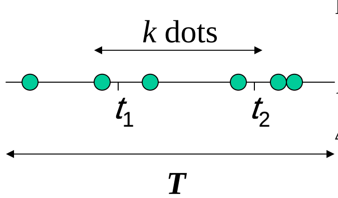
Lecture 2

- Poisson Process
- Traffic models
  - Circuit switched voice
  - Packet data

Poisson Process

- Let us randomly place n dots (uniform distribution) on the interval $(0,T)$. What is the probability that there are k dots on the interval (t_1,t_2)?

$$\Pr\{k \text{ dots on the interval } (t_1,t_2)\} = \binom{n}{k} p^k (1-p)^{n-k}$$



$$p = \frac{\Delta t}{T}$$

$$\Delta t = t_2 - t_1$$

Consider a limit

$$n \to \infty, \quad T \to \infty \quad np = n\frac{\Delta t}{T} \to \lambda \Delta t$$

Poisson Theorem

$$\Pr\{k \text{ dots on the interval } (t_1,t_2)\} = \binom{n}{k} p^k (1-p)^{n-k} \to e^{-\lambda \Delta t} \frac{(\lambda \Delta t)^k}{k!}$$

Poisson Process

- If Δt is small, then

$$\Pr\{\text{Only one dot on the interval } \Delta t\} = \lambda \Delta t e^{-\lambda \Delta t} \approx \lambda \Delta t$$

- That is,

$$\lim_{\Delta t \to 0} \frac{\Pr\{\text{Only one dot on the interval } \Delta t\}}{\Delta t} = \lambda$$

- Events that there are k dots on interval (t_1,t_2) and (t_3,t_4), $t_1 < t_2 < t_3 < t_4$ are independent of each other

$$\Pr\{k_1 \text{ dot on } (t_1, t_2) \text{ and } k_2 \text{ dots on } (t_3, t_4)\}$$

$$= \Pr\{k_1 \text{ dots on } (t_1, t_2)\} \Pr\{k_2 \text{ dots on } (t_3, t_4)\}$$

- Poisson Process:

$$\Pr\{k \text{ dots on the interval } \Delta t\} = e^{-\lambda \Delta t} \frac{(\lambda \Delta t)^k}{k!}$$

Poisson process

- Poisson process can be utilized to model arrivals from *independent* sources
  - Call arrivals in voice telephony
  - Packet session arrivals in data networks
  - Handovers from neighboring cells (approximately)
- Poisson model, in general, is not valid for modeling arrivals from correlated sources, such as
  - Packets generated by single computer during a packet session

Circuit switched voice traffic

Call arrival process

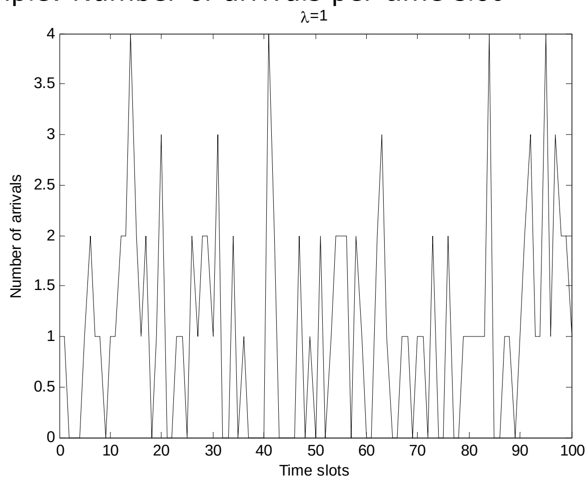
- Call arrival process:
  - $T \to \infty$ denotes the time interval from the big bang till the end of time
  - n denotes the total number of calls that arrive on time interval T.
- Probability that k calls arrive during the time interval of length Δt

$$\Pr\{k \text{ calls arrive during } \Delta t\} = e^{-\lambda \Delta t} \frac{(\lambda \Delta t)^k}{k!} \qquad \text{(Poisson Process)}$$

$$\Pr\{\text{Only one call arrives during } \Delta t\} \approx \lambda \Delta t \quad \text{when } \Delta t \text{ is very small}$$

Call arrival process

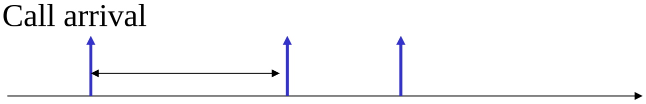
- Example: Number of arrivals per time slot



Call arrival process

- Inter-arrival time T of calls

Call arrival



Inter-arrival time T

$$\Pr\{T \le t\} = \Pr\{\text{At least one call arrives during } t\}$$

$$= 1 - \Pr\{\text{No calls arrive during } t\} = 1 - e^{-\lambda t}$$

$$p(t) = \frac{d}{dt}\Pr\{T \le t\} = \lambda e^{-\lambda t}$$

- Expected inter-arrival time

$$E\{T\} = \int_0^\infty \lambda t e^{-\lambda t} dt = \frac{1}{\lambda}\int_0^\infty x e^{-x} dx = \frac{1}{\lambda}$$

Call arrival process

- Let T denote the interarrival time
- Probability that next call arrives in time t

$$F(t)=\Pr\{T\le t\}=1-e^{-\lambda t}$$

- Given that new call has not arrived in time t, the probability that it will arrive in time t+Δt is given by

$$\Pr\{T\le t+\Delta t\,|\,T>t\}=\frac{\Pr\{t<T\le t+\Delta t\}}{\Pr\{T>t\}}$$

$$=\frac{\Pr\{T\le t+\Delta t\}-\Pr\{T\le t\}}{1-\Pr\{T\le t\}}=\frac{1-e^{-\lambda(t+\Delta t)}-\left(1-e^{-\lambda t}\right)}{1-\left(1-e^{-\lambda t}\right)}$$

$$=\frac{e^{-\lambda t}-e^{-\lambda(t+\Delta t)}}{e^{-\lambda t}}=1-e^{-\lambda\Delta t}=F(\Delta t)$$ which is independent of t!

Hence, the arrival process is *memoryless*.

Call departures

- Also call departures can be modeled with Poisson process
- Mean call holding time τ follows then exponential distribution

$$p(\tau)=\mu e^{-\mu\tau}$$

- Mean call holding time

$$E\{\tau\}=\int_0^\infty \tau\mu e^{-\mu\tau}d\tau=\frac{1}{\mu}$$

- Probability that a call in progress ends during a time interval Δt:

$$\Pr\{\text{Call ends during }\Delta t\}=\mu\Delta t e^{-\mu\Delta t}\approx\mu\Delta t$$

Call arrivals and departures

- Call arrivals per time unit

$$\lim_{\Delta t \to 0} \frac{\Pr\{\text{Only one call arrive during } \Delta t\}}{\Delta t} = \lambda$$

- Call departures per time unit

$$\lim_{\Delta t \to 0} \frac{\Pr\{\text{Only one call departures during } \Delta t\}}{\Delta t} = \mu$$

- Call arrivals and departures constitute a continuous time birth-death process.

Call arrivals and departures

- Steady state probabilities of continuous Markov chain could be derived using discrete time Markov chain where the time interval Δt is small
- State probability of a birth dead call arrival-departure process

$$\pi_i = (1 - \lambda - \mu)\pi_i + \lambda\pi_{i-1} - \mu\pi_{i+1}$$

Probability that the system is in State i+1 and a call departs

Probability that system is in state i-1 and one call arrives

Probability that the system is in state i and no call arrives or departs

$$\sum_i \pi_i = 1$$

Probability that the system is one of its possible states

Call arrivals and departures

- The steady state probabilities are the same as in the discrete time birth-death process
  - State probabilities

$$\pi_k = \left(\frac{\lambda}{\mu}\right)^k \pi_0$$

$$\pi_0 = \left(1 - \frac{\lambda}{\mu}\right)$$

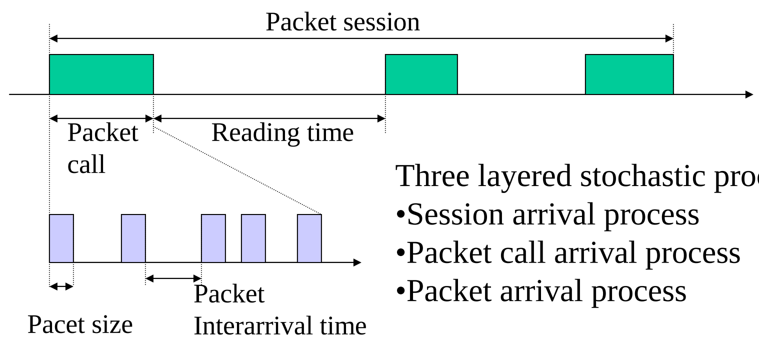
  - Number of calls in progress

$$N = \sum_{k=0}^{\infty} k\pi_k = \frac{\frac{\lambda}{\mu}}{1-\frac{\lambda}{\mu}} = \frac{\lambda}{\mu - \lambda}$$

Packet switched traffic

Packet switched traffic

- Non-real-time services
  - SMS, WWW, FTP, Email,...
- Traffic model



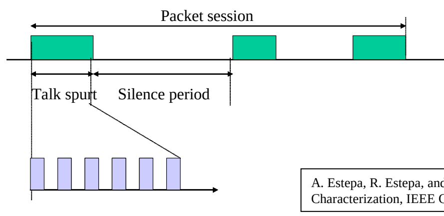
Three layered stochastic processes
•Session arrival process
•Packet call arrival process
•Packet arrival process

VoIP

- Traffic generated by VoIP source resembles circuit switched voice.
- Simple VoIP without voice activity detection and single rate speech codec:
  - Call interarrival time: exponential
  - Call holding time: exponential
  - Packet interarrival time: fixed
  - Packet size: fixed
- The traffic characteristics of the above model are the same as in case of circuit switched voice

VoIP

- For example in [9] an empirical measurement showed that the talk spurt lengths were exponentially distributed with a mean of 1026 ms and the silence lengths also exponentially distributed with a mean of 1171 ms resulting in an activity factor of 47.17%.
- During the silence periods, comfort noise will sometimes be generated which means that during a silence period the application will send data in a very low rate.



	Distribution	Mean
Session level		
Call holding time (sec)	exponential/lognormal	100-300
Packet level		
AMR Frame length (ms) Rate (kbps)	 Fixed Fixed	 20 12.2
iLBC Frame length (ms) Rate (kbps)	 Fixed Fixed	 20/30 15.20/13.33

A. Estepa, R. Estepa, and J. Vozmediano, A new Approach for VoIP Trac Characterization, IEEE Communications Letters, Vol. 8, pp 644-646, 2004.

File transfer

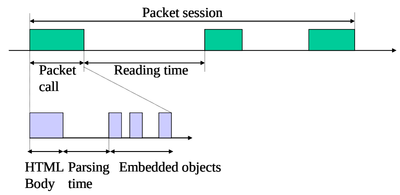
- Commonly utilized models
  - Packet session interarrival: Exponential/geometric (Poisson arrival)
  - Number of arriving packet sessions: Poisson
  - Reading time distribution: Exponential/geometric
  - Packet interarrival time: Exponential/geometric or log-normal
  - Packet length: Fixed
  - Length of the packet session: Lognormal/Pareto distributed

	Distribution	Mean
Activity level		
File size (Mbytes)	(Truncated) Lognormal	≈ 2
Reading time (sec)	Exponential	180

3GPP2-TSGC5, HTTP and FTP Trac Model for 1xEV-DV Simulations

Web browsing

- Web trac modeling is quite complex due to large number of embedded objects such as pictures and java applets embedded within web pages.
- The retrieval of a web page (packet call) starts with a HTTP GET request to which the server answers by sending the main HTML page.
- After receiving the main HTML page the Web browser has to parse the page to find references of where the embedded objects within the page can be downloaded from and after that retrieve each individual object.
- A complete Web session consists of multiple such web page retrievals (packet calls)



Component	Distribution	Parameters
Activity level		
Main object size (bytes)	Truncated Lognormal	Mean = 10 710 Std. dev. = 25 032 Minimum = 100 Maximum = 2 000 000
Embedded object size (bytes)	Truncated Lognormal	Mean = 7 758 Std. dev. = 126 168 Minimum = 50 Maximum = 2 000 000
Number of embedded objects	Pareto	Mean = 5.64 Maximum = 53
Reading time (sec)	Exponential	Mean = 30
Parsing time (sec)	Exponential	Mean = 0.13

3GPP2-TSGC5, HTTP and FTP Trac Model for 1xEV-DV Simulations

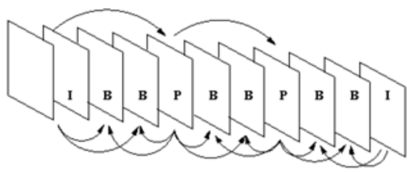
Streaming video

- A MPEG video stream consists of three types of frames:
  - I frames that encode still images,
  - smaller P frames that construct an image by using information from the previous I and P frames
  - and the smallest of the three, B frames that use information from both the previous and the next I and P frames for image construction.
- The inter-arrival-time of frames is constant and usually corresponds to a rate of 30 frames per second.

Streaming video

- The temporal order of P and B frames between two consecutive I frames can be described with a Group-of-Pictures (GOP) pattern.
  - Quite often this pattern is fixed so that the GOP can be described with two parameters: the
  - I-to-I distance (N) and the I-to-P frame distance (M)

Example: N=9, M=3



Streaming video

- MPEG trac can be modeled by examining the trac on the Session Level and the Packet Level. The duration of the stream depends on whether it is generated from a Video Conference, a video stream or a broadcast and can be modeled in many ways.
- There has been a considerable amount of research on how to model frame sizes of VBR video traffic and it can be roughly divided to the following basic categories
  - Models based on Markov chains (most simple models that only describe short term dependencies)
  - Models based on Auto-Regressive (AR) processes (more accurate but also more complex)
  - Self-similar or fractal models (most accurate but very complex)

L. Derong, E. I. Sara, and S. Wei. Nested auto-regressive processes for MPEGencoded video traffic modelling. IEEE Transactions on Circuits and Systems for Video Technology, 11:169 183, February 2001.

M. Krunz and S. Tripathi, On the Characterization of VBR MPEG Stream, ACM, pp. 192-202, 1997

O. Rose, Simple and ecient models for variable bit rate MPEG video trac. Technical report, Institute of Computer Science, University of Wirzburg, July 1995.

Characteristic of Internet traffic

Characteristics of packet traffic

- Empirical studies of different data networks indicate that Packet traffic exhibits extended temporal correlations, i.e., **long-range dependence (LRD)**, and hence when viewed within some range of (sufficiently large) time scales, the traffic appears to be **fractal-like** or **self-similar**, in the sense that a segment of the traffic measured at some time scale looks or behaves just like an appropriately scaled version of the traffic measured over a different time scale

Empirical results:
Ethernet (LAN):
W. E. Leland, M. S. Taqqu, W. Willinger, and D. V. Wilson, "On the Self-Similar Nature of Ethernet Traffic (Extended Version), IEEE/ACM Transactions on Networking, Volume: 2, Issue: 1, February 1994.

Internet traffic:
V. Paxson and S. Floyd, "Wide Area Traffic: The Failure of Poisson Modeling," IEEE/ACM Transactions on Networking, Volume: 3, Issue: 3, June 1995, Pages:226 – 244

An overview:
A. Erramilli, M. Roughan, D. Veitch, D. and W. Willinger, "Self-similar traffic and network dynamics," *Proceedings of the IEEE* , Volume: 90 , Issue: 5 , May 2002 Pages:800 - 819

Characteristics of packet traffic

- The self-similar behavior of data traffic is caused by the high-variability of the of individual sessions that make up the aggregate traffic.
- Self-similar scaling behavior over sufficiently large time scales can be observed, if the durations (in time) or sizes (in bytes) of the individual sessions or IP flows that generate the aggregate traffic have a *heavy-tailed distribution with infinite variance* (that is, range from extremely short/small to extremely long/large with non-negligible probability).
- The LRD nature of network traffic is mainly caused by *user/application characteristics* such as Poisson arrivals of sessions and heavy-tailed distributions for the session durations/sizes, and has little to do with the network, i.e., with the protocol-specific mechanisms that determine the actual flow of packets as they traverse the Internet.

=> Self-similarity and LRD cannot be avoided by changing the protocols.

Characteristics of packet traffic

- Let X(t) denote stationary traffic rate process
  - Mean, variance, and auto-covariance:

$$\mu = E\{X(t)\}$$

$$v = E\{(X(t)-\mu)^2\}$$

$$c_X(k) = E\{(X(t+k)-\mu)(X(t)-\mu)\}$$

  - The process is said to exhibit (asymptotically) long-range dependencies (LRD) if

$$c_X(k) \sim c_\gamma k^{-\beta}, \quad c_\gamma > 0, 0 < \beta < 1$$

otherwise if

$$c_X(k) \sim \phi^k, \quad |\phi| < 1$$

it is said to exhibit short-range dependencies (SDR).

~ denotes 'asymptotically' That is, the ratio of the two sides tends to one in the limit of large k

Characteristics of packet traffic

- Consider the sample mean estimator

$$\bar{X}_n = \frac{1}{n}\sum_{k=1}^{n} X(k)$$

- This process has stationary increments, and

$$\operatorname{var}\{\bar{X}_n\} \sim \frac{2c_\gamma}{(1-\beta)(2-\beta)} n^{-\beta}$$

$$\frac{\operatorname{var}\{\bar{X}_n\}}{\operatorname{var}\{\bar{X}_m\}} \sim \left(\frac{n}{m}\right)^{\beta}$$

  if held exactly, implies that the process is self-similar.
- The confidence interval of the mean estimator is proportional to $\sqrt{c_\gamma}$ while the scaling exponent β defines the character of LRF effect.

Characteristics of packet traffic

- The unique continuous Gaussian process with LRD characteristics and with stationary increments is the well-known *fractional Brownian motion* (fBm).
- The fBm family of processes is indexed by single parameter $H=(1-\beta)/2$ (Hurst parameter).
  - If $0 \le H < 1/2$ the process is SDR
  - If $H = 1/2$ the process is memoryless
  - If $1/2 < H \le 1$, the process is LDR
- Data traffic can be modeled with fBm $H \in (1/2,1)$

Characteristics of packet traffic

- The aggregate traffic of *m* sources can be modeled as
  $A(t) = mt + \sqrt{am}Z(t), \quad a > 0$
  where Z(t) is zero mean fBm process
- Assuming fixed transmission rate C, the resulting queuing process is known as fractional Brownian storage.

$$X(t) = sum_{s \le t}\left\{A(t) - A(s) - (t-s)C\right\}$$

I. Norros, "On the Use of Fractional Brownian Motion in the Theory of Connectionless Networks," IEEE Journal on Selected Areas in Communications, Volume: 13, Issue: 6, August 1995, Pages: 953 – 962.

Characteristics of packet traffic

- Let ε=Pr{X>x} denote the probability, that the buffer capacity x is exceeded
- Then self-similar arrival process with H>1/2, we have

$$C = m + f^{-1}(\varepsilon)a^{1/2H}x^{-(1-H)/H}m^{1/2H}$$

where

$$f(x) = \Pr\left\{\sup_{t \ge 0}\left\{Z(t) - xt\right\} > 1\right\}$$

- The required excess capacity due to burstiness of the arrival process increases slower than linearly in *m* so that multiplexing gain is obtained by using links with higher capacity.

Characteristics of packet traffic

- Consider an on-off traffic source model, where the on and off times follow different heavy-tailed probability density with infinite variance ($1<\alpha<2$)

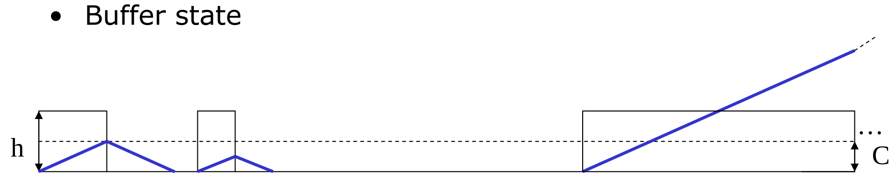
$$\Pr\{T_{on} > x\} \sim c_\alpha x^{-\alpha} \quad \text{Pareto distribution}$$

$$E\{T_{on}\} = \frac{1}{\mu} \quad E\{T_{off}\} = \frac{1}{\upsilon}$$

- During the on-period packets are emitted with rate h
- Average rate of the on-off source $\lambda = h\nu/(\nu+\mu)$.
- Aggregating many such on–off sources results in aggregate link traffic that exhibits self-similar scaling behavior.
- Even if $\lambda < C$, where C denotes the link capacity, the queue can blow up.

Characteristics of packet traffic

- Buffer state



  - Average rate of the on-off source $\lambda = h\nu/(\nu+\mu) < C$
  - However, due to the large variance, there is no upper bound for how long the on-period takes.

Characteristics of packet traffic

- Self-similarity concerns large time scales and leaves the small scale behavior, such as strong short term correlation of the arrivals, unspecified.
- TCP uses feedback to react to network congestion.
- Because TCP feedback modifies the self-similarity in the offered traffic, "open loop" modeling approaches will not accurately predict TCP performance.
- Chaotic maps have been suggest to model the complex the source behavior (A. Erramilli, *et. al.* 2002).
- TCP tends to saturate the bottleneck links. Since wireless links usually are the bottlenecks, *saturated traffic conditions* where each transmitter is always assumed to have packet ready for transmission is commonly utilized to analyze the performance of MAC schemes in wireless systems.