S72.3280
Tutorial 5
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Exercise 1

Assume a the channel is described as a tapped delay line.
The channel response from moments
-2 … 2 is $h = [0.1 \quad 0.5 \quad 1 \quad 0.5 \quad 0.1]$

We are interested in equalisation of the bit value at the moment 0. $h[0] = 1$

We know loglikelihood ($L_{c,k}$) values for the bits (from the decoder output)

$L_{c,k} = [-3.1 \quad 0.3 \quad 4.1 \quad -2.1 \quad 1.1]$

The received symbol at moment $y(0) = -0.1$
Equalize the bit at the position 0 by using the soft bit values

Solution 1

We calculate the soft bit values from the loglikelihood values of the bits

$$\hat{u} = 1p(u = 1) - 1p(0 = -1)$$

$$= \frac{e^{L_{c,k}}}{1 + e^{L_{c,k}}} - \frac{e^{0}}{1 + e^{L_{c,k}}} = \tanh\left(\frac{L_{c,k}}{2}\right)$$

For our bits this gives a vector.
The channel model is given as

$$y_0 = \sum_{l=-2}^{2} h_l \hat{x}_{-l} + n$$

The equalized estimate of the bit becomes then

$$h_0 \overline{x}_0 = y_0 - \sum_{\substack{l=-2 \\ l \neq 0}}^{2} h_l \hat{x}_{-l}$$

	-2	-1	0	1	2
Lc	-3.1	0.3	4.1	-2.1	1.1
Soft_bits	-0.23	-0.067	0.91	-0.44	0.082

The estimate contains additional noise and remaining interference. By assuming these to have Gaussian distribution we calculate the variance of the equalized signal to be

$$\sigma^2 = \sigma_n^2 + \sum_{\substack{l=-2 \\ l \neq 0}}^{2} h_l \left(1 - \left(\hat{x}_{-l}\right)^2\right) = 0.50$$

The loglikelihood value is calculated from the Gaussian distribution

$$L_{out} = \frac{2h(0)}{\sigma^2} \overline{x} = 0.26$$

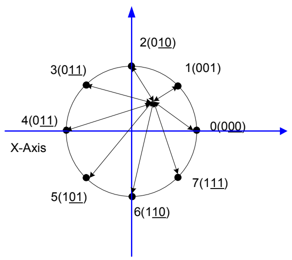
If needed we can calculate also a output soft bit value $\tanh\left(\frac{L_{out}}{2}\right) = 0.12$

Exercise 2

A system uses 8 psk modulation (figure below) and the channel is modelled as AWGN with SNR= 4 dB.

The received signal value is $y = 0.7+0.6\,j$.

Calculate a posterior probability for each possible constellation point.



Solution 2

The a posterior probability is calculated as

$p(Y \mid X)$

where Y is the received noisy symbol value and X stands for a possible constellation point.

For the constellation points we have 8 possible positions in the complex plain.

$x_i = x_{i,real} + j \cdot x_{i,imag}$

where i is the index of the particular constellation point.

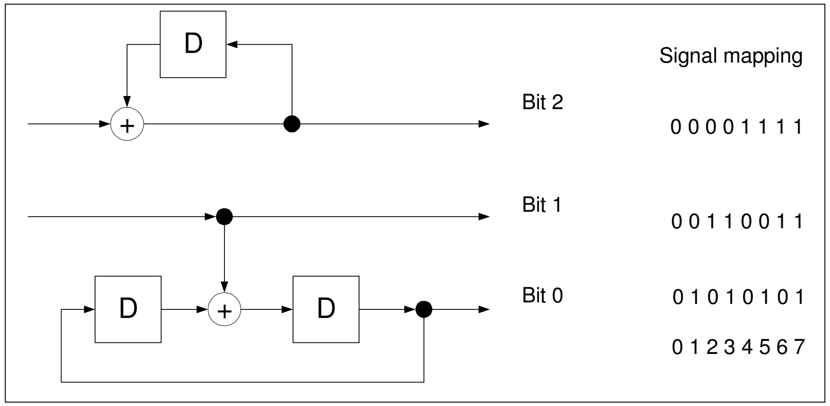
The probability that some particular constellation point was transmitted can be evaluated as a multiplication of two independent Gaussian distributions – one for the real part and one for the imaginary part.

$$p(y \mid x_i) = \frac{1}{\sqrt{2\pi}\sigma} e^{-\frac{(y_{real}-x_{i,real})^2}{2\sigma^2}} \cdot \frac{1}{\sqrt{2\pi}\sigma} e^{-\frac{(y_{imag}-x_{i,imag})^2}{2\sigma^2}} = \frac{1}{2\pi\sigma^2} e^{-\frac{|y-x_i|^2}{2\sigma^2}}$$

x_i	$p(y \mid x_i)$	$\ln(p(y \mid x_i))$	$\ln \frac{p(y \mid x_i)}{\sum_i p(y \mid x_i)}$
1	0.13	*-2.026*	*-2.34*
0.701+0.7071j	1.19	*0.1768*	*-0.1408*
0+1j	0.048	*-3.0307*	*-3.34*
-0.701+0.7071j	0.000057	*-9.7697*	*-10.09*
-1	0.000…	*-16.0925*	*16.41*
-0.701-0.7071j	0.000…	*-18.2953*	*18.61*
0-1j	0.000…	*-15.0878*	*-15.41*
0.701-0.7071j	0.000237	*-8.3488*	*-8.67*

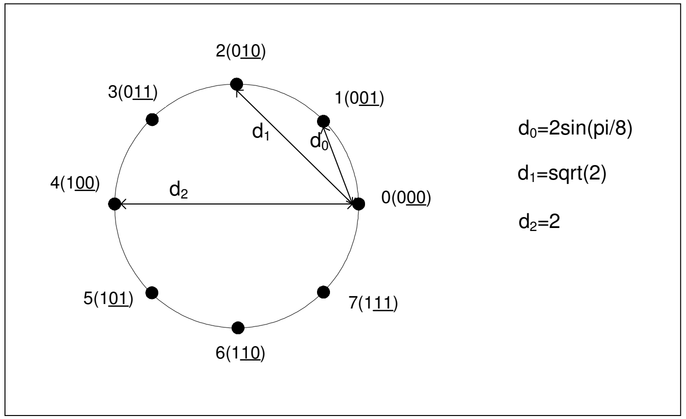
Exercise 3

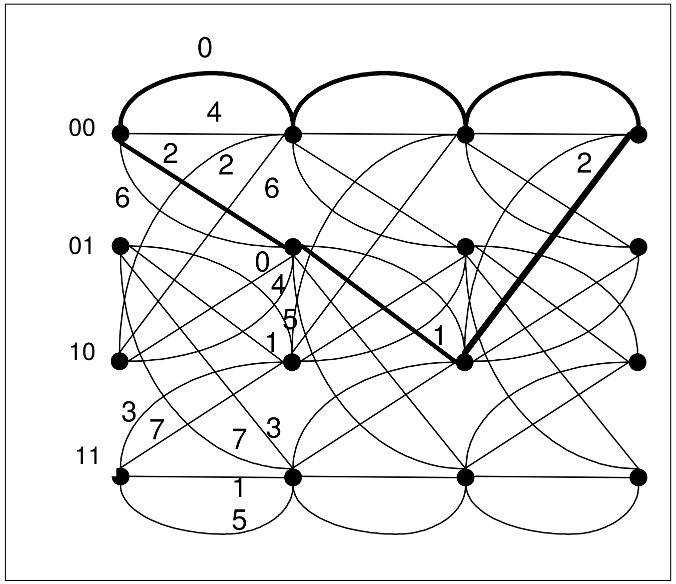
TCM mapping



Given the trellis encoder on the figure above.

Label the signals and determine the squared inter signal distance and average signal energy.





The transition trellis of the code is

The minimum free distance is minimal from the parallel paths and diverging paths.

$$d_{free} = \min\left\{d_2; \sqrt{d_1^2 + d_0^2 + d_1^2}\right\}$$

$$= \min\left\{2; \sqrt{2 + \left(2\sin\frac{\pi}{8}\right)^2 + 2}\right\} = 2$$