
S-72.3235 Network Access 3 cr

Medium Access Protocols for Wireless Networks

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<http://tll.tkk.fi/en/Studies/S-72.3235>

Contents & objectives

- The course aims at providing the students the *fundamentals* packet oriented wireless communication systems.
- The focus is on the *performance analysis* of **medium access control protocols** MAC.
- Commonly utilized MAC protocols will be briefly reviewed and their performance discussed.

Course material

- Book:
 - R. Rom and M. Sidi, Multiple Access Protocols - Performance and analysis, Springer-Verlag, 1989
<http://www.comnet.technion.ac.il/rom/PDF/MAP.pdf>
 - Articles:
 - G. Bianchi, "Performance Analysis of the IEEE 802.11 Distributed Coordination Function," IEEE JOURNAL ON SELECTED AREAS IN COMMUNICATIONS, VOL. 18, NO. 3, MARCH 2000
<http://ieeexplore.ieee.org/iel5/49/18172/00840210.pdf>
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Tentative schedule

| | Week | Lecture | Exercises | Computer exercises | Homework |
|--|------|---|-------------------------|--------------------|-------------------|
| | 3 | L1 Introduction, stochasitic processes | | | |
| | 4 | L2 Traffic models M/G/1 queues | E1 Queuing theory | | |
| | 4 | L3 Conflifc free access | | | |
| | 5 | L4 Dynamic conflict free access & IEEE 802.16 | E2 Conflict free MAC | | |
| | 5 | L5 ALOHA | | | |
| | 6 | L6 Random access in cellular systems | E3 ALOHA protocols | | |
| | 6 | L7 CSMA & IEEE802.11 | | | Analytical work |
| | 7 | | E4 CSMA | C1 Computer #1 | |
| | 7 | L8 IEEE 802.11 and 11e | | | |
| | 8 | | E5 Backoff & Bursting | C2 Computer #2 | Simulation work |
| | 8 | L9 Collision resolution | | | |
| | 9 | | E6 Collision resolution | C3 Computer #3 | |
| | 9 | L10 IEEE 802.15.4 | | | Homework deadline |
| | 10 | | | | |

Homework

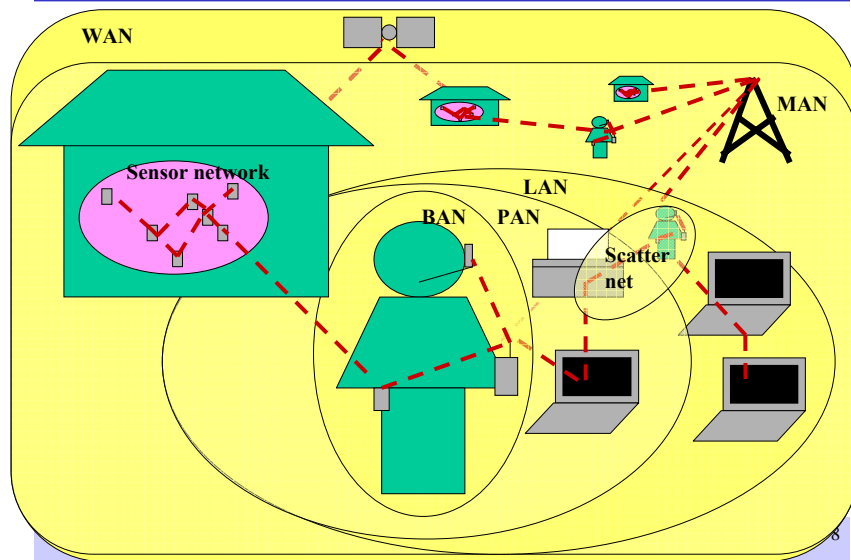
- There are two homework problems
 - One paper and pencil type of problem
 - One computer simulation problem
- Homework problems are not mandatory, but highly recommended.
- They can give up to 10 extra points to the exam

Lecture 1.

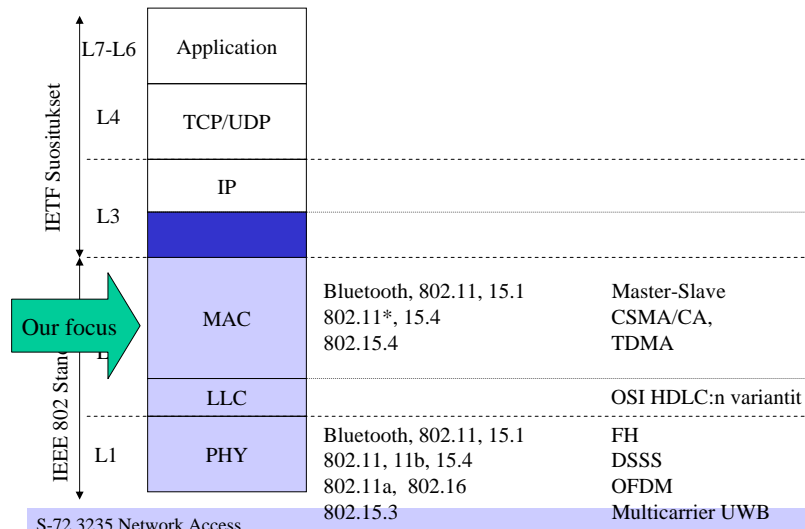
- Introduction to medium access control
- Recapitulation of stochastic processes and queuing theory

Medium access control

Wireless data networks

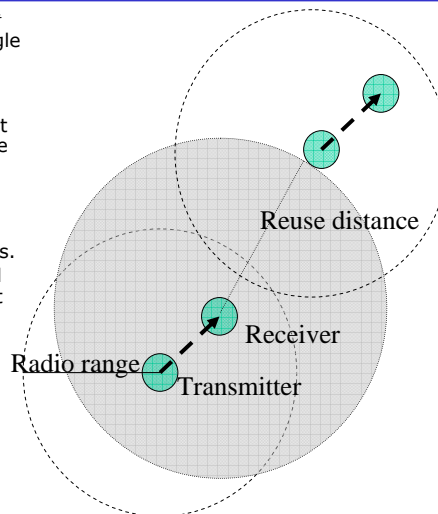


Protocol architecture



Medium access control

- Wireless transmission is *broadcast* in nature. That is more than a single receiver can potentially receive every transmitted message.
- Transmissions over a broadcast channel interfere, in the sense that one transmission coinciding in time with another may cause none of them to be received.
- The success of a transmission between a pair of nodes is not independent of other transmissions.
- To make a transmission successful interference must be avoided or at least controlled.
- The channel is a shared resource whose allocation is critical for proper operation of the network.
- The schemes used for channel access are known in literature as *Multiple Access Protocols (MAC)*.



Medium access control protocols

- The task of the Medium Access Control (MAC) protocol is to divide the resources between the radio links such that
 - Interference is avoided or kept at controlled level
 - Utilization of the radio resources is maximized
 - Quality of service QoS differentiation among the flow classes is achieved
 - Fairness inside a QoS class is maintained

Medium access control protocols

- The operation of the MAC protocol can be
 - *Centralized* such that single entity controls the resource division among the radio links leading to *conflict free access*
 - *Decentralized* such that each link makes transmission decisions independently leading to *contention based access*
- Contention schemes differ in principle from conflict-free schemes
 - A transmitting user is not guaranteed to be successful.
 - The protocol must prescribe a way to resolve conflicts once they occur so that all messages are eventually transmitted successfully.

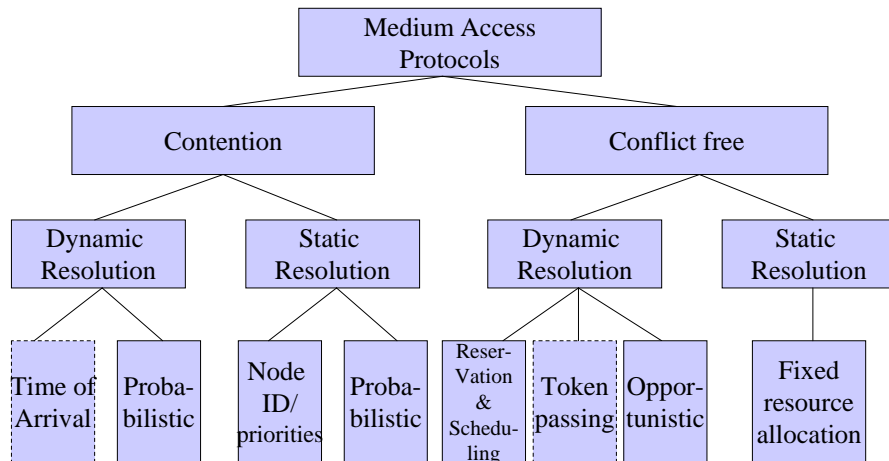
Conflict free access

- In conflict free protocols, the resource allocation can be
 - *Static* - not dependent on the traffic or channel conditions (TDMA, FDMA, F/TDMA, OFDMA, CDMA, OFCDMA,...)
 - *Dynamic* – based on demand and/or channel conditions
 - Token passing
 - Channel reservation (satellite systems, IEEE802.15.4,...)
 - Dynamic scheduling (UMTS R99, WiMAX,...)
 - Channel dependent “opportunistic” scheduling (e.g. CDMA2000 1xEV-DO/DV, HSDPA, LTE)

Contention based access

- In contention based protocols, the conflicts caused by colliding packets (interference) must be resolved. Conflict resolution methods can be divided into
 - *Static* - the actual behavior is not influenced by the dynamics of the system. The transmission schedule for the interfering users can be
 - Fixed: based on node IDs or priorities
 - Probabilistic: schedule is chosen from a fixed distribution (p-persistent CSMA)
 - *Dynamic* – the actual behavior of the system depends on system dynamics.
 - Transmission schedule could be determined by the time of the arrival
 - Probabilistic: Transmission schedule depends on the number of colliding packets (BEB in IEEE802.3 and IEEE802.11)

Classification of MAC protocols



Medium access control protocols

- The issues affecting the performance of the channel access
 - **Connectivity**
 - Can all the nodes hear each other or are there hidden terminals?
 - What is the network topology? Single hop, multi-hop (mesh/ad hoc)
 - **Channel type**
 - What is the required Signal-to-Interference ratio for correct reception? Is there possibility for capture in case of collisions?
 - Do protocol messages get lost due to fading?
 - **Synchronism**
 - Is the network synchronized, i.e. slotted or can transmissions start and end at arbitrary time instances.
 - **Feedback information**
 - Can collisions be detected? Can the colliding nodes be identified?
 - How much information can be shared among the nodes?
 - Is correct reception acknowledged by the receiver?
 - **Traffic**
 - Is the message size fixed or does it vary? Is packets generated randomly or with steady rate? Can transmission buffers assumed to be saturated (TCP tends to saturate buffers) or are they likely to be empty at times?
 - **User population**
 - Is the number of users fixed or random? Can it be known by the system?
 - **Buffering capability**
 - How many packets can the nodes buffer? Will packets be lost due to buffer overflow?

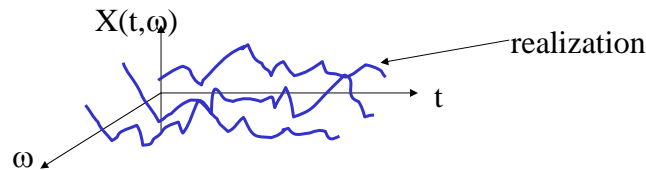
Other relevant courses

- Simulation tools
 - S-38.3148 Network simulation 5 cr
- Mathematical tools
 - S-38.3143 Queue Theory 5 cr
- Traffic modeling and performance analysis
 - S-38.3141 Teletraffic theory 5 cr
- Conflict free access
 - S-72.3260 Radio Resource Management Methods 3 cr

Stochastic processes and queuing theory

Stochastic processes

- A stochastic process is a set of indexed random variables
 $\{X(t, \omega), t \in T, \omega \in \Omega\}$
- The index set $t \in T$ is called *parameter space* of the process
- Each individual random variable is a mapping from the sample space to set of real (or complex) numbers.
- A parameterized set $X(t)$ corresponding to a sample ω is called *realization/trajectory/path* of the process.



Stochastic processes

- *State-space* of the process is a set of values that $X(t)$ may obtain.
- State space is *discrete*, if the number of states is finite or numerable. The corresponding stochastic process is called discrete time process/sequence/chain
 $\{X(t_k)\}, t \in \{t_0, t_1, t_2, \dots\}$
- State space is *continuous*, if the number of states is innumerable. The corresponding stochastic process is called continuous time process/sequence/chain
 $X(t), t \in (0, \infty]$

Markov-processes

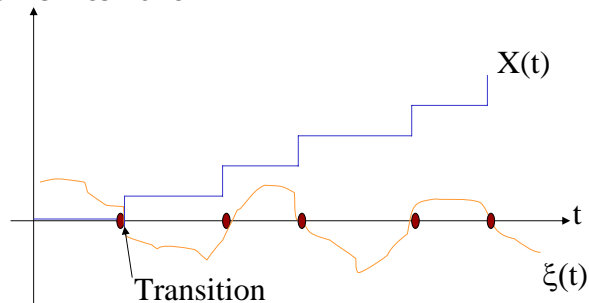
- *Markov property*: The state of the process at time t_{n+1} depends only on its state at the previous time instance t_n
$$\Pr\{X(t_{n+1}) = x_{n+1} \mid X(t_n) = x_n, X(t_{n-1}) = x_{n-1}, \dots, X(t_1) = x_1\}$$
$$= \Pr\{X(t_{n+1}) = x_{n+1} \mid X(t_n) = x_n\}$$
- *Markov-Process*: The process stays in a state x_n random time interval after which it changes its state randomly according to certain state transition probabilities.
- *Markov-Process* has the *Markov property*, if the state time distribution of the process is memoryless. That is, transition is allowed to take place every time instant.
 - *Continuous time Markov-Process*: State time distribution is exponential
 - *Discrete time Markov-Process*: State time distribution is geometric

Other related processes

- *Semi-Markov process*: State time distribution can be arbitrary. At the instance of state transitions, the process behaves as Markov chain.
 - *Imbedded Markov-chain*, Semi-Markov process observed at state transition times.
- *Random walk/Process with independent increments*: Location of a particle moving in space: Next position = Previous position + random variable
$$S_n = S_{n-1} + X_n, \quad S_0 = 0$$
where X_1, X_2, \dots is a sequence of independent identically distributed random variables, n is the number of state transitions

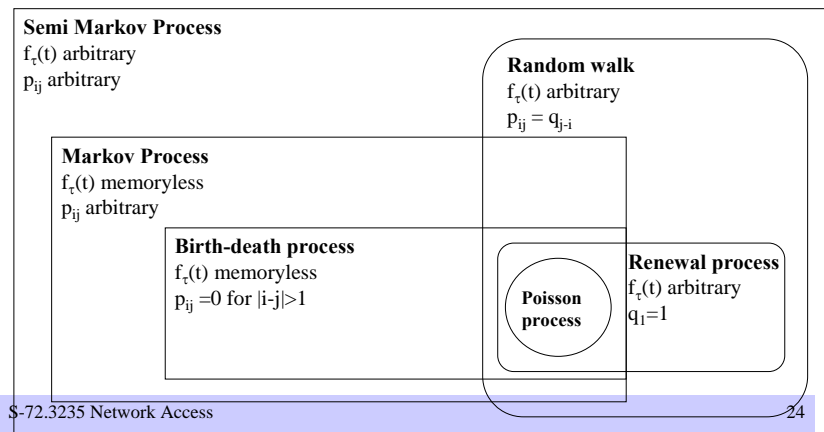
Other related processes

- *Renewal/recurrent process*: Related to the random walk, but instead of position, we interested in counting the number of transitions $X(t)$ that take place as a function of time t . I.e. $X(t)$ is a random variable that states the number of transitions that have taken place in time interval t .



Classification of stochastic processes

- $f_{\tau}(t)$ probability density function of time spent in a state
- p_{ij} transition probability
- q_i state transition rate



Discrete-time Markov chains

- **Definition:** The sequence of random variables X_1, X_2, \dots forms a discrete-time Markov chain if for all n ($n=1, 2, \dots$) and all possible values of the random variables we have that

$$\Pr\{X_{n+1} = x_{n+1} | X_n = x_n, X_{n-1} = x_{n-1}, \dots, X_1 = x_1\} = \Pr\{X_{n+1} = x_{n+1} | X_n = x_n\}$$

The state variable $x_n=i$ implies that the state of the system was E_i at time slot n .

Discrete-time Markov chains

- Markov chain is said to be *homogenic* (stationary), if state transition probabilities are independent of time index.

$$\Pr\{X_{n+1} = j | X_n = i\} = \Pr\{X_{m+1} = j | X_m = i\} = p_{ij} \forall m, n$$

- For homogenic Markov chain, the state transition probability from state $E_i: X_n=j$ to state $E_j: X_{n+1}=i$ can be defined as:

$$p_{ij} \triangleq \Pr\{X_n = j | X_{n-1} = i\}$$

Discrete-time Markov chains

- Assume that the state space is independent of the time index n . Probability that the system is in state E_j at time instant $n+m$ ($X_{m+n}=j$) conditioned that it was in state E_i at time m ($X_n=i$) is

$$p_{ij}^n \triangleq \Pr\{X_{m+n} = j | X_m = i\} = \sum_{k=0}^{\infty} p_{ik}^n p_{kj}^m \quad \text{Chapman-Kolmogorov equation}$$

- If there exists an integer m_0 such that $p_{ij}^{m_0} > 0$, the Markov chain is said to be *irreducible*.
- Let A denote the set of all states in a Markov chain.
 - A subset $A_1 \subset A$ is said to be *closed* if no one-step transition is possible from any single state in A_1 to its complement $A_1^c = A \setminus A_1$.
 - If A_1 consist of a single state E_j , the state is called *absorbing state*. If A_1 is closed and does not contain any proper closed subsets, then A_1 forms *irreducible sub-Markov chain*.

Discrete-time Markov chains

- The chain is irreducible if

$$p_{ij} > 0 \forall i, j$$

- E_3 is absorbing state if

$$p_{ij} > 0, \quad i \neq 3$$

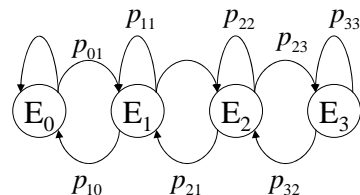
$$p_{33} = 1$$

$$p_{32} = 0$$

- E_2 and E_3 form an irreducible sub-Markov chain if

$$p_{ij} > 0, \quad (i, j) \neq (2, 1)$$

$$p_{21} = 0$$



Discrete-time Markov chains

- Probability that the chain returns to state E_i :

$$f_i^{(n)} = \Pr\{X_{n+m} = i | X_m = i\} = p_{ii}^{(n)} \quad \text{with } n \text{ steps}$$

$$f_i = \sum_{n=1}^{\infty} f_i^{(n)} \quad \text{at all}$$

- If $f_i = 1$ Markov chain is called *recurrent*; otherwise it is called *transient*.
- If the initial state is revisited in regular time intervals, the chain is said to be *periodic*; otherwise it is called *aperiodic (non-periodic)*.
- *Mean recurrence time* of state E_i

$$M_i = \sum_{n=1}^{\infty} n f_i^{(n)} \quad \begin{matrix} = \infty & \text{Recurrent null} \\ < \infty & \text{Recurrent nonnull} \end{matrix}$$

Discrete-time Markov chains

- Markov chain is Ergodic stochastic process if it is aperiodic, recurrent $f_i = 1$ and recurrent nonnull $M_i < \infty$
- Probability that the system is in state E_i at time instant n
 $\pi_i^{(n)} = \Pr\{X_n = i\}$ **State probability**

Theorem. In irreducible, aperiodic, homogeneous Markov chain, the limit value

$$\pi_i = \lim_{n \rightarrow \infty} \pi_i^{(n)}$$

fulfills either

a) $f_i < 1$ tai $M_i = \infty \Rightarrow \pi_i = 0$

b) $f_i = 1, M_i < \infty$

$$\pi_i = \frac{1}{M_i} = \sum_j \pi_j p_{ji} \quad \sum_i \pi_i = 1$$

for all i (for all states E_i)

Discrete-time Markov chains

- If the number of states is finite, the state probabilities can be solved from the following set of linear equations

$$\pi_i = \frac{1}{M_i} = \sum_j \pi_j P_{ji}$$

$$\sum_i \pi_i = 1$$

- Define

$\boldsymbol{\pi} = (\pi_1 \dots \pi_m)$ Row vector containing state probabilities

$\mathbf{P} = (p_{ij})$ Non-negative state transition probability

$\boldsymbol{\pi} = \boldsymbol{\pi}\mathbf{P}$ Equation for left eigenvalues of P

State probabilities are defined by the (left) Perron-eigenvector of the state transition matrix P that fulfills

$$\sum_i \pi_i = 1$$

Discrete-time Markov chains

- The state transition matrix P has the following properties
 - P is a nonnegative matrix $P \geq 0$
 - The largest eigenvalue in modulus $\rho(P)$ is equal to 1:

$$\lambda_i \mathbf{x}_i = \mathbf{x}_i \mathbf{P}$$

$$\rho(\mathbf{P}) = \max_i \{|\lambda_i|\} = 1$$

$$\exists i, \lambda_i = 1$$
 - The row and column sums of P are equal to 1

$$\sum_j p_{ji} = 1, \sum_i p_{ji} = 1$$
 - If the chain is irreducible, then also P is an irreducible matrix, and the Perron eigenvector can be taken to be strictly positive

$$\boldsymbol{\pi} = \boldsymbol{\pi}\mathbf{P} \Rightarrow \boldsymbol{\pi} > 0$$

Discrete-time Markov chains

- The state probability can be solved simply by using the power method for solving the Perron eigenvector

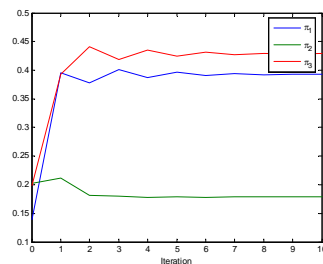
$$\boldsymbol{\pi}(n+1) = \frac{\boldsymbol{\pi}(n)\mathbf{P}}{\boldsymbol{\pi}(n)\mathbf{P}\mathbf{1}^T}$$

$$\boldsymbol{\pi}(0) > 0$$

$$\mathbf{1} = (1 \ 1 \ \dots \ 1)$$

- Example

```
P=[.1 .1 .8;.3 .3 .4;.7 .2 .1];
Pi=rand(1,3);
I=ones(size(Pi));
for k=1:10
    Pi(k+1,:) = Pi(k,:) * P / (Pi(k,:) * P * I');
end;
plot(0:10, Pi)
```



S-72.3235 Network Access

33

Some useful tools

Characteristic function and moment generating
function

Probability generating function

Characteristic / moment generating function

- Moment generating function = Fourier-transformation of the probability density function

$$\psi(\omega) = E\{e^{i\omega X}\} = \int_{-\infty}^{\infty} e^{i\omega x} p(x) dx, \quad i = \sqrt{-1}$$

- Inverse Fourier-transform

$$p(x) = \frac{1}{2\pi} \int_{-\infty}^{\infty} \psi(i\omega) e^{-i\omega x} d\omega$$

- k^{th} derivative of the characteristic function

$$\frac{d^k}{d\omega^k} \psi(\omega) = \int_{-\infty}^{\infty} (ix)^k e^{i\omega x} p(x) dx$$

- k^{th} moment

$$\overline{X^k} = E\{X^k\} = \lim_{\omega \rightarrow 0} (-i)^k \frac{d^k}{d\omega^k} \psi(i\omega)$$

Laplace transform

- Consider random variable X with support $[0, \infty]$. That is, $X \geq 0$
- The pdf of the variable is $p(x)$
- Laplace-transform of the pdf

$$P^*(s) = E\{e^{-sX}\} = \int_{-\infty}^{\infty} e^{-sx} p(x) dx$$

- Characteristic function

$$\psi(s) = P^*(i\omega)$$

- k^{th} moment

$$\overline{X^k} = E\{X^k\} = \lim_{s \rightarrow 0} (-1)^k \frac{d}{ds^k} P^*(s)$$

Probability generating function

- Discrete random variable
 $\Pr\{X = k\} = p_k \quad k=0,1,2,3,\dots$
- Probability generating function = Z-transform of the probability

$$G(z) = E\{z^X\} = \sum_{k=0}^{\infty} z^k p_k, \quad z \in \mathbb{C}$$

- Properties of G(z)

$$G(1) = \sum_{k=0}^{\infty} p_k = 1$$

$$|G(z)| < \sum_{k=0}^{\infty} |z^k| p_k < \sum_{k=0}^{\infty} p_k = 1, \quad |z| \leq 1$$

Probability generating function

- First derivative yields expected value:

$$E\{X\} = \sum_{k=0}^{\infty} k p_k = \frac{d}{dz} G(z) \Big|_{z=1}$$

- 2nd derivative yields 2nd moment

$$\frac{d^2}{dz^2} G(z) \Big|_{z=1} = \sum_{k=1}^{\infty} k(k-1) z^{k-2} p_k \Big|_{z=1} = \sum_{k=1}^{\infty} k^2 p_k - \sum_{k=1}^{\infty} k p_k = E\{X^2\} - E\{X\}$$

$$E\{X^2\} = G''(1) - G'(1)$$

$$\text{var } X = G''(1) - G'(1) - [G'(1)]^2$$

Probability generating function

- Let $\{X_i\}$ be a set of independent identically distributed discrete random variables. $\Pr\{X_i=k\}=p_k$ for all i .

$$G_{X_i}(z) = E\{z^{X_i}\} = \sum_{k=0}^{\infty} z^k p_k \triangleq G_X(z) \quad \forall i$$

- Let N be a discrete random variable independent of $\{X_i\}$. $\Pr\{N=k\}=q_k$

$$G_N(z) = E\{z^N\} = \sum_{k=0}^{\infty} z^k q_k$$

Probability generating function

- Consider a random sum

$$S_N = \sum_{i=1}^N X_i$$

- Probability generating function of S_N :

$$E\{z^{S_N} | N\} = E\left\{z^{\sum_{i=1}^N X_i} \middle| N\right\} = \prod_{i=1}^N E\{z^{X_i}\} = [G_X(z)]^N$$

$$G_{S_N}(z) = E\{z^{S_N}\} = E\left\{[G_X(z)]^N\right\} = \sum_{k=0}^{\infty} [G_X(z)]^k q_k = G_N(G_X(z))$$

- Wald's Lemma $E\{S_N\} = E\{N\}E\{X_i\}$

$$G_{S_N}'(z) = G_N'(G_X(z))G_X'(z)$$

$$E\{S_N\} = G_{S_N}'(1) = G_N'(G_X(1))G_X'(1) = G_N'(1)G_X'(1) = E\{N\}E\{X_i\}$$