Lecture 3

Conflict free access

Contents

- Fixed resource allocation
  - FDMA
  - TDMA
  - Generalized TDMA
  - CDMA
- Dynamic resource allocation
  - Reservation
  - Packet scheduling

M/G/1

- Poisson arrival process
- Single server
- General interarrival time distribution
  - λ arrival rate of the packets
- General service time distribution
  - Service time of the packets are independent and identically distributed random variables
  - $\tilde{X}_i$ service time of customer i
  - $X^*(s)$ Laplace transform of the service time distribution $E\{\exp(-sX_i)\}$
  - $\overline{X}$ mean service time $E\{X_i\}$
  - $\overline{X^2}$ second moment of service time $E\{X_i^2\}$

M/G/1

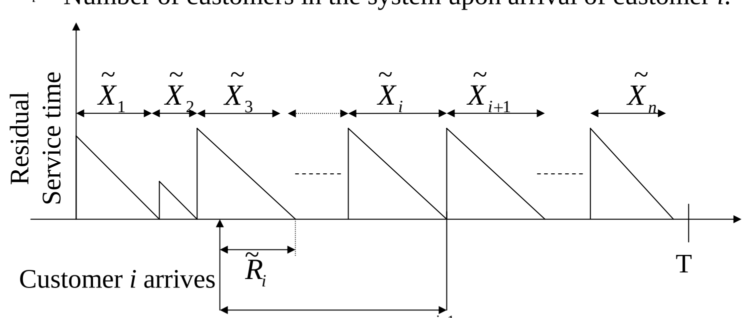
- Define

W_i Waiting (queuing) time of customer *i*

R_i Residual service time when customer *i* arrives

$\tilde{X}_i$ Service time of customer *i*

N_i Number of customers in the system upon arrival of customer *i*.



$$\tilde{W}_i = \tilde{R}_i + \sum_{j=i-N_i}^{i-1} \tilde{X}_i$$

M/G/1

- Expected waiting (queuing) time

$$E\{\tilde{W}_i\} = E\{\tilde{R}_i\} + E\left\{\sum_{j=i-N_i}^{i-1} \tilde{X}_i\right\} = R + \lambda \overline{X} W$$

$$E\{R_i\} = \lim_{N\to\infty} E\left\{\frac{1}{N}\frac{1}{2}\sum_{i=1}^{N} \tilde{X}^2\right\} = \frac{1}{2}\lambda \overline{X^2}$$

$$W = \frac{1}{2}\frac{\lambda \overline{X^2}}{1-\rho}, \quad \rho = \lambda \overline{X} \qquad \text{Pollaczek-Khinchin}$$

- Total time spend in the system = waiting time + service time

$$D = W + \overline{X} = \overline{X} + \frac{1}{2}\frac{\lambda \overline{X^2}}{1-\rho}, \quad \rho = \lambda \overline{X}$$

ρ = Utilization
Fraction of time the server is busy

M/G/1

- Number of customers in the system N
- We can model the system at departure time and extend the results to all points of time:
  - in the case of Poisson arrival the distribution of N at departure times is the same as at arbitrary points of time (PASTA)
  - the $\tilde{N}_t$ follows a discrete time Markov process at departure times:
- $\tilde{N}_k$: number of customers after the departure of a customer k
- $\tilde{V}_k$: number of arrivals during the service time of customer k, $\tilde{V}_k$ ~ Poisson(λ)

$$\tilde{N}_{k+1} = \begin{cases} \tilde{N}_k - 1 + \tilde{V}_{k+1} & N_k \geq 1 \\ \tilde{V}_{k+1} & N_k = 0 \end{cases}$$

M/G/1

- Expressing the steady state of the Markov-chain describing N, we get the z-transform of the distribution of N $(Q(z)=E\{z^N\})$

$$Q(z)=X^*(\lambda-\lambda z)\frac{(1-\rho)(1-z)}{X^*(\lambda-\lambda z)-z}$$

Pollaczek-Khinchintransform

- L-transform of the waiting time W pdf

$$W^*(s)=\frac{s(1-\rho)}{s-\lambda-\lambda X^*(s)}$$

- L-transform of the time spend in the system D=W+X pdf

$$D^*(s)=W^*(s)X^*(s)=X^*(s)\frac{s(1-\rho)}{s-\lambda+\lambda X^*(s)}$$

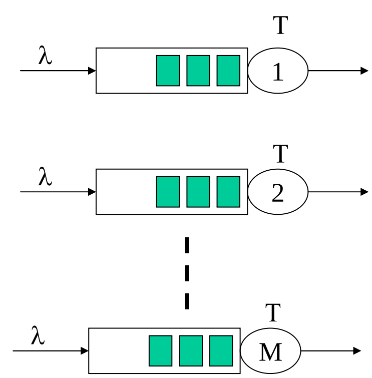
Fixed resource allocation

Fixed resource allocation

- System parameters
  - Fixed data rate R
  - Fixed message size P
  - Number of user M
  - If the whole bandwidth would be allocated just to single user, the transmission time of the packet would be P/R
  - Each user generates packets according to Poisson process with intensity λ

FDMA

- FDMA system
  - Bandwidth is divided equally among the M users
  - The system can be modeled as M parallel M/D/1 queueing systems
    - Data rate per user R/M
    - Service time T=MP/R
    - Laplace transform of the service time distribution: X(s)=E{exp(-sT)}= exp(-sT)



FDMA

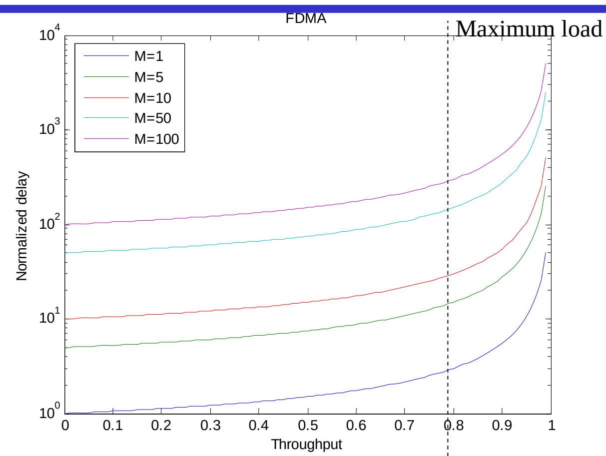
- Expected packet delay

$$D = W + \bar{X} = \bar{X} + \frac{1}{2}\frac{\lambda \overline{X^2}}{1-\lambda \bar{X}} = T + \frac{1}{2}\frac{\lambda T^2}{1-\lambda T} = T\left(1+\frac{1}{2}\frac{\rho}{1-\rho}\right) = \frac{MP}{R}\left(1+\frac{1}{2}\frac{\rho}{1-\rho}\right)$$

- Conflict free system: Throughput S = Utilization $\rho = \lambda T$
- Normalized delay = Expected delay / Deleay in single user system

$$\hat{D}_{FDMA} = \frac{D}{\mathrm{P/R}} = M\left(1+\frac{1}{2}\frac{S}{1-S}\right) = M\,\frac{2-S}{2(1-S)} = \frac{M}{2}\left(1+\frac{1}{1-S}\right)$$

FDMA



Delay is rather insensitive to the throughput

Delay increases quickly and cannot be tolerated

FDMA

- Delay distribution

$$D^*(s) = X^*(s)\frac{s(1-\rho)}{s-\lambda-\lambda X^*(s)} = \frac{s(1-\rho)e^{-sT}}{s-\lambda-\lambda e^{-sT}} = \frac{s(1-\rho)}{\lambda+(s-\lambda)e^{sT}}$$

- Moments

$$E\{D^m\} = (-1)^m \frac{d}{ds} D^*(s)\Big|_{s=0}$$

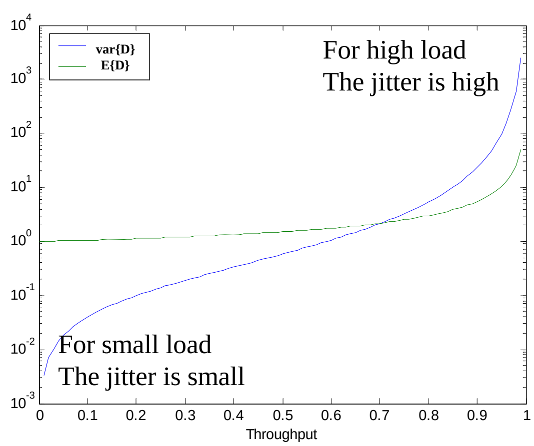
Calculation of the moments typically require (repeated) application of the Hôpital's rule

$$\lim_{s\to 0}\frac{f(s)}{g(s)} = \lim_{s\to 0}\frac{f'(s)}{g'(s)}$$

$$f(0)=0, g(0)=0$$

Variance of the packet delay

- Normalized on T=1



Variance of the packet delay

Symbolic tools such as Maple or Mathematica can be utilized to find the moments

Here is example for calculating variance of the packet delay using Matlab's Symbolic Toolbox (i.e. Maple)

```
syms s T lambda
rho=lambda*T;
D=s*(1-rho)/(lambda+(s-lambda)*exp(s*T));
dD=diff(D);         %Derivative
d2D=diff(dD);       %Second derivative
m1=-limit(dD,s,0);  %First moment
m2=limit(d2D,s,0);  %Second moment
varD=simplify(m2-m1^2) %Variance
S=0:.01:.99;        %Throughput vector
%Numerical solution
mean_delay=double(subs(subs(m1,T,1),lambda,S));
var_delay=double(subs(subs(varD,T,1),lambda,S));
%Plot results
semilogy(S,var_delay,S,mean_delay)
legend('var(D)','E{D}',2)
xlabel('Throughput')
```

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Flexible spectrum use

- World Radio Conference 2007 will identify new specrum for next generation communication systems (IMT-A)
- Tentative spectrum requirements may be ~100 MHz/operator
  - Very high peak data rate requirements (100 Mbit/s vehicular, 1Gbit/s nomadic)
- The available spectrum may not allow full spectrum to all operators
- Benefits from availability of full spectrum for a user
  - user impatience
  - the hardware is there, why not let the user benefit from it
- Spectrum sharing between operators becomes a viable option to investigate

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Flexible spectrum use

- Assume that available bandwidth is 100MHz allowing peak data rate of 100 Mbit/s for vehicular users
- The number of competing operators is 2
- Option 1:
  - Both operators get 50 MHz band and are not allowed to share the spectrum
- Option 2:
  - Operators share the spectrum (e.g. by sharing the hardware)

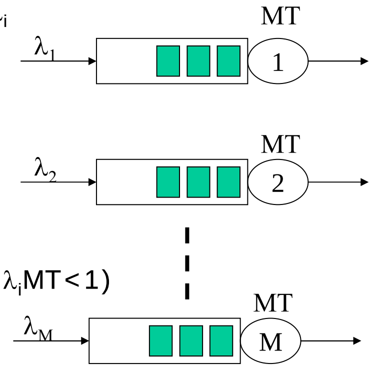
Licensed bands

- M operators
- Offered traffic of operator i: λ_i
- Packet delay

$$D_i = \overline{X} + \frac{1}{2}\frac{\lambda_i \overline{X^2}}{1-\lambda_i \overline{X}} = MT\left(1+\frac{1}{2}\frac{\lambda_i (MT)}{1-\lambda_i (MT)}\right)$$

- Maximum offered traffic ($S_i = \lambda_i MT < 1$)

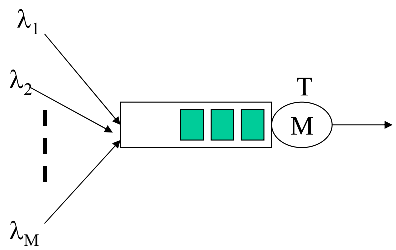
$$\lambda_i \le \frac{1}{MT}$$



Shared spectrum

- Aggregate traffic
  $\lambda = \Sigma_i \lambda_i$
- Packet delay

$$D = \bar{X} + \frac{1}{2}\frac{\lambda \overline{X^2}}{1-\lambda \bar{X}} = T\left(1+\frac{1}{2}\frac{\sum_i \lambda_i T}{1-\sum_i \lambda_i T}\right) = T\left(1+\frac{1}{2}\frac{\sum_i S_i}{1-\sum_i S_i}\right)$$

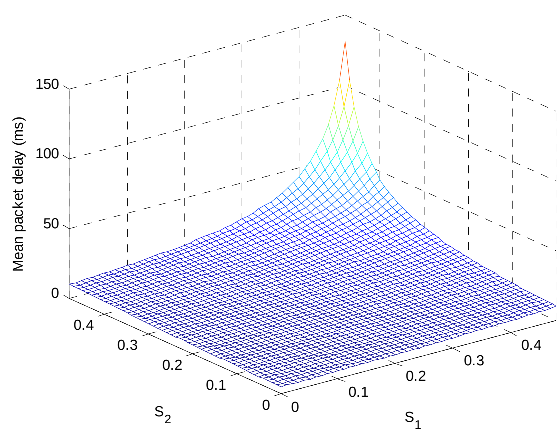


- Maximum offered traffic ($S=S_1+S_2<1$)

$$\sum_i \lambda_i < \frac{1}{T}$$

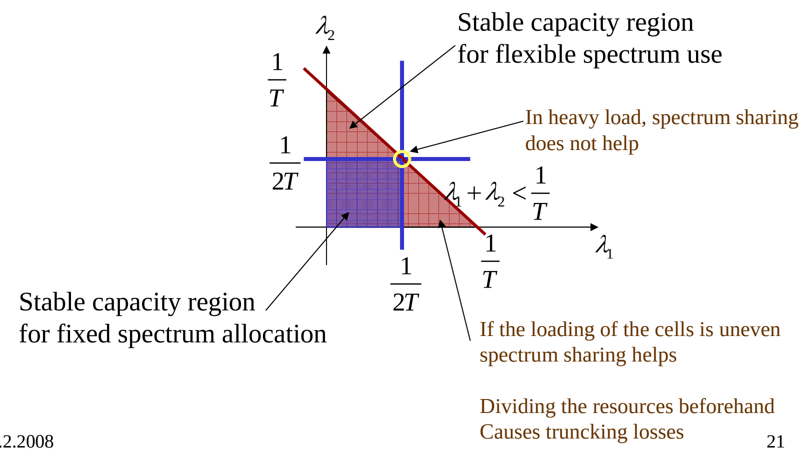
Shared spectrum

- Two operators share the spectrum ideally based on traffic
- Packet size: 65535 octets



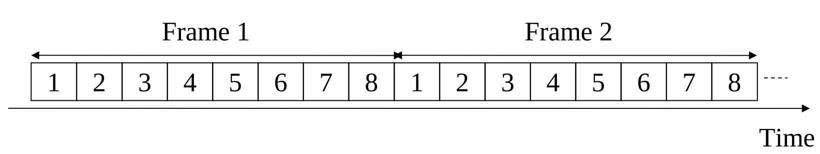
Shared spectrum

- M=2
  - Capacity region in terms of offered traffic



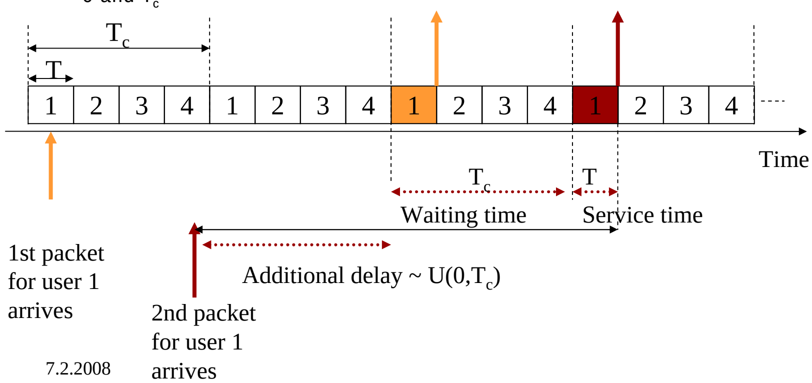
TDMA

- In time division multiple access, the time axis is divided into time slots which are preassigned to the different users.
- The slot assignment follow predefined pattern that repeat itself in a cycle or frame



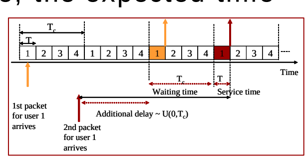
TDMA

- Assume that the MAC layer packet size is selected to be equal to the slot size T=P/R
- The frame size is T_c=MT
- Each packet in the queue will add T_c to the waiting time
- Arrival can take place at any time during the frame
- Poisson arrivals: Arrivals are uniformly distributed over the time axis => Remaining time till the beginning of next frame is uniformly distributed between 0 and T_c



TDMA

- Waiting time till the end of the frame is uniformly distributed between 0 and T_c. Hence, the expected time is



$$W_c = \int_0^{T_c} t \frac{1}{T_c} dt = \frac{1}{2} T_c = \frac{1}{2} MT$$

- Waiting time in the queue

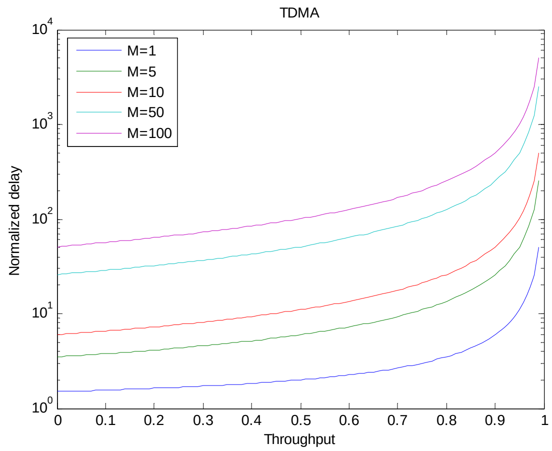
$$W_q = \frac{\rho}{2(1-\rho)} T_c = \frac{\rho}{2(1-\rho)} MT, \quad \rho = \lambda T_c = \lambda MT$$

- Service (transmission time) is T
- Expected packet delay

$$D = W_c + W_q + T = \frac{1}{2} MT + \frac{\rho}{2(1-\rho)} MT + T = T\left[1 + \frac{M}{2(1-\rho)}\right]$$

TDMA

- Normalized packet delay $\hat{D}_{TDMA} = \frac{D}{\text{P/R}} = 1 + \frac{1}{2(1-S)}, \quad S = \lambda \frac{MP}{R}$



TDMA vs FDMA

- Comparison between FDMA and TDMA

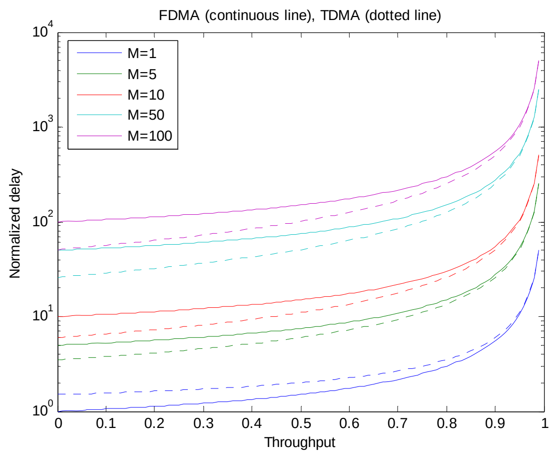
$$\hat{D}_{FDMA} = \frac{M}{2}\left(1 + \frac{1}{1-S}\right)$$

$$\hat{D}_{TDMA} = 1 + \frac{1}{2(1-S)}$$

$$\hat{D}_{FDMA} = \hat{D}_{TDMA} + \frac{P}{R}\left[\frac{M}{2} - 1\right] \geq \hat{D}_{TDMA}$$

  - **The expected delay of the TDMA is always less than that of FDMA for M>1**.
  - The difference grows linearly as a function of users
  - For high load the factor 1/(1-S) dominates, and the relative difference becomes smaller

TDMA vs FDMA



TDMA

- In general the messages are much bigger than MAC layer slot capacity.
- Hence transmission of a single message requires multiple MAC layer frames.
- Let $\tilde{L}$ denote the length of the message in time slots.
- The generating function of $\tilde{L}$ (z-trandform of its pdf) is thus

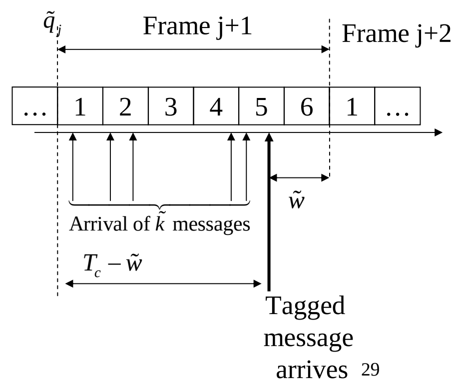
$$L(z) = \sum_{l=1}^{\infty} \Pr\left\{\tilde{L} = l\right\} z^l$$

$$\overline{L} = \frac{d}{dz} L(z)\bigg|_{z=1}, \quad \overline{L^2} + \overline{L} = \frac{d^2}{dz^2} L(z)\bigg|_{z=1}$$

- Messages arrive according to Poisson process with intensity λ

TDMA

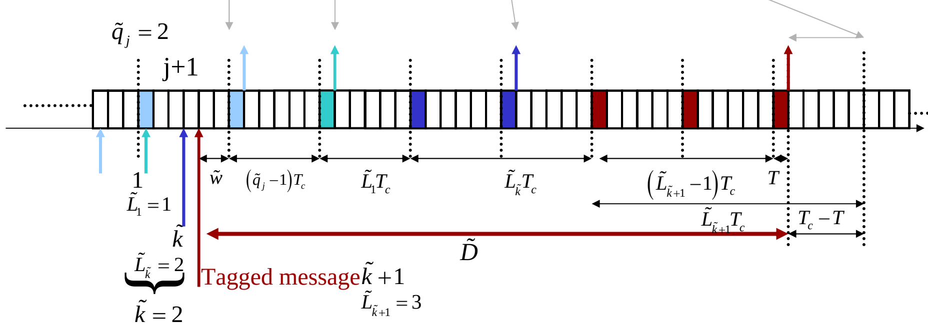
- Message delay $\tilde{D}$ is defined as the time elapsing between the message arrival epoch until the transmission of the last packet of the message is completed.
- Consider an arbitrary tagged message arriving at $T_c - \tilde{w}$ seconds after the beginning of the $(j+1)$st frame.
- Assume that the tagged message is the $(\tilde{k}+1)$st message arriving during that frame.
- The number of messages $\tilde{k}$ arriving before the tagged message follows Poisson distribution with parameter $\lambda(T_c - \tilde{w})$
- Let $\tilde{q}_j$ denote the number of packets waiting transmission at the beginning of the (j+1)st frame



TDMA

- Delay

$$\tilde{D} = \tilde{w} + \underbrace{\max\{\tilde{q}_j - 1, 0\} T_c + T_c \sum_{i=1}^{\tilde{k}} \tilde{L}_i + \tilde{L}_{\tilde{k}+1} T_c}_{\tilde{D}_{M/G/1}} + T - T_c$$



TDMA

- Uniformly distributed time till the beginning of the next frame
  - Probability distibution function $f_{\tilde{w}}(t) = \frac{1}{T_c}, \quad 0 \le t \le T$
  - Characteristic function (L-transform)

$$W^*(s) = E\left\{e^{-s\tilde{w}}\right\} = \int_0^\infty \frac{1}{T_c} e^{-st} dt = \frac{1-e^{-sT_c}}{sT_c}$$

- M/D/1 queue with service time expressed in terms of frame lengths $\rho=\lambda T_c$

$$D_{M/G/1}{}^*(s) = L^*(s)\frac{s(1-\rho)}{s-\lambda-\lambda L^*(s)}$$

- Characteristic function of the constant T-T_c

$$T^*(s) = E\left\{e^{-s(T-T_c)}\right\} = e^{-s(T-T_c)}$$

TDMA

- Packet delay $\tilde{D} = \tilde{w} + \tilde{D}_{M/G/1} + T - T_c$
- Characteristic function of the packet delay

$$\begin{aligned} D^*(s) &= E\left\{e^{-s\tilde{D}}\right\} = E\left\{e^{-s\tilde{w}}\right\} E\left\{e^{-s\tilde{D}_{M/G/1}}\right\} E\left\{e^{-s(T-T_c)}\right\} \\ &= W^*(s) D^*_{M/G/1}(s) T^*(s) \\ &= \frac{1-e^{-sT_c}}{sT_c} L^*(s) \frac{s(1-\rho)}{s-\lambda-\lambda L^*(s)} e^{-s(T-T_c)} \\ &= \frac{1-\rho}{T_c} \frac{1-e^{-sT_c}}{s-\lambda-\lambda L^*(s)} e^{s(T_c-T)} L^*(s) \end{aligned}$$

That is,

$$D^*(s) = \frac{1-\rho}{T_c} \frac{1-e^{-sT_c}}{s-\lambda-\lambda L^*(s)} e^{s(T_c-T)} L^*(s)$$

TDMA

- Expected message delay

$$D = E\left\{\tilde{w} + \tilde{D}_{M/G/1} + T - T_c\right\}$$

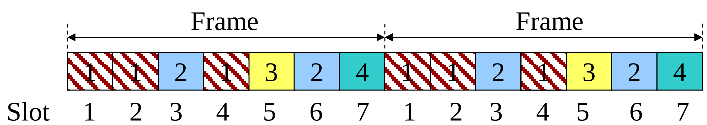
$$= \frac{d}{ds} D^*(s)\Big|_{s=0} = T_c\left(\bar{L} - \frac{1}{2}\right) + \frac{\lambda T_c^2 \overline{L^2}}{2\left(1 - \lambda T_c \bar{L}\right)} + T$$

- Normalized delay

$$\hat{D} = \frac{D}{T} = M\left(\bar{L} - \frac{1}{2}\right) + \frac{M\,\overline{L^2}}{\bar{L}} \frac{\rho}{1-\rho} + 1$$

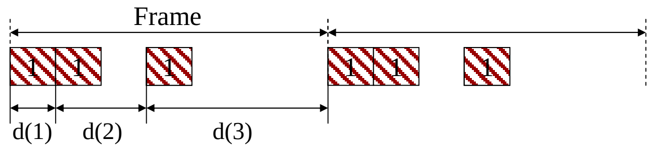
Generalized TDMA

- The allocation of single slot within the frame to each user is reasonable if the communication requirements are homogeneous.
- In data transmission, the required capacity of the users vary and it thus makes sense to allocate different number of slots to users based on their needs.
- Example:
  - User 1: 3 slots, User 2: 2 slots, User 3 and 4: 1 slot

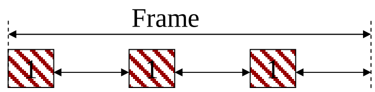


Generalized TDMA

- Let d(k) denote the time between two consecutive slots allocated to an user



- Hofri and Rosberg has shown that the message delay can be minimized by allocating the slots uniformly d(k)=d for all k (if possible)



Generalized TDMA

- Slot length: T=1
- Number of slots in a frame: M=24
- Number of slots allocated to an user: K=4
- Message size is uniformly distributed between 1 and 4
- It can be seen from the figure that the impact of slot assignment is not very high

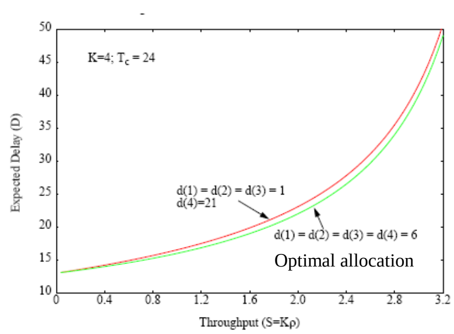


FIGURE 2.8: Throughput Delay for Generalized TDMA
$L(z) = (z+z^2+z^3+z^4)/4$

Radio channel

- Fast fading cause packet loss
- In pedestrian channel, the transmission time interval (TTI) / slot length is shorter than the coherence time of the channel and thus the packet loss process is correlated.
- In data channels, the effect of fading is mitigated by using Automatic Repeat reQuest (ARQ). This translates the packet loss to packet delay.
- Second generation cellular systems utilized traditional ARQ in which erroneous packet is discarded by the receiver and retransmitted by the transmitter
- Modern wireless networks utilizes hybrid ARQ, in which the receiver combines the original packet with the retransmitted copy of the packet (chase combining) or additional code bits provided by the transmitter (incremental redundancy)

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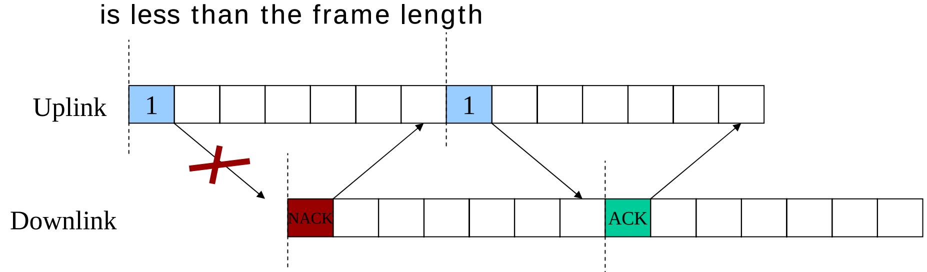
ARQ

- Assume for simplicity that the message is one MAC layer packet long and the MAC packet contains error detection coding such that the receiver is able to detect lost packets.
- In FDD systems, we can utilize time offset between uplink and downlink such that the ARQ feedback delay is less than the frame length



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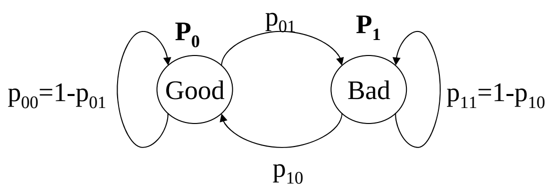
ARQ

- We could model the channel state as a Markov chain, in which each state corresponds to different block error rate BLER. (Gilbert-Elliot channel model)
- The simplest channel model consists of two sates: Good (no error) and Bad (packet lost)



- Block error probability $P_{BLER} = P_1$
- Average length of an error burst $B = \frac{1}{p_{10}}$

ARQ

- In steady state

$$\begin{cases} p_{01}P_0 = P_1 p_{10} \\ P_0 + P_1 = 1 \end{cases}$$

$$\Rightarrow P_1 = \frac{p_{01}}{p_{10}} P_0, \quad P_0 = \frac{1}{1+\frac{p_{01}}{p_{10}}}$$

- The service time distribution of the packet is geometric

$$\Pr\{L=l\} = \begin{cases} P_0 & l=1 \\ P_1 (1-p_{10})^{l-1} p_{10} & l = 2,3,4... \end{cases}$$

Channel was bad during the first TX attempt — stayed bad for *l*-1 slots — and finally became good again

ARQ

- Transmission time in terms of frames

$$\overline{L} = P_0 + P_1 \sum_{l=2}^{\infty} l\left(1-p_{10}\right)^{l-1} p_{10} = P_0 - p_{10}P_1 + P_1 \sum_{l=0}^{\infty} l\left(1-p_{10}\right)^{l-1} p_{10}$$

$$= P_0 - p_{10}P_1 + P_1 \frac{1}{p_{10}}$$

$$\overline{L^2} = P_0 + P_1 \sum_{l=2}^{\infty} l^2\left(1-p_{10}\right)^{l-1} p_{10} = P_0 - p_{10}P_1 + P_1 \sum_{l=0}^{\infty} l^2\left(1-p_{10}\right)^{l-1} p_{10}$$

$$= P_0 - p_{10}P_1 + P_1 \left(\frac{1}{p_{10}} + \frac{2\left(1-p_{10}\right)}{p_{10}^{\;2}}\right)$$

- Packet delay

$$D = T_c\left(\overline{L} - \frac{1}{2}\right) + \frac{\lambda T_c^2\, \overline{L^2}}{2\left(1-\lambda T_c \overline{L}\right)} + T$$

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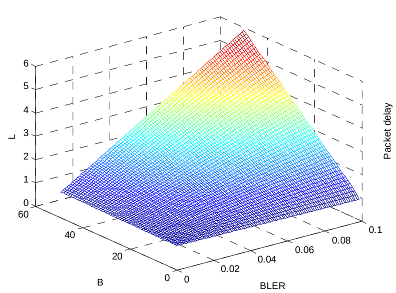
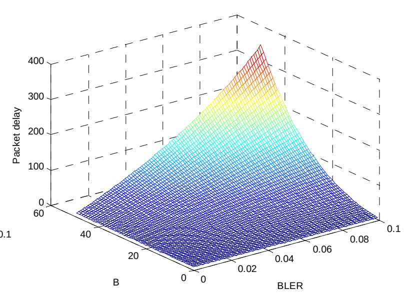
ARQ

- Retransmissions contribute to the load $\rho = \lambda T_c \overline{L}$





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CDMA

- Single rate CDMA system
  - Spreading factor is S=W/R
  - Transmit power control is utilized to requlate the receive power Q
  - Noise power is ν
  - Other-to-own cell interference ratio is i
  - Required Signal-to-noise+interference ratio (SINR) at the receiver is Γ

$$\Gamma = S\frac{Q}{(1+i)(M-1)Q+\upsilon}$$

$$\Rightarrow Q = \frac{\Gamma S^{-1}}{1+\Gamma S^{-1}}\frac{\upsilon}{1-(1+i)\frac{\Gamma S^{-1}}{1+\Gamma S^{-1}}M}$$

CDMA

- Maximum tolerable noise raise over thermal (RoT) (also known as noise raise) is η

$$\frac{(1+i)(M-1)Q+\upsilon}{\upsilon} \le \eta \Rightarrow M \le \frac{\eta-1}{\eta}\frac{1}{(1+i)\frac{\Gamma S^{-1}}{1+\Gamma S^{-1}}}$$

Hence, the RoT constraint defines the number of parallel channels.

- Now for given chip rate W, SINR-target Γ and RoT η, the system appears as M parallel channels with each having roughly portion 1/M of the overall bandwidth

$$R = \frac{\eta-1}{\eta}\frac{W}{\Gamma}\frac{1}{1-\frac{1}{M}}\frac{1}{M} \sim \frac{1}{M} \text{ for large } M$$

Hence, with fixed rate and number of channels, the packet delay performance of CDMA is similar to FDMA.

CDMA vs FDMA

- In modern CDMA systems, the data rate of the user is controlled by packet scheduler and the data rate can be set based on the current interference level (i(t) and M(t))
- This avoids the trunking losses caused by the fixed channel allocation used in FDMA
- Hence, CDMA is more flexible than FDMA

CDMA vs TDMA

- If dynamic conflict free packet scheduler is used also in TDMA type of slotted system, then both CDMA (simultanous transmissions of several users) and TDMA (allocating all the resources to signle user at the time) approaches has their merits. In brief, CDMA provides gains if the interference power is large and uncontrollable (uplink) while TDMA is beneficial if the interference can be kept small (downlink).
- More detailed analysis is provided in the course ***S-72.3260 Radio Resource Management Methods***