S72.3280

Tutorial 5

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Exercise 1

Assume a the channel is described as a tapped delay line.
The channel response from moments
-2 … 2 is $h = \begin{bmatrix} 0.1 & 0.5 & 1 & 0.5 & 0.1 \end{bmatrix}$

We are interested in equalisation of the bit value at the moment 0. $h[0] = 1$

We know loglikelihood ($L_{c,k}$) values for the bits (from the decoder output)

$$L_{c,k} = \begin{bmatrix} -3.1 & 0.3 & 4.1 & -2.1 & 1.1 \end{bmatrix}$$

The received symbol at moment $y(0) = -0.1$

Equalize the bit at the position 0 by using the soft bit values

Solution 1

We calculate the soft bit values from the loglikelihood values of the bits

$$\hat{u} = 1p(u=1) - 1p(0=-1)$$

$$= \frac{\mathrm{e}^{L_{c,k}}}{1+e^{L_{c,k}}} - \frac{\mathrm{e}^{0}}{1+e^{L_{c,k}}} = \tanh\left(\frac{L_{c,k}}{2}\right)$$

For our bits this gives a vector.

The channel model is given as

$$y_0 = \sum_{l=-2}^{2} h_l \hat{x}_{-l} + n$$

The equalized estimate of the bit becomes then

$$h_0 \overline{x}_0 = y_0 - \sum_{\substack{l=-2 \\ l \neq \grave{} 0}}^{2} h_l \hat{x}_{-l}$$

	-2	-1	0	1	2
Lc	-3.1	0.3	4.1	-2.1	1.1
Soft_bits	-0.23	-0.067	0.91	-0.44	0.082

The estimate contains additional noise and remaining interference. By assuming these to have Gaussian distribution we calculate the variance of the equalized signal to be

$$\sigma^2 = \sigma_n^2 + \sum_{\substack{l=-2 \\ l\neq`0}}^{2} h_l \left(1-\left(\hat{x}_{-l}\right)^2\right) = 0.50$$

The loglikelihood value is calculated from the Gaussian distribution

$$L_{out} = \frac{2h(0)}{\sigma^2}\overline{x} = 0.26$$

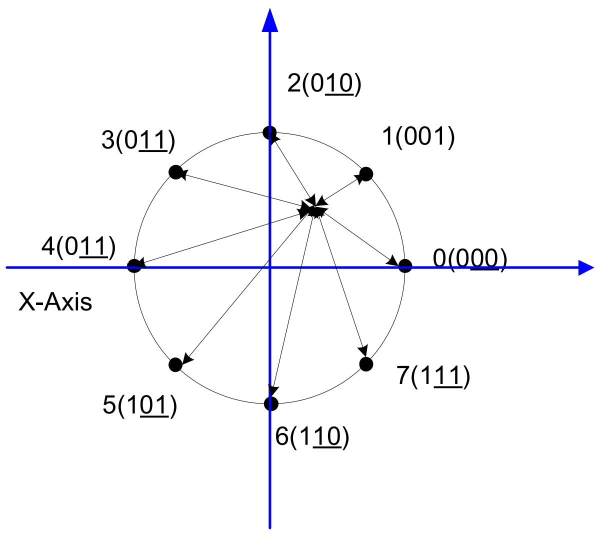
If needed we can calculate also a output soft bit value $\tanh\left(\frac{L_{out}}{2}\right) = 0.12$

Exercise 2

A system uses 8 psk modulation (figure below) and the channel is modelled as AWGN with SNR= 4 dB.

The received signal value is $r = 0.7+0.6\ j$.

Calculate a posterior probability for each possible constellation point.



Solution 2

The a posterior probability is calculated as

$$p(Y \mid X)$$

where Y is the received noisy symbol value and X stands for a possible constellation point.

For the constellation points we have 8 possible positions in the complex plain.

$$x_i = x_{i,real} + j \cdot x_{i,imag}$$

where i is the index of the particular constellation point.

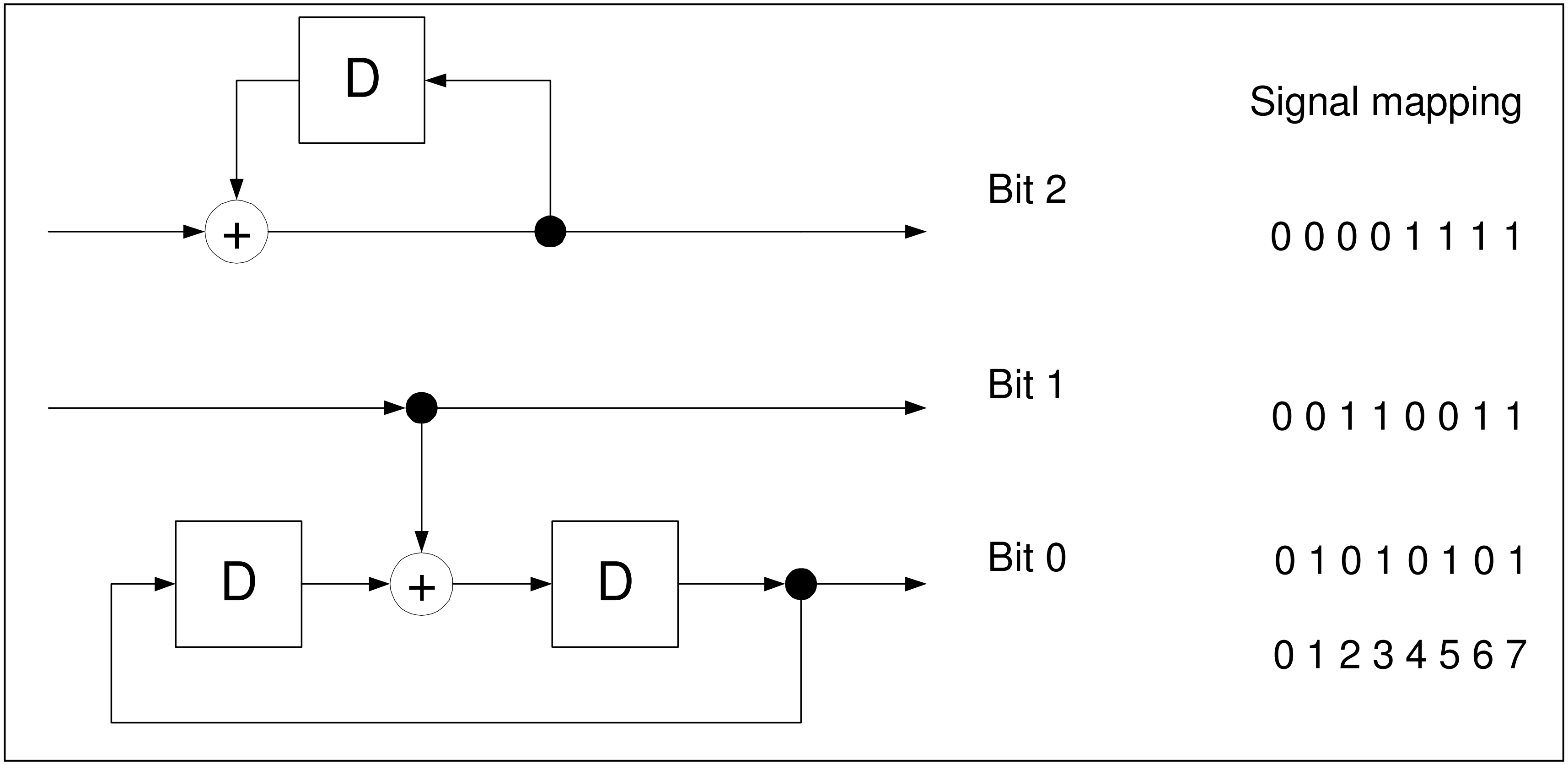
The probability that some particular constellation point was transmitted can be evaluated as a multiplication of two independent Gaussian distributions – one for the real part and one for the imaginary part.

$$p\left(y \mid x_i\right) = \frac{1}{\sqrt{2\pi}\sigma} e^{-\frac{\left(y_{real} - x_{i,real}\right)^2}{2\sigma^2}} \cdot \frac{1}{\sqrt{2\pi}\sigma} e^{-\frac{\left(y_{imag} - x_{i,imag}\right)^2}{2\sigma^2}} = \frac{1}{2\pi\sigma^2} e^{-\frac{\left|y - x_i\right|^2}{2\sigma^2}}$$

x_i	$p(y \mid x_i)$	$\ln(p(y \mid x_i))$	$\ln \frac{p(y \mid x_i)}{\sum_i p(y \mid x_i)}$
1	0.13	*-2.026*	*-2.34*
0.701+0.7071j	1.19	*0.1768*	*-0.1408*
0+1j	0.048	*-3.0307*	*-3.34*
-0.701+0.7071j	0.000057	*-9.7697*	*-10.09*
-1	0.000…	*-16.0925*	*16.41*
-0.701-0.7071j	0.000…	*-18.2953*	*18.61*
0-1j	0.000…	*-15.0878*	*-15.41*
0.701-0.7071j	0.000237	*-8.3488*	*-8.67*

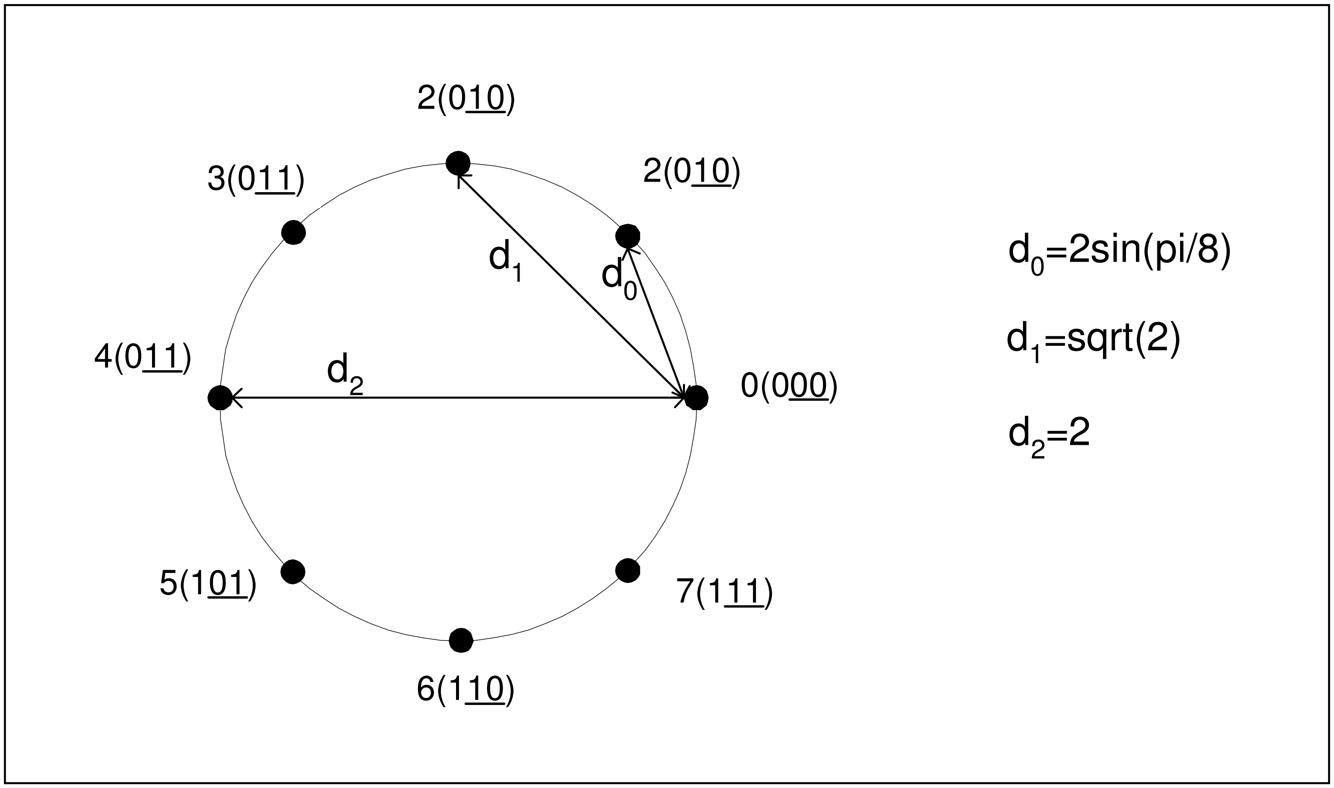
Exercise 3

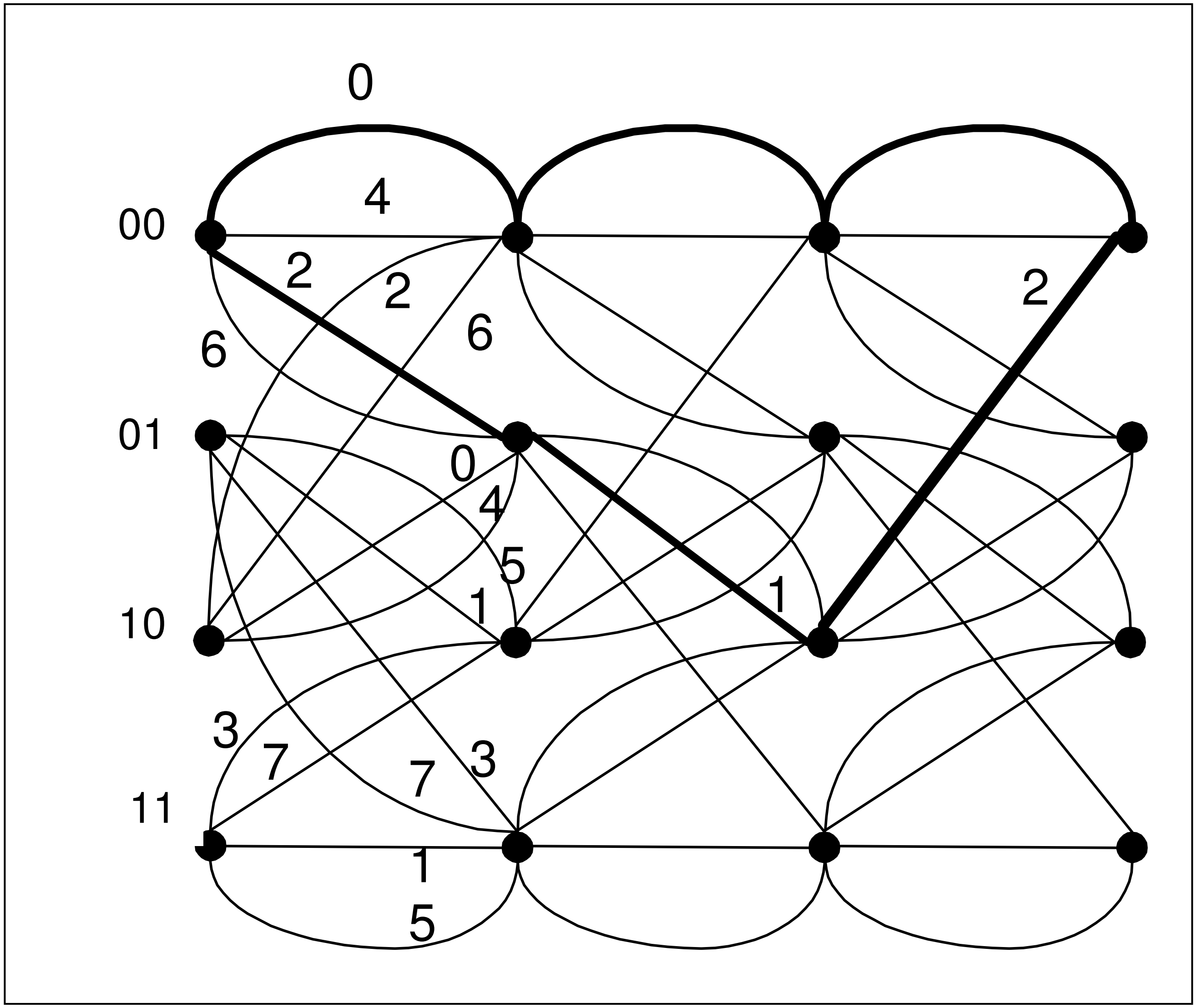
TCM mapping



Given the trellis encoder on the figure above.

Label the signals and determine the squared inter signal distance and average signal energy.


2(010)
3(011)
2(010)
d1
d0
4(011)
d2
0(000)
5(101)
7(111)
6(110)
d0=2sin(pi/8)
d1=sqrt(2)
d2=2



The transition trellis of the code is

The minimum free distance is minimal from the parallel paths and diverging paths.

$$d_{free} = \min\left\{d_2; \sqrt{d_1^2 + d_0^2 + d_1^2}\right\}$$

$$= \min\left\{2; \sqrt{2 + \left(2\sin\frac{\pi}{8}\right) + 2}\right\} = 2$$

Exercise 4

By using encoder in previous exercise

Encode the bit sequence [1 0 0 1 1 1]

Draw the path trough the trellis for both decoders.

The transmitted complex bits are $\begin{bmatrix} i & -\frac{1}{\sqrt{2}}(1+i) & i \end{bmatrix}$

the received symbol values are

[0.1546 + 1.1110i -0.6984 - 1.1119i   0.5504 + 1.0893i]

Calculate the estimate of the ML sequence by using Viterbi algorithm

Solution 4

We arrange the input bits into pairs where in the first row is every second bit and in the second row every first bit.

0 1 1

1 0 1

We feed these pairs of bits into the encoder and generate output

time

0 1 0

1 0 1 $\Rightarrow 252$

0 1 0

Calculate the maximum likelihood path probability trough the trellis

Since we know that we start from state 00 our calculations are simplified. We have to evaluate the probabilities only for transitions from state 00.

We are using Viterbi algorithm and accordingly to that sum together the path weights in the trellis. At each merging trellis node we remember only the path that has higher weight.

The weights to the edges are assigned accordingly to the probabilities of corresponding symbols. Since I and Q branches are independent we can calculate the weight as multiplication of two independent Gaussian probabilities.

$$p(y_1, y_2 \mid x_1, x_2) = \frac{1}{2\pi\sigma\sigma}\exp\left(-\frac{(y_1 - x_1)^2}{2\sigma^2} - \frac{(y_2 - x_2)^2}{2\sigma^2}\right)$$

Were we assume that the noise variance is same for both branches. Taking logarithm and removing the terms that are equal for every symbol we can express the weight as:

$$\frac{1}{\sigma^2}\left(x_1 m_1 + x_2 m_2\right).$$

We have to calculate this value for each possible transition. Since term $\frac{1}{\sigma^2}$ is scaling constant common for all the symbols we drop it from subsequent calculations.

In first stage we have four possible transitions from 00 state to 00 state with symbols [0 2] and corresponding weights [0.1546 – 0.1546] and to state 01 with symbols [4 6] and weights [1.111 – 1.111].

In interval 2 we have transitions from state 00 and 01.

00 -> 00

Initial state 00 to	Sumbols	Weight	Weight along the path
00	[0 2]	-0.6984 0.6984	-0.5438 0.8530
01	[4 6]	-1.1119 1.1119	-0.9573 1.2665

Initial state 01 to	Symbols	Weight [symbol1 symbol2]	Weight along the path
10	[1 5]	[-1.2801 1.2801]	[-0.1691 2.3911]
11	[3 7]	[-0.2924 0.2924]	[-0.8186 1.4034]

In the stage 3 we have to calculate the weight for all the transitions. In this stage we have to select also the maximum value from different merging paths.

The weights for each symbol are.

0.5504 1.1594 1.0893 0.3811 -0.5504 -1.1594 -1.0893 -0.3811

The weights after merging we get in each state the following weights

Sate	00	01	10	11
Weight	3.48	2.94	2.42	2.56

We see that the weight at the state 00 is highest and the path corresponding to path ending in this state is trough the states [00 01 10 00] and symbol sequence

[2 5 2]. This symbol sequence is generated by the bit pairs

$$\begin{bmatrix} 0 & 1 & 1 \\ 1 & 0 & 1 \end{bmatrix}$$

We see that the decoder has found the correct sequences.