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Advanced Radio Transmission Methods

Part1: Linear Receivers for interfering channels

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Introduction to ISI and equaliztion

Baseband receiver

1. receiver (pulse shaping) filter
2. sampling
3. channel estimation in signal space
4. Inter-symbol interference mitigation in signal space
5. (soft) symbol detection
6. decoder

- Here we concentrate on 4
  - understanding equalization based on matrix operations in signal space
  - as opposed to pre-sampling frequency domain & filtering treatment prevalent in literature

Signal model in complex signal space

- There are N_r received signals, $\mathbf{y}$ is a $N_\mathrm{r} \times 1$ vector
- There are N_t transmitted symbols, $\mathbf{x}$ is a $N_\mathrm{t} \times 1$ vector
  - the symbols are independent, and normalized to have power 1: $\mathrm{E}\left\{\mathbf{x}\mathbf{x}^\mathrm{H}\right\} = \mathbf{I}$
  - The identity matrix is denoted by $\mathbf{I}$
- The noise $\mathbf{n}$ is additive white Gaussian, a $N_\mathrm{r} \times 1$ vector
  - noise covariance is $\mathrm{E}\left\{\mathbf{n}\mathbf{n}^\mathrm{H}\right\} = \mathrm{N}_0\ \mathbf{I}$
  - noise PDF of each component : $p(n) = \frac{1}{\pi \mathrm{N}_0} \mathrm{e}^{-|n|^2/\mathrm{N}_0}$
  - joint noise PDF: $p(\mathbf{n}) = \frac{1}{(\pi \mathrm{N}_0)^{N_\mathrm{r}}} \mathrm{e}^{-|\mathbf{n}|^2/\mathrm{N}_0}$ where $|\mathbf{n}|^2 = \mathbf{n}^\mathrm{H}\mathbf{n}$
- channel $\mathbf{H}$ is a $N_\mathrm{r} \times N_\mathrm{t}$ matrix, includes:
  - physical channel
  - pulse shaping filtering at Tx and Rx
  - Tx power level

Signal space example

- three received signals, two transmitted symbols:

Vector signal model

$$\begin{bmatrix} y_1 \\ y_2 \\ y_3 \end{bmatrix} = \begin{bmatrix} h_{11} & h_{12} \\ h_{21} & h_{22} \\ h_{31} & h_{32} \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} + \begin{bmatrix} n_1 \\ n_2 \\ n_3 \end{bmatrix}$$

consists of the three equations:

$$\begin{aligned} y_1 &= h_{11}x_1 + h_{12}x_2 + n_1 \\ y_2 &= h_{21}x_1 + h_{22}x_2 + n_2 \\ y_3 &= h_{31}x_1 + h_{32}x_2 + n_3 \end{aligned}$$

Signal model II

- The task of the receiver is to estimate the transmitted symbols from this set of equations.
- Based on the same signal model receivers for a muiltitude of cases may be understood
  - Multple input, multiple output channels (MIMO)
  - multiuser detection
  - equalization (ISI)

MIMO

- signal model represents reception in one symbol period
- N_t transmit antennas
- N_r receive antennas
- h_{mn} is channel between Tx antenna n and Rx antenna m
- Multiple symbols transmitted simultaneously: x_n from Tx antenna n
  - intentional non-orthogonality
  - interference between simultaneously transmitted symbols

Multiuser detection, multiple Rx antennas

- signal model represents reception in one symbol period
- N_t users *synchronously* transmitting on the same channel
- N_r receive antennas
- h_{mn} is channel between user n and Rx antenna m
- Multiple symbols received simultaneously: x_n from user n
  - intentional non-orthogonality (e.g. CDMA)
  - interference between users

Multiuser detection, CDMA

- signal model represents reception over multiple chip periods
- one spread symbol transmitted per user
- N_t users *synchronously* transmitting on the same channel
- N_r received chips, N_r = spreading factor
- h_{mn} is channel times spreading code of user n in received chip m
- Multiple symbols received simultaneously: x_n from user n
  - intentional non-orthogonality
  - interference between users

Channel convolution, ISI and equalization

Transmitted Base band signal

- Follow path of individual symbol from transmitter, through air interface to base band receiver.

Notation:

- Transmitter pulse shaping filter $f_\mathrm{t}(t)$
- Receiver (pulse shaping) filter $f_\mathrm{r}(t)$
- channel impulse response $c(t)$
- symbol interval T

Transmitted base band (equivalent low-pass) signal,
independent symbols transmitted with interval T using pulse shape f_t

$$x(t) = \sqrt{P} \sum_m x_m \; f_\mathrm{t}(t - mT)$$

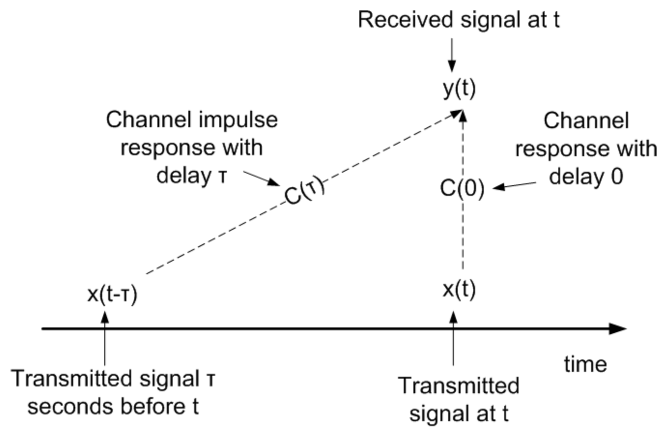
- transmit power P, symbols x_m normalized to unit power
- transmit (and receive) filter should have compact support - causality

Received base band signal

$$y_b(t) = \int_{-\infty}^{\infty} d\tau \; x(t-\tau) \; c(\tau) + n(t) = (x * c)(t) + n(t)$$

- Received BB signal is a convolution of channel and transmitted signal
- at each time instant
  - received signal sum of multiple delayed copies of transmitted signals
  - delays caused by channel impulse response
- channel impulse response is causal
- propagation time difference absorbed into definition of c(t)

Channel convolution



AWGN process

- AWGN process $n(t)$ has the auto-correlation

$$\mathrm{E}\left\{n(t)\ n(t')^*\right\} = \mathrm{N}_0\ \delta(t-t')$$

  - $\delta(t-t')$ is the Dirac delta-function:

$$\int_{-\infty}^{\infty} dt\ f(t)\ \delta(t-t') = f(t')$$

- below, limits of integrals generally dropped

Example: Flat fading (single-tap) channel

- channel impulse response

$$c(t) = \delta(t-d)$$

  - d is propagation delay between transmitter and receiver
- received signal becomes

$$y_b(t) = x(t-d) + n(t)$$

  - delayed copy of transmitted signal

Receive (pulse shaping) filtered signal

- received signal after filtering

$$\begin{aligned} y(t) &= \int d\tau'\ y_b(t-\tau')f_{\mathrm{r}}(\tau') \\ &= \int d\tau\ \underbrace{\int d\tau'\ f_{\mathrm{r}}(\tau')\ x(t-\tau'-\tau)}_{\text{Rx filtered Tx signal } x_{\mathrm{r}}(t-\tau)}\ c(\tau) + \underbrace{\int d\tau'\ n(t-\tau')f_{\mathrm{r}}(\tau')}_{\text{Rx filtered noise } n_{\mathrm{r}}(t)} \end{aligned}$$

- the part due to Rx filtered Tx signal:

$$x_{\mathrm{r}}(t) = \sqrt{P}\sum_m x_m \int d\tau'\ f_{\mathrm{r}}(\tau')\ f_{\mathrm{t}}(t-mT-\tau') \equiv \sqrt{P}\sum_m x_m\ f(t-mT)$$

  - convolution of the Rx and Tx filters: combined Rx-Tx pulse shaping filter

$$f(t) = (f_{\mathrm{r}} * f_{\mathrm{t}})\,(t) = \int d\tau'\ f_{\mathrm{r}}(\tau')\ f_{\mathrm{t}}(t-\tau')$$

Receive (pulse shaping) filtered signal II

- The received signal after pulse shaping is

$$y(t) = \sum_m x_m \sqrt{P} \int d\tau\, f(t-mT-\tau)\, c(\tau) + n_{\mathrm{r}}(t) \equiv \sum_m x_m\, h(t-mT) + n_{\mathrm{r}}(t)$$

- convolution of channel impulse response and combined Tx-Rx pulse shaping filters is

$$h(t) = \sqrt{P}\ \left(f_{\mathrm{t}} * f_{\mathrm{r}} * c\right)(t) = \sqrt{P} \int d\tau \int d\tau'\ f_{\mathrm{t}}(t-\tau-\tau')\ f_{\mathrm{r}}(\tau')\ c(\tau)$$

Sampling

- After Rx filtering, signal is sampled:

$$\begin{aligned} y_k &= y(kT'+\Delta) = \sum_m x_m\ h(kT'-mT+\Delta)\ +\ n_{\mathrm{r}}(kT'+\Delta) \\ &= \sum_m h_{km}\ x_m\ +\ n_k \end{aligned}$$

- sampling interval $T' \leq T$
  - symbol-spaced Tapped Delay Line: $T' = T$
  - fractionally spaced TDL: $T' < T$
- sampling instances $kT' + \Delta$,
  - Δ reflects the insecurity in timing
- channel coefficients h_{km}
- noise samples n_k

Symbol-spaced sampling

- One received sample per transmitted symbol

$$y_k = h_{kk} x_k + \underbrace{\sum_{m \neq k} h_{km}\ x_m}_{\text{ISI}}\ +\ n_k$$

- if $h_{km} \neq h\delta_{km}$, there is Inter-Symbol-Interference (ISI)
- With symbol-spaced sampling, the channel taps are

$$h_{km} = \sqrt{P} \int d\tau\ f\Big((k-m)T + \Delta - \tau\Big)\ c(\tau)$$

- In flat fading $h_{km} = \sqrt{P}\ f\Big((k-m)T + \Delta - d\Big)$
- Nyqvist criterion: $f\Big((k-m)T + \Delta - d\Big) = \delta_{km}$

Tx-Rx filter design

- The combined Tx & Rx filters should be designed to minimize ISI
- The Tx filter should be designed to optimize bandwidth usage
- If f_t is rectangular pulse, there is no ISI
  - ⇒ extensive out-of band emissions at Tx
  - ⇒ pulse shaping at Tx is required
- family of pulse shapes f
  satisfying Nyqvist criterion:
  Raised Cosine pulses

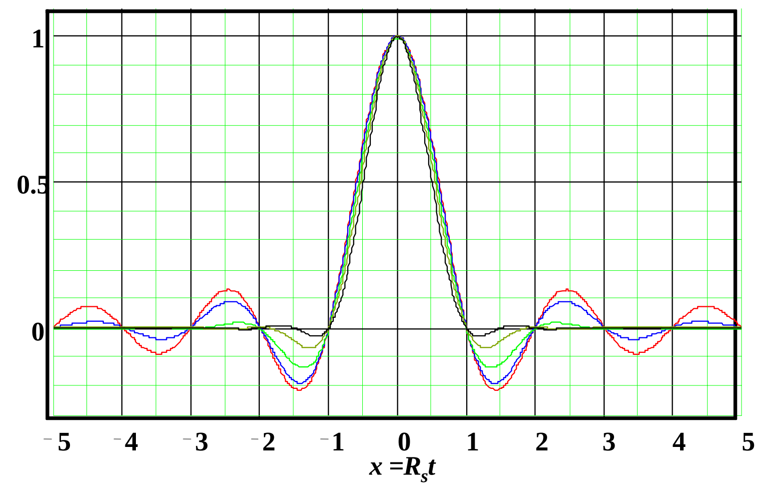
Raised Cosine pulse shaping

$$f_{\mathrm{RC}}(t) = \frac{\sin(\pi t/T)}{\pi t/T} \cdot \frac{\cos(\alpha \pi t/T)}{1-(2\pi t/T)^2}$$

- If f is a Raised Cosine pulse, and $\Delta = d$
  - no ISI
  - $h_{km} = \sqrt{P}\delta_{km}$.
- vulnerability to sampling inaccuracy
- When non roll-off ($\alpha = 0$) f is sinc-pulse
  - ISI with non-exact timing infinite
- With roll-off ($\alpha > 0$),
  - RC tolerates some timing inaccuracy ("eye open").

Raised cosine pulse

Raised cosine filter impulse responses: α = 0, 0.25, 0.5, 0.75, 1



Root-Raised Cosine (RRC)

- noise power at Rx is minimized if f distributed evenly between Tx and Rx: root-raised cosine $f_{\mathrm{t}} = f_{\mathrm{r}} = f_{\mathrm{RRC}}$

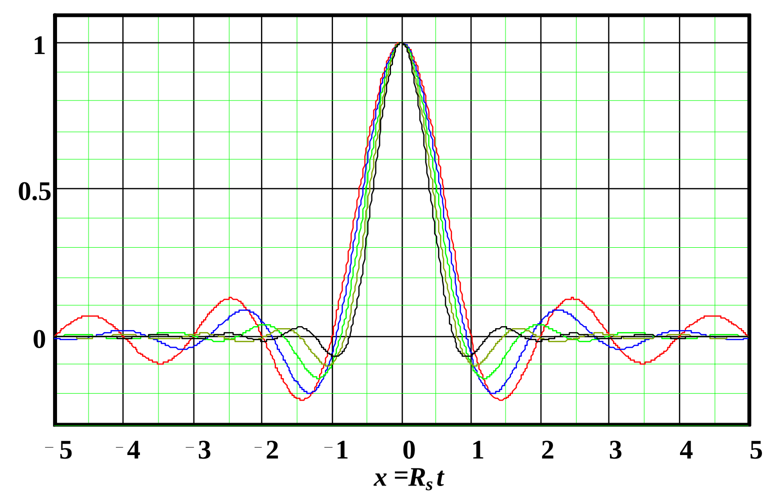
$$f_{\mathrm{RRC}} = \frac{\sqrt{1/T}}{1-(4\alpha t/T)^2}\left(\frac{\sin(\pi(1-\alpha)t/T)}{\pi t/T} + \frac{4\alpha}{\pi}\cos(\pi(1+\alpha)t/T)\right)$$

$$f_{\mathrm{RRC}} * f_{\mathrm{RRC}} = f_{\mathrm{RC}}$$

- RRCs with delay kT are orthogonal functions
- with RRC at Tx \& Rx
  - matched filter:
    - receiver matched to orthogonal Tx waveform
      - no noise colouring (see two slides ahead)

RRC pulse

Root raised cosine filter impulse responses: α = 0, 0.25, 0.5, 0.75, 1



Noise sample covariance

★ sampled, Rx-filtered noise process

$$n_k = n_{\rm r}(kT+\Delta) = (n * f_{\rm r})\,(kT+\Delta)$$

★ noise covariance

$$\begin{aligned}\mathrm{E}\{n_m n_k^*\} &= \int d\tau \int d\tau'\; f_{\rm r}(\tau)\; f_{\rm r}(\tau')\; \underbrace{\mathrm{E}\{n(mT+\Delta-\tau)\; n^*(kT+\Delta-\tau')\}}_{=\mathrm{N}_0\;\delta\left((m-k)T\right)} \\ &= \mathrm{N}_0\,(f_{\rm r} * f_{\rm r}^*)\left((m-k)T\right)\end{aligned}$$

★ If $f_{\rm r}$ is RRC, we have

$$\mathrm{E}\{n_m n_k^*\} = \mathrm{N}_0\;\delta_{mk}$$

• RRC filtering at Rx does not color the noise

Optimal filtering in flat fading

- In frequency flat channel, RRC filtering at Tx and Rx is optimum
  - removes ISI
  - maximizes SNR at the output of the sampler
  - does not color noise
  - consequence of RRC orthogonality
- If some other Rx filter is used, one may have to whiten the noise before further processing.

Frequency selective channel

- In frequency selective fading, Rx-filtering with possible noise whitening is done before sampling
- If Rx-filter is RRC, noise samples are uncorrelated, and base band detector may operate directly on them
- Combined Tx-Rx still best to be RC, so that the filters do not needlessly increase ISI
- Filtering w.r.t. to the channel impulse response may be done before sampling, or after sampling in signal space
- In latter case, the channel taps h_{km} estimated in signal space
  - h_{km} includes channel and filter impulse responses
- Equalization of ISI done on the samples in signal space

Fractional spaced sampling

- Symbol-spaced sampling works when the receiver knows the sampling time perfectly.
- If imperfect or random sampling timing
  - oversampling (at least Nyqvist rate) required to fully be able to reproduce the band-limited signal.
  - the additional samples of Nyqvist sampling compared to symbol-spaced sampling can be understood to be required to estimate transmitter pulse timing
- receivers for oversampled signals: fractional spaced equalizers
  - not discussed here

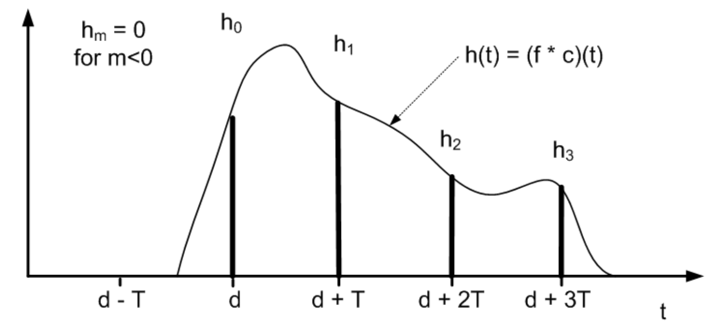
Signal model for Equalization

- In signal model after sampling assume that
  - channel taps are constant $h_{km} = h_{k-m}$
  - causality realized as $h_m = 0$ for $m < 0$
  - there are L non-zero channel taps $\{h_m\}_{m=0}^{L-1}$
- we have

$$y_k = h_0 \ x_k + \sum_{m=1}^{L-1} h_m \ x_{k-m} \ + \ n_k$$

where n_k is AWGN.

Tapped Delay Line



Signal model for Equalization: finite block

- For finite block of N_t symbols, this can be written in the vector form

$$\mathbf{y} = \mathbf{H}\mathbf{x} + \mathbf{n}$$

where the channel matrix is a Toeplitz matrix.
- Example: block of $N_t = 5$ Rx symbols, $L = 3$ channel taps

$$\begin{bmatrix} y_1 \\ y_2 \\ y_3 \\ y_4 \\ y_5 \end{bmatrix} = \begin{bmatrix} h_2 & h_1 & h_0 & 0 & 0 & 0 & 0 \\ 0 & h_2 & h_1 & h_0 & 0 & 0 & 0 \\ 0 & 0 & h_2 & h_1 & h_0 & 0 & 0 \\ 0 & 0 & 0 & h_2 & h_1 & h_0 & 0 \\ 0 & 0 & 0 & 0 & h_2 & h_1 & h_0 \end{bmatrix} \begin{bmatrix} x_{-1} \\ x_0 \\ x_1 \\ x_2 \\ x_3 \\ x_4 \\ x_5 \end{bmatrix} + \begin{bmatrix} n_1 \\ n_2 \\ n_3 \\ n_4 \\ n_5 \end{bmatrix}$$

- E.g.: $y_5 = h_0 x_5 + \underbrace{h_1 x_4 + h_2 x_3}_{\text{ISI}} + n_5$

Signal model for Channel Equalization

- signal model represents reception of sequence of symbol periods
- N_t symbols in a transmission block that is equalized
- N_r samples at the receiver
- h_{mn} is channel between transmitted symbol n and received sample m
- Multiple symbols received simultaneously: x_n from user n
  - non-orthogonality caused by the channel (and possibly filters)
  - inter-symbol interference