if degradation caused by decision threshold wandering must be less than 1 dB on the BEP-level 10^{-9}.

Usable formulas:

$$\int_{-\infty}^{\infty} \mathrm{Q}(x)dx = x\,\mathrm{Q}(x) - \frac{\mathrm{e}^{-x^2/2}}{\sqrt{2\pi}} + C, \quad \mathrm{Q}(6.00)=10^{-9}$$

$$x\,\mathrm{Q}(x) \cong \frac{\mathrm{e}^{-x^2/2}}{\sqrt{2\pi}}\left[1 - \frac{1}{x^2} + \frac{3}{x^4} - \ldots\right]$$

P6 In a bipolar binary transmission system the transmit filter generates a rectangular waveform, and the receiver filter is a sub-optimal RC-low-pass filter with the time constant τ.

a) Optimize the filter in single symbol transmission and estimate the degradation compared to a matched filter.

b) In the reception of a symbol sequence inter-symbol interference (ISI) is generated. By using guard intervals of duration ΔT the impact of ISI can be reduced. How much must the data rate be reduced, if the peak ISI should be lower than 1% of the signal sample value using the time constant obtained in part a)?

c) Optimize the receiver filter taking into account with respect to peak ISI, and calculate the degradation compared to the single symbol matched filter receiver

d) Calculate how much the data rate should be reduced with the requirement in part b) with the filter time constant from part c).

P7 A bipolar binary PAM-signal is transmitted using rectangular pulses over an echo channel the impulse response of which is $h_C(t) = \delta(t) + 0{,}4\cdot\delta(t-T)$, where $T = 1/R_b$. The receiver filter is matched to the transmitted pulse assuming the channel to be ideal with the impulse response $h_C(t) = \delta(t)$, giving the bit error probability $P_b = Q\left(\sqrt{\frac{2E_x}{N_o}}\right)$, where E_X is the rectangular pulse energy and $N_O/2$ the two-sided power spectral density of the AWGN. The receiver makes symbol by symbol decisions.

a) Sketch the waveform of the decision circuit input pulse.

b) Derive the BEP-expression in the echo channel.

c) Estimate the performance degradation in dB caused by the echo.

P8. A digital transmission system in an AWGN-channel is power-limited but not bandwidth-limited. The received power is 1 pW and the one-sided noise power spectral density is 10^{-20} W/Hz. Using orthogonal signalling where the number of symbols M is a integer power of two:

a) Determine the minimum M to transmit 10 Mbit/s if the BEP-requirement is 10^{-6} .

b) Calculate how many times must the signal bandwidth be increased from the value in a) when the bit rate is doubled but the BEP requirement remains unchanged?

P9 A raised-cosine spectrum pulse has a half amplitude bandwidth of 1200 Hz and the spectrum upper limit is 1500 Hz. The pulse is used for transmission of a 4-state digital signal.

a) Determine the roll-off factor.

b) What is the bit rate of the transmitted signal?
The signal is transmitted over a multiple echo channel with the impulse response

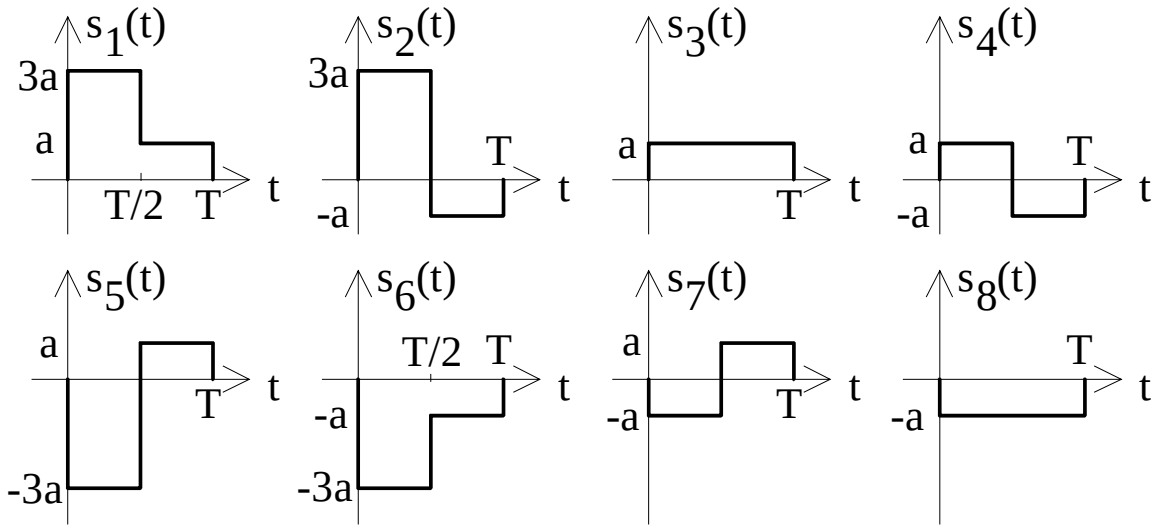
$$h(t) = -0.2\delta(t+T) + \delta(t) + 0.4\delta(t-T) - 0.3\delta(t-2T)$$

where T is the inverse of the symbol rate.

c) Which is the symbol sequence causing the maximum intersymbol interference (ISI), and what is the magnitude of the maximum ISI? Sampling is assumed to take place at same time instant as in the ideal channel.

d) What is the occurrence probability of the symbol sequence causing the maximum ISI?

Homework 1. Submit your solution at latest on 15th December, 2005



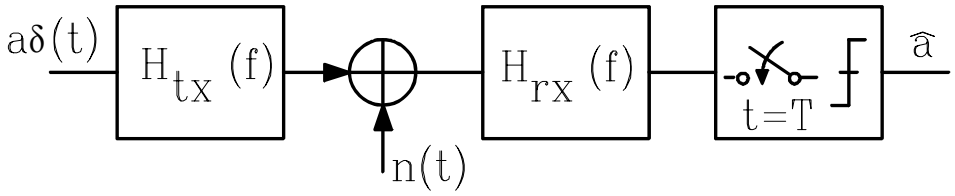
In the transmission of a symbol taking 8 different values, the pulse shapes shown in the above figure are used.

a) Select the minimum number (=2) of base function and determine and draw the constellation diagram including the receiver decision areas, when all symbol values have equal probability of occurrence.

b) Derive the expression of symbol error probability as a function of average E_b/N_o.

c) How many dB better is this transmission system better than a bipolar 8PAM-system when the required average energy/bit for achieving the symbol error probability 10^{-6} are compared?

Homework 2



$$h_{tx}(t) = \frac{1}{\tau}\exp\left(-t/\tau\right)\cdot \mathrm{u}(t)$$

The figure shows the basic block diagram of a bipolar binary transmission system in an AWGN-channel. Only a single binary symbol is transmitted.

a) Present the expression and graph of the impulse response of a matched receiver filter.

b) The receiver filter is made implementable by removing the part on the negative time axis. Derive expressions for the amplitude of the signal sample and the variance (average power) of the noise sample.

c) How large must the value of the sampling instant T be, that the degradation compared to an ideal matched filter should not exceed 1 dB? Γιωε τηε ρεσυλτ ωιτη ηελπ οφ τηε τιμε χονσταντ τ.