S-72.3235 Network Access 3 cr

Medium Access Protocols for Wireless Networks

Lecturer: Prof. Riku Jäntti
Course assistants:
B.Sc. Mirza Alam & M.Sc. Shekar Nethi

http://tll.tkk.fi/en/Studies/S-72.3235

Contents & objectives

- The course aims at providing the students the *fundamentals* packet oriented wireless communication systems.
- The focus is on the *performance analysis* of medium access control protocols MAC.
- Commonly utilized MAC protocols will be briefly reviewed and their performance discussed.

Course material

- Book:
  - R. Rom and M. Sidi, Multiple Access Protocols - Performance and analysis, Springer-Verlag, 1989
    http://www.comnet.technion.ac.il/rom/PDF/MAP.pdf
- Articles:
  - G. Bianchi, "Performance Analysis of the IEEE 802.11 Distributed Coordination Function," IEEE JOURNAL ON SELECTED AREAS IN COMMUNICATIONS, VOL. 18, NO. 3, MARCH 2000
    http://ieeexplore.ieee.org/iel5/49/18172/00840210.pdf

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Tentative schedule

	Week	Lecture	Exercises	Computer exercises	Homework
	3	L1 Introduction, stochasitc processes			
	4	L2 M/G/1 queues	E1 Queuing theory		
	4	L3 Conflic free access			
	5	L4 Dynamic conflict free access	E2 Conflict free MAC		
	5	L5 ALOHA			
	6	L6 Random access in cellular	E3 ALOHA protocols		
	6	L7 CSMA & IEEE802.11			Analytical work
	7		E4 CSMA	C1 Computer #1	
	7	L8 IEEE 802.11 and 11e			
	8		E5 Backoff & Bursting	C2 Computer #2	Simulation work
	8	L9 Collision resolution			
	9		E6 Collision resolution	C3 Computer #3	
	9	L10 IEEE 802.15.4			Homework deadline
	10				

Homework

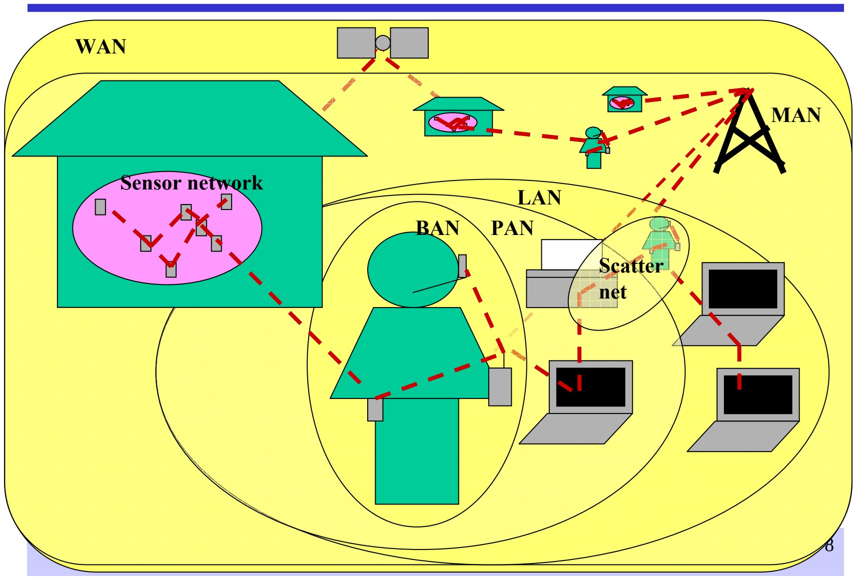
- There are two homework problems
  - One paper and pencil type of problem
  - One computer simulation problem
- Homework problems are not mandatory, but highly recommended.
- They can give up to 10 extra points to the exam

Lecture 1.

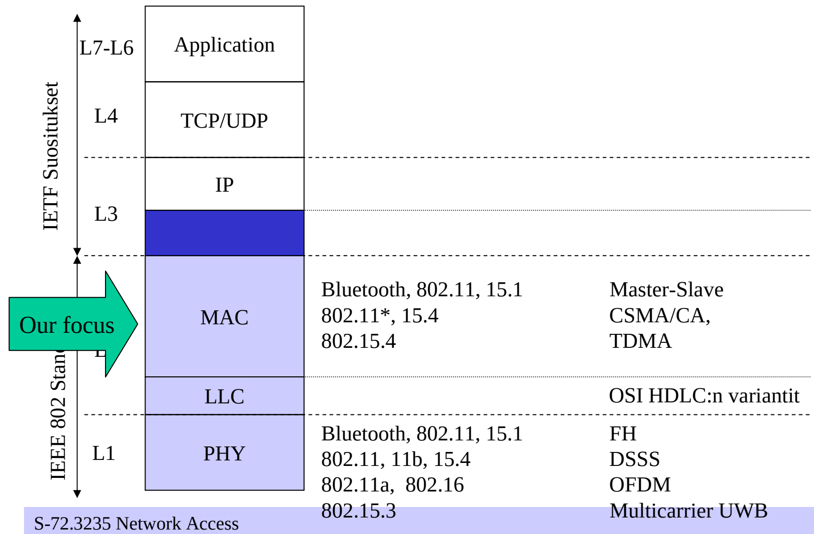
- Introduction to medium access control
- Recapitulation of stochastic processes and queuing theory
- The Poisson process
- Traffic models
  - Circuit switched
  - Packet switched

Medium access control

Wireless data networks

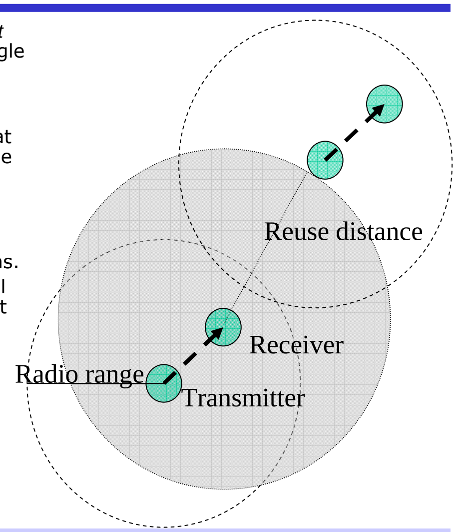


Protocol architecture



Medium access control

- Wireless transmission is *broadcast* in nature. That is more than a single receiver can potentially receive every transmitted message.
- Transmissions over a broadcast channel interfere, in the sense that one transmission coinciding in time with another may cause none of them to be received.
- The success of a transmission between a pair of nodes is not independent of other transmissions.
- To make a transmission successful interference must be avoided or at least controlled.
- The channel is a shared resource whose allocation is critical for proper operation of the network.
- The schemes used for channel access are known in literature as *Multiple Access Protocols (MAC)*.



Medium access control protocols

- The task of the Medium Access Control (MAC) protocol is to divide the resources between the radio links such that
  - Interference is avoided or kept at controlled level
  - Utilization of the radio resources is maximized
  - Quality of service QoS differentiation among the flow classes is achieved
  - Fairness inside a QoS class is maintained

Medium access control protocols

- The operation of the MAC protocol can be
  - *Centralized* such that single entity controls the resource division among the radio links leading to *conflict free access*
  - *Decentralized* such that each link makes transmission decisions independently leading to *contention based access*
- Contention schemes differ in principle from conflict-free schemes
  - A transmitting user is not guaranteed to be successful.
  - The protocol must prescribe a way to resolve conflicts once they occur so that all messages are eventually transmitted successfully.

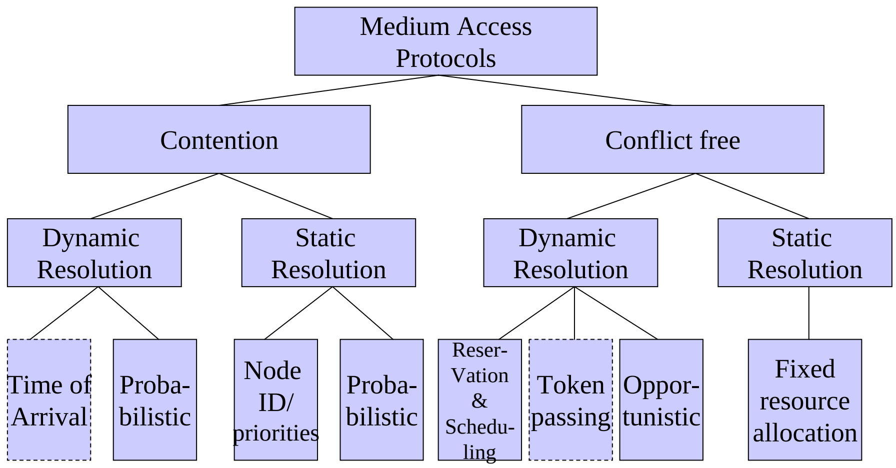
Conflict free access

- In conflict free protocols, the resource allocation can be
  - *Static* - not dependent on the traffic or channel conditions (TDMA, FDMA, F/TDMA, OFDMA, CDMA, OFCDMA,…)
  - *Dynamic* – based on demand and/or channel conditions
    - Token passing
    - Channel reservation (satellite systems, IEEE802.15.4,…)
    - Dynamic scheduling (UMTS R99, WiMAX,…)
    - Channel dependent "opportunistic" scheduling (e.g. CDMA2000 1xEV-DO/DV, HSDPA, LTE)

Contention based access

- In contention based protocols, the conflicts caused by colliding packets (interference) must be resolved. Conflict resolution methods can be divided into
  - *Static* - the actual behavior is not influenced by the dynamics of the system. The transmission schedule for the interfering users can be
    - Fixed: based on node IDs or priorities
    - Probabilistic: schedule is chosen from a fixed distribution (p-persistent CSMA)
  - *Dynamic* – the actual behavior of the system depends system dynamics.
    - Transmission schedule could be determined by the time of the arrival
    - Probabilistic: Transmission schedule depends on the number of colliding packets (BEB in IEEE802.3 and IEEE802.11)

Classification of MAC protocols



Medium access control protocols

- The issues affecting the performance of the channel access
  - **Connectivity**
    - Can all the nodes hear each other or are there hidden terminals?
    - What is the network topology? Single hop, multi-hop (mesh/ad hoc)
  - **Channel type**
    - What is the required Signal-to-Interference ratio for correct reception? Is there possibility for capture in case of collisions?
    - Do protocol messages get lost due to fading?
  - **Synchronism**
    - Is the network synchronized, i.e. slotted or can transmissions start and end at arbitrary time instances.
  - **Feedback information**
    - Can collisions be detected? Can the colliding nodes be identified?
    - How much information can be shared among the nodes?
    - Is correct reception acknowledged by the receiver?
  - **Traffic**
    - Is the message size fixed or does it vary? Is packets generated randomly or with steady rate? Can transmission buffers assumed to be saturated (TCP tends to saturate buffers) or are they likely to be empty at times?
  - **User population**
    - Is the number of users fixed or random? Can it be known by the system?
  - **Buffering capability**
    - How many packets can the nodes buffer? Will packets be lost due to buffer overflow?

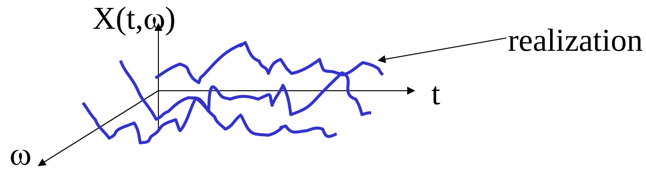
Other relevant courses

- Simulation tools
  - S-38.3148 Network simulation 5 cr
- Mathematical tools
  - S-38.3143 Queue Theory 5 cr
- Traffic modeling and performance analysis
  - S-38.3141 Teletraffic theory 5 cr
- Conflict free access
  - S-72.3260 Radio Resource Management Methods 3 cr

Stochastic processes and queuing theory

Stochastic processes

- A stochastic process is a set of indexed random variables
  $\{X(t,\omega), t \in T, \omega \in \Omega\}$
- The index set $t \in T$ is called *parameter space* of the process
- Each individual random variable is a mapping from the sample space to set of real (or complex) numbers.
- A parameterized set X(t) corresponding to a sample ω is called *realization/trajectory/path* of the process.



Stochastic processes

- *State-space* of the process is a set of values that $X(t)$ may obtain.
- State space is *discrete*, if the number of states is finite or numerable. The corresponding stochastic process is called discrete time process/sequence/chain
  $\{X(t_k)\}, \quad t \in \{t_0, t_1, t_2, \ldots\}$
- State space is *continuous*, if the number of states is innumerable. The corresponding stochastic process is called continuous time process/sequence/chain
  $X(t), \quad t \in (0, \infty]$

Markov-processes

- *Markov property*: The state of the process at time t_{n+1} depends only on its state at the previous time instance t_n

$$\Pr\left\{X\left(t_{n+1}\right)=x_{n+1}\middle|X\left(t_n\right)=x_n, X\left(t_{n-1}\right)=x_{n-1},\ldots,X\left(t_1\right)=x_1\right\}$$

$$=\Pr\left\{X\left(t_{n+1}\right)=x_{n+1}\middle|X\left(t_n\right)=x_n\right\}$$

- *Markov-Process:* The process stays in a state x_n random time interval after which it changes it state randomly according to certain state transition probabilities.
- *Markov-Process* has the *Markov property*, if the state time distribution of the process is memoryless. That is, transition is allowed to take place every time instant.
  - *Continuous time Markov-Process*: State time distribution is exponential
  - *Discrete time Markov-Process*: State time distribution is geometric

Other related processes

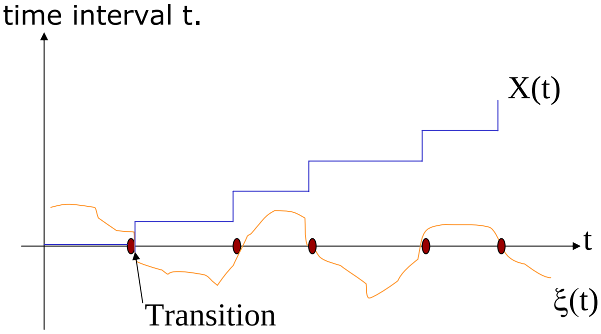
- *Semi-Markov process*: State time distribution can be arbitrary. At the instance of state transitions, the process behaves as Markov chain.
  - *Imbedded Markov-chain*, Semi-Markov process observed at state transition times.
- *Random walk/Process with independent increments*: Location of a particle moving in space: Next position=Previous position + random variable

  $S_n = S_{n-1} + X_n, \quad S_0 = 0$

  where $X_1, X_2, \ldots$ is a sequence of independent identically distributed random variables, n is the number of state transitions

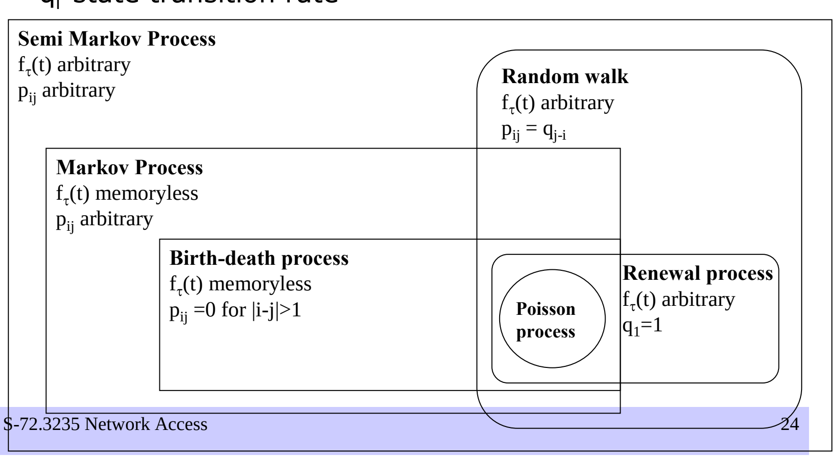
Other related processes

- *Renewall/recurrent process*: Related to the random walk, but instead of position, we interested in counting the number of transitions X(t) that take place as a function of time t. I.e. X(t) is a random variable that states the number of transitions that have taken place in time interval t.



Classification of stochastic processes

- $f_\tau(t)$ probability density function of time spent in a state
- p_{ij} transition probability
- q_i state transition rate



Discrete-time Markov chains

- **Definition:** *The sequence of random variables X_1,X_2,… forms a discrete-time Markov chain if for all n (n=1,2,…) and all possible values of the random variables we have that*

$$\Pr\left\{X_{n+1}=x_{n+1}\middle|X_n=x_n,X_{n-1}=x_{n-1},...,X_1=x_1\right\}=\Pr\left\{X_{n+1}=x_{n+1}\middle|X_n=x_n\right\}$$

The state variable x_n=i implies that the state of the system was E_i at time slot *n*.

Discrete-time Markov chains

- Markov chain is said to be *homogenic* (stationary), if state transition probabilities are independent of time index.

$$\Pr\left\{X_{n+1}=j\middle|X_n=i\right\}=\Pr\left\{X_{m+1}=j\middle|X_m=i\right\}=p_{ij}\ \forall m,n$$

- For homogenic Markov chain, the state transition probability from state E_i: X_n=j to state E_j: X_{n+1}=i can be defined as:

$$p_{ij}\triangleq\Pr\left\{X_n=j\middle|X_{n-1}=i\right\}$$

Discrete-time Markov chains

- Assume that the state space is independent of the time index n. Probability that the system is in state E_j at time instant n+m (X_{m+n}=j) conditioned that it was in state E_i at time m (X_n=i) is

$$p_{ij}^{n} \triangleq \Pr\left\{X_{m+n} = j \middle| X_m = i\right\} = \sum_{k=0}^{\infty} p_{ik}^{n} p_{kj}^{m}$$ **Chapman-Kolmogorov equation**

- If there exists an integer m_0 such that $p_{ij}^{m_0}$>0, the Markov chain is said to be *irreducible*.
- Let *A* denote the set of all states in a Markov chain.
  - A subset $A_1 \subset A$ is said to be *closed* if no one-step transition is possible from any single state in A_i to its complement $A_i^C = A \setminus A_i$.
  - If A_1 consist of a single state E_j, the state is called *absorbing state*. If A_1 is closed and does not contain any proper closed subsets, then A_1 forms *irreducible sub-Markov chain*.

Discrete-time Markov chains

- The chain is irreducible if

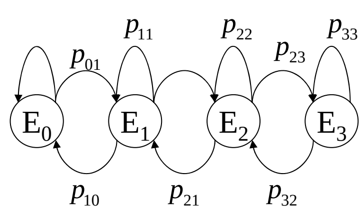
$$p_{ij} > 0 \forall i, j$$

- E 3 is absorbing state if

$$p_{ij} > 0, \quad i \neq 3$$
$$p_{33} = 1$$
$$p_{32} = 0$$

- E_2 and E_3 form an irreducible sub-Markov chain if

$$p_{ij} > 0, \quad (i, j) \neq (2,1)$$
$$p_{21} = 0$$



Discrete-time Markov chains

- Probability that the chain returns to state E_i:

$$f_i^{(n)} = \Pr\{X_{n+m} = i \mid X_m = i\} = p_{ii}^{(n)} \qquad \text{with } n \text{ steps}$$

$$f_i = \sum_{n=1}^{\infty} f_i^{(n)} \qquad \text{at all}$$

- If $f_i = 1$ Markov chain is called *recurrent*; otherwise it is called *transient*.
- If the initial state is revisited in regular time intervals, the chain is said to be periodic; otherwise it is called aperiodic (non-periodic).
- *Mean recurrence time* of state E_i

$$M_i = \sum_{n=1}^{\infty} n f_i^{(n)} \quad \begin{matrix} = \infty \text{ Recurrent null} \\ < \infty \text{ Recurrent nonnull} \end{matrix}$$

Discrete-time Markov chains

- Markov chain is Ergodic stochastic process if it is aperiodic, recurrent $f_i = 1$ and recurrent nonnull $M_i < \infty$
- Probability that the system is in state E_i at time instant n

$\pi_i^{(n)} = \Pr\{X_n = i\}$ **State probability**

Theorem. In irreducible, aperiodic, homogeneous Markov chain, the limit value

$$\pi_i = \lim_{n\to\infty} \pi_i^{(n)}$$

fulfills either

a) $f_i < 1 \text{ tai } M_i = \infty \quad \Rightarrow \pi_i = 0$

b) $f_i = 1, M_i < \infty$

$$\pi_i = \frac{1}{M_i} = \sum_j \pi_j p_{ji} \qquad \sum_i \pi_i = 1$$

for all i (for all states E_i)

Discrete-time Markov chains

- If the number of states is finite, the state probabilities can be solved from the following set of linear equations

$$\pi_i = \frac{1}{M_i} = \sum_j \pi_j p_{ji}$$

$$\sum_i \pi_i = 1$$

- Define

$\boldsymbol{\pi} = \begin{pmatrix} \pi_1 & \dots & \pi_m \end{pmatrix}$ Row vector containing state probabilities

$\mathbf{P} = \left(p_{ij}\right)$ Non-negative state transition probability

$\boldsymbol{\pi} = \boldsymbol{\pi}\mathbf{P}$ Equation for left eigenvalues of P

State probabilities are defined by the (left) Perron-eigenvector of the state transition matrix P that fulfills

$$\sum_i \pi_i = 1$$

Discrete-time Markov chains

- The state transition matrix P has the following properties
  - P is a nonnegative matrix P$\geq$0
  - The largest eigenvalue in modulus ρ(P) is equal to 1:

    $\lambda_i \mathbf{x}_i = \mathbf{x}_i \mathbf{P}$

    $\rho\left(\mathbf{P}\right) = \max_i \left\{\left|\lambda_i\right|\right\} = 1$

    $\exists i, \quad \lambda_i = 1$
  - The row and column sums of P are equal to 1

    $\sum_j p_{ji} = 1, \quad \sum_i p_{ji} = 1$
  - If the chain is irreducible, then also P is an irreducible matrix, and the Perron eigenvector can be taken to be strictly positive

$$\boldsymbol{\pi} = \boldsymbol{\pi}\mathbf{P} \Rightarrow \boldsymbol{\pi} > 0$$

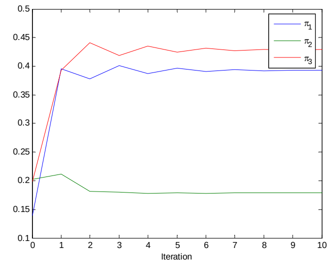
Discrete-time Markov chains

- The state probability can be solved simply by using the power method for solving the Perron eigenvector

$$\boldsymbol{\pi}(n+1) = \frac{\boldsymbol{\pi}(n)\mathbf{P}}{\boldsymbol{\pi}(n)\mathbf{P}\mathbf{1}^T}$$

$$\boldsymbol{\pi}(0) > 0$$

$$\mathbf{1} = (1 \quad 1 \quad \ldots \quad 1)$$



- Example

```
P=[.1 .1 .8;.3 .3 .4;.7 .2 .1];
Pi=rand(1,3);
I=ones(size(Pi));
for k=1:10
    Pi(k+1,:)=Pi(k,:)*P/(Pi(k,:)*P*I');
end;
plot(0:10,Pi)
```

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Some useful tools

Characteristic function and moment generating function

Probability generating function

Characteristic / moment generating function

- Moment generating function = Fourier-transformation of the probability density function

$$\psi(\omega) = \mathrm{E}\left\{e^{i\omega X}\right\} = \int_{-\infty}^{\infty} e^{i\omega x} p(x)dx, \quad i = \sqrt{-1}$$

- Inverse Fourier-transform

$$p(x) = \frac{1}{2\pi}\int_{-\infty}^{\infty} \psi(i\omega) e^{-i\omega x} d\omega$$

- k^{th} derivative of the characteristic function

$$\frac{d^k}{d\omega^k}\psi(\omega) = \int_{-\infty}^{\infty} (ix)^k e^{i\omega x} p(x)dx$$

- k^{th} moment

$$\overline{X^k} = \mathrm{E}\left\{X^k\right\} = \lim_{\omega\to 0}(-i)^k \frac{d^k}{d\omega^k}\psi(i\omega)$$

Laplace transform

- Consider random variable X with support [0,∞]. That is, X≥0
- The pdf of the variable is p(x)
- Laplace-transform of the pdf

$$P^*(s) = \mathrm{E}\left\{e^{-sX}\right\} = \int_{-\infty}^{\infty} e^{-sx} p(x)dx$$

- Characteristic function

$$\psi(s) = P^*(i\omega)$$

- k^{th} moment

$$\overline{X^k} = E\left\{X^k\right\} = \lim_{s\to 0}(-1)^k \frac{d}{ds^k} P^*(s)$$

Probability generating function

- Discrete random variable

$$\Pr\{X=k\}=p_k \quad k=0,1,2,3,\dots$$

- Probability generating function = Z-transform of the probability

$$G(z)=\mathrm{E}\{z^X\}=\sum_{k=0}^{\infty} z^k p_k, \quad z\in\mathbb{C}$$

- Properties of G(z)

$$G(1)=\sum_{k=0}^{\infty} p_k = 1$$

$$|G(z)|<\sum_{k=0}^{\infty}|z^k|\,p_k<\sum_{k=0}^{\infty} p_k=1, \quad |z|\le 1$$

Probability generating function

- First derivative yields expected value:

$$\mathrm{E}\{X\}=\sum_{k=0}^{\infty} kp_k=\frac{d}{dz}G(z)\Big|_{z=1}$$

- 2nd derivative yields 2nd moment

$$\frac{d^2}{dz^2}G(z)\Big|_{z=1}=\sum_{k=1}^{\infty} k(k-1)z^{k-2}p_k\Big|_{z=1}=\sum_{k=1}^{\infty}k^2p_k-\sum_{k=1}^{\infty}kp_k=\mathrm{E}\{X^2\}-\mathrm{E}\{X\}$$

$$\mathrm{E}\{X^2\}=G''(1)-G'(1)$$

$$\mathrm{var}\,X=G''(1)-G'(1)-[G'(1)]^2$$

Probability generating function

- Let $\{X_i\}$ be a set of independent identically distributed discrete random variables. $\Pr\{X_i=k\}=p_k$ for all i.

$$G_{X_i}(z) = \mathrm{E}\left\{z^{X_i}\right\} = \sum_{k=0}^{\infty} z^k p_k \triangleq G_X(z) \quad \forall i$$

- Let N be a discrete random variable independent of $\{X_i\}$. $\Pr\{N=k\}=q_k$

$$G_N(z) = \mathrm{E}\left\{z^N\right\} = \sum_{k=0}^{\infty} z^k q_k$$

Probability generating function

- Consider a random sum

$$S_N = \sum_{i=1}^{N} X_i$$

- Probability generating function of S_N:

$$\mathrm{E}\left\{z^{S_N} \middle| N\right\} = \mathrm{E}\left\{ z^{\sum_{i=1}^{N} X_i} \middle| N \right\} = \prod_{i=1}^{N} \mathrm{E}\left\{z^{X_i}\right\} = \left[G_X(z)\right]^N$$

$$G_{S_N}(z) = \mathrm{E}\left\{z^{S_N}\right\} = \mathrm{E}\left\{\left[G_X(z)\right]^N\right\} = \sum_{k=0}^{\infty}\left[G_X(z)\right]^k q_k = G_N\left(G_X(z)\right)$$

- Wald's Lemma $\mathrm{E}\{S_N\}=\mathrm{E}\{N\}\mathrm{E}\{X_i\}$

$$G_{S_N}'(z) = G_N'\left(G_X(z)\right) G_X'(z)$$

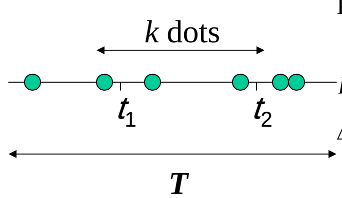
$$\mathrm{E}\left\{S_N\right\} = G_{S_N}'(1) = G_N'\left(G_X(1)\right) G_X'(1) = G_N'(1) G_X'(1) = \mathrm{E}\left\{N\right\}\mathrm{E}\left\{X_i\right\}$$

Poisson process

Poisson Process

- Let us randomly place n dots (uniform distribution) on the interval $(0,T)$. What is the probability that there are k dots on the interval (t_1,t_2)?

$$\Pr\{k \text{ dots on the interval } (t_1,t_2)\} = \binom{n}{k} p^k (1-p)^{n-k}$$



$$p = \frac{\Delta t}{T}$$

$$\Delta t = t_2 - t_1$$

Consider a limit

$$n \to \infty, \quad T \to \infty \quad np = n\frac{\Delta t}{T} \to \lambda \Delta t$$

Poisson Theorem

$$\boxed{\Pr\{k \text{ dots on the interval } (t_1,t_2)\} = \binom{n}{k} p^k (1-p)^{n-k} \to e^{-\lambda \Delta t}\frac{(\lambda \Delta t)^k}{k!}}$$

Poisson Process

- If Δt is small, then

  $$\Pr\{\text{Only one dot on the interval } \Delta t\} = \lambda \Delta t e^{-\lambda \Delta t} \approx \lambda \Delta t$$

- That is,

  $$\lim_{\Delta t \to 0} \frac{\Pr\{\text{Only one dot on the interval } \Delta t\}}{\Delta t} = \lambda$$

- Events that there are k dots on interval (t_1,t_2) and (t_3,t_4), t_1<t_2<t_3<t_4 are independent of each other

  $$\Pr\{k_1 \text{ dot on } (t_1, t_2) \text{ and } k_2 \text{ dots on } (t_3, t_4)\}$$

  $$= \Pr\{k_1 \text{ dots on } (t_1, t_2)\} \Pr\{k_2 \text{ dots on } (t_3, t_4)\}$$

- Poisson Process:

  $$\Pr\{k \text{ dots on the interval } \Delta t\} = e^{-\lambda \Delta t} \frac{(\lambda \Delta t)^k}{k!}$$

Poisson process

- Poisson process can be utilized to model arrivals from *independent* sources
  - Call arrivals in voice telephony
  - Packet session arrivals in data networks
  - Handovers from neighboring cells (approximately)
- Poisson model, in general, is not valid for modeling arrivals from single or correlated sources, such as
  - Packets generated by single computer during a packet session

Circuit switched voice traffic

Call arrival process

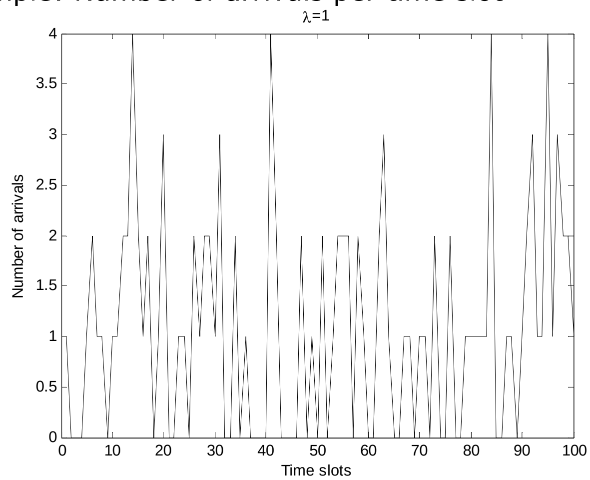
- Call arrival process:
  - $T \to \infty$ denotes the time interval from the big bang till the end of time
  - n denotes the total number of calls that arrive on time interval T.
- Probability that k calls arrive during the time interval of length Δt

$$\Pr\{k \text{ calls arrive during } \Delta t\} = e^{-\lambda \Delta t} \frac{(\lambda \Delta t)^k}{k!} \qquad \text{(Poisson Process)}$$

$$\Pr\{\text{Only one call arrives during } \Delta t\} \approx \lambda \Delta t \quad \text{when } \Delta t \text{ is very small}$$

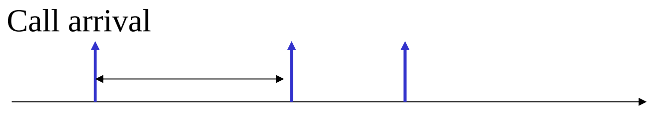
Call arrival process

- Example: Number of arrivals per time slot



Call arrival process

- Inter-arrival time T of calls



$$\Pr\{T \le t\} = \Pr\{\text{At least one call arrives during } t\}$$

$$= 1 - \Pr\{\text{No calls arrive during } t\} = 1 - e^{-\lambda t}$$

$$p(t) = \frac{d}{dt}\Pr\{T \le t\} = \lambda e^{-\lambda t}$$

- Expected inter-arrival time

$$E\{T\} = \int_0^\infty \lambda t e^{-\lambda t} dt = \frac{1}{\lambda}\int_0^\infty x e^{-x} dx = \frac{1}{\lambda}$$

Call arrival process

- Let T denote the interarrival time
- Probability that next call arrives in time t

$F(t) = \Pr\{T \le t\} = 1 - e^{-\lambda t}$

- Given that new call has not arrived in time t, the probability that it will arrive in time t+Δt is given by

$$\Pr\{T \le t+\Delta t \mid T > t\} = \frac{\Pr\{t < T \le t+\Delta t\}}{\Pr\{T > t\}}$$

$$= \frac{\Pr\{T \le t+\Delta t\} - \Pr\{T \le t\}}{1 - \Pr\{T \le t\}} = \frac{1 - e^{-\lambda(t+\Delta t)} - \left(1 - e^{-\lambda t}\right)}{1 - \left(1 - e^{-\lambda t}\right)}$$

$= \frac{e^{-\lambda t} - e^{-\lambda(t+\Delta t)}}{e^{-\lambda t}} = 1 - e^{-\lambda \Delta t} = F(\Delta t)$ which is independent of t!

Hence, the arrival process is *memoryless*.

Call departures

- Also call departures can be modeled with Poisson process
- Mean call holding time τ follows then exponential distribution

$$p(\tau) = \mu e^{-\mu\tau}$$

- Mean call holding time

$$E\{\tau\} = \int_0^\infty \tau \mu e^{-\mu\tau} d\tau = \frac{1}{\mu}$$

- Probability that a call in progress ends during a time interval Δt:

$$\Pr\{\text{Call ends during } \Delta t\} = \mu \Delta t e^{-\mu\Delta t} \approx \mu \Delta t$$

Call arrivals and departures

- Call arrivals per time unit

$$\lim_{\Delta t\to 0}\frac{\Pr\{\text{Only one call arrive during } \Delta t\}}{\Delta t}=\lambda$$

- Call departures per time unit

$$\lim_{\Delta t\to 0}\frac{\Pr\{\text{Only one call departures during } \Delta t\}}{\Delta t}=\mu$$

- Call arrivals and departures constitute a continuous time birth-death process.

Call arrivals and departures

- Steady state probabilities of continuous Markov chain could be derived using discrete time Markov chain where the time interval Δt is small
- State probability of a birth dead call arrival-departure process

$$\pi_i=(1-\lambda-\mu)\pi_i+\lambda\pi_{i-1}-\mu\pi_{i+1}$$

Probability that the system is in State i+1 and a call departs

Probability that system is in state i-1 and one call arrives

Probability that the system is in state i and no call arrives or departs

$$\sum_i \pi_i=1$$

Probability that the system is one of its possible states

Call arrivals and departures

- The steady state probabilities are the same as in the discrete time birth-death process
  - State probabilities

$$\pi_k = \left(\frac{\lambda}{\mu}\right)^k \pi_0$$

$$\pi_0 = \left(1 - \frac{\lambda}{\mu}\right)$$

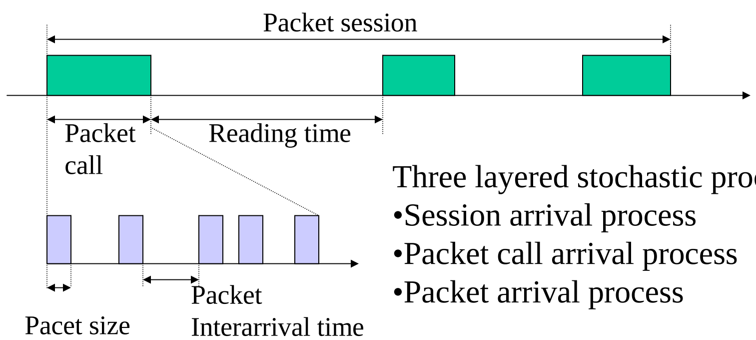
  - Number of calls in progress

$$N = \sum_{k=0}^{\infty} k\pi_k = \frac{\frac{\lambda}{\mu}}{1 - \frac{\lambda}{\mu}} = \frac{\lambda}{\mu - \lambda}$$

Packet switched traffic

Packet switched traffic

- Non-real-time services
  - SMS, WWW, FTP, Email,...
- Traffic model



Three layered stochastic processes
•Session arrival process
•Packet call arrival process
•Packet arrival process

Packet switched traffic

- Commonly utilized models
  - Packet session interarrival: Exponential/geometric
  - Number of arriving packet sessions: Poisson
  - Reading time distribution: Exponential/geometric
  - Packet interarrival time: Exponential/geometric or log-normal
  - Packet length: Fixed
  - Length of the packet session: Parento distributed

Characteristics of packet traffic

- Empirical studies of different data networks indicate that Packet traffic exhibits extended temporal correlations, i.e., **long-range dependence (LRD)**, and hence when viewed within some range of (sufficiently large) time scales, the traffic appears to be **fractal-like** or **self-similar**, in the sense that a segment of the traffic measured at some time scale looks or behaves just like an appropriately scaled version of the traffic measured over a different time scale

Empirical results:
Ethernet (LAN):
W. E. Leland, M. S. Taqqu, W. Willinger, and D. V. Wilson, "On the Self-Similar Nature of Ethernet Traffic (Extended Version), IEEE/ACM Transactions on Networking, Volume: 2, Issue: 1, February 1994.

Internet traffic:
V. Paxson and S. Floyd, "Wide Area Traffic: The Failure of Poisson Modeling," IEEE/ACM Transactions on Networking, Volume: 3, Issue: 3, June 1995, Pages:226 – 244

Recent overview:
A. Erramilli, M. Roughan, D. Veitch, D. and W. Willinger, "Self-similar traffic and network dynamics," *Proceedings of the IEEE* , Volume: 90 , Issue: 5 , May 2002 Pages:800 - 819

Characteristics of packet traffic

- The self-similar behavior of data traffic is caused by the high-variability of the of individual sessions that make up the aggregate traffic.
- Self-similar scaling behavior over sufficiently large time scales can be observed, if the durations (in time) or sizes (in bytes) of the individual sessions or IP flows that generate the aggregate traffic have a *heavy-tailed distribution with infinite variance* (that is, range from extremely short/small to extremely long/large with non-negligible probability).
- The LRD nature of network traffic is mainly caused by *user/application characteristics* such as Poisson arrivals of sessions and heavy-tailed distributions for the session durations/sizes, and has little to do with the network, i.e., with the protocol-specific mechanisms that determine the actual flow of packets as they traverse the Internet.

=> Self-similarity and LRD cannot be avoided by changing the protocols.

Characteristics of packet traffic

- Let X(t) denote stationary traffic rate process
  - Mean, variance, and auto-covariance:

$$\mu = E\{X(t)\}$$

$$v = E\left\{(X(t)-\mu)^2\right\}$$

$$c_X(k) = E\left\{(X(t+k)-\mu)(X(t)-\mu)\right\}$$

  - The process is said to exhibit (asymptotically) long-range dependencies (LRD) if

$$c_X(k) \sim c_\gamma k^{-\beta}, \quad c_\gamma > 0,\ 0 < \beta < 1$$

otherwise if

$$c_X(k) \sim \phi^k, \quad |\phi| < 1$$

it is said to exhibit short-range dependencies (SDR).

~ denotes 'asymptotically'
That is, the ratio of the two sides tends to one in the limit of large k

Characteristics of packet traffic

- Consider a mean estimator

$$\bar{X}_n = \frac{1}{n}\sum_{k=1}^{n} X(k)$$

- This process has stationary increments, and

$$\operatorname{var}\{\bar{X}_n\} \sim \frac{2c_\gamma}{(1-\beta)(2-\beta)} n^{-\beta}$$

$$\frac{\operatorname{var}\{\bar{X}_n\}}{\operatorname{var}\{\bar{X}_m\}} \sim \left(\frac{n}{m}\right)^{\beta}$$

if held exactly, implies that the process is self-similar.

Characteristics of packet traffic

- Let Y(n) denote the number of bytes arrived during the interval (0,n)

$$Y(n)=\sum_{k=1}^{n}X(k)$$

- It follows that

$$Y(t)\overset{d}{\sim}a^{H}Y(at)$$

have the same distribution.
H denotes the Hurst parameter.

Characteristics of packet traffic

- The unique continuous Gaussian process with LRD characteristics and with stationary increments is the well-known *fractional Brownian motion* (fBm)
- Data traffic can be modeled with fBm H∈(1/2,1)
- Traffic model

$$Y(t)=\mu_Y t+\sigma_Y W(t)$$

W(t) is fBm process with zero mean and Hurst parameter H.

- **Multiplexing independent flows have the effect of reducing σ_Y which in turn reduces the temporal burstiness.**

I. Norros, "On the Use of Fractional Brownian Motion in the Theory of Connectionless Networks," IEEE Journal on Selected Areas in Communications, Volume: 13, Issue: 6, August 1995, Pages: 953 – 962.

Characteristics of packet traffic

- Consider an on-off traffic source model, where the on and off times follow different heavy-tailed probability density with infinite variance ($1<\alpha<2$)

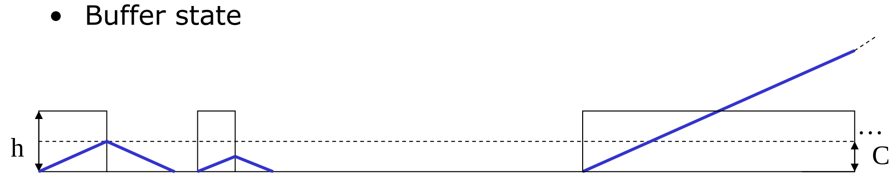
$$\Pr\{T_{on} > x\} \sim c_\alpha x^{-\alpha} \quad \text{Pareto distribution}$$

$$E\{T_{on}\} = \frac{1}{\mu} \quad E\{T_{off}\} = \frac{1}{\upsilon}$$

- During the on-period packets are emitted with rate h
- Average rate of the on-off source $\lambda=h\nu/(\nu+\mu)$.
- Aggregating many such on–off sources results in aggregate link traffic that exhibits self-similar scaling behavior.
- Even if $\lambda < C$, where C denotes the link capacity, the queue can blow up.

Characteristics of packet traffic

- Buffer state



  - Average rate of the on-off source $\lambda=h\nu/(\nu+\mu)<C$
  - However, due to the large variance, there is no upper bound for how long the on-period takes.

Characteristics of packet traffic

- Self-similarity concerns large time scales and leaves the small scale behavior, such as strong short term correlation of the arrivals, unspecified.
- TCP uses feedback to react to network congestion.
- Because TCP feedback modifies the self-similarity in the offered traffic, “open loop” modeling approaches will not accurately predict TCP performance.
- Chaotic maps have been suggest to model the complex the source behavior (A. Erramilli, *et. al.* 2002).
- TCP tends to saturate the bottleneck links. Since wireless links usually are the bottlenecks, *saturated traffic conditions* where each transmitter is always assumed to have packet ready for transmission is commonly utilized to analyze the performance of MAC schemes in wireless systems.