



S-72.3320 Advanced Digital Communication (4 cr)

# S.72-3320 Advanced Digital Communication (4 cr)

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- ◆ **Study modules:** Examination /Tutorials (voluntary) /Project work
- ◆ **NOTE:** Half of exam questions directly from tutorials
- ◆ Project work guidelines available at the course homepage

# Practicalities

- ◆ **References:** (no need to buy these, supplementary material distributed later by Edita)
  - A. B. Carlson: Communication Systems (4th ed.)
  - J. G. Proakis, Digital Communications (4th ed.)
  - L. Ahlin, J. Zander: Principles of Wireless Communications
- ◆ **Prerequisites:** S-72.1140 Transmission Methods, (recommended S-72.1130 Telecommunication Systems)
- ◆ **Homepage:** <http://www.comlab.hut.fi/studies/3320/>
- ◆ **Timetables:**
  - Lectures: Tuesdays 12-14 S3, Fridays 10-12 S2
  - Tutorials: Fridays 14-16 S1

# Timetable – spring 2006

- ◆ *27.1 Basics of Spread Spectrum Communications*
- ◆ *31.1 Fading Multipath Radio Channels*
- ◆ *3.2 no lecture*
- ◆ *7.2. Digital Transmission over a Fading Channel*
- ◆ *10.2 Cyclic Codes*
- ◆ *14.2 OFDM in Wideband Fading Channel*
- ◆ *17.2 Convolutional Codes*
- ◆ *21.2 Fiber-optic Communications*
- ◆ *24.2 Optical Networking*



S-72.3320 Advanced Digital Communication (4 cr)

*Spread spectrum and  
Code Division Multiple Access (CDMA)  
communications*

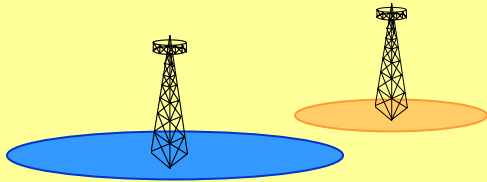
# Spread Spectrum (SS) Communications

## - Agenda Today

- ◆ Basic principles and block diagrams of spread spectrum communication systems
- ◆ Characterizing concepts
- ◆ Types of SS modulation: principles and circuits
  - direct sequence (DS)
  - frequency hopping (FH)
- ◆ Error rates
- ◆ Spreading code sequences; generation and properties
  - Maximal Length (a linear, cyclic code)
  - Gold
  - Walsh
- ◆ Asynchronous CDMA systems

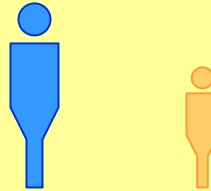
# How Tele-operators\* Market CDMA

## Coverage



For Coverage, CDMA saves wireless carriers from deploying the 400% more cell site that are required by GSM

## Capacity



CDMA's capacity supports at least 400% more revenue-producing subscribers in the same spectrum when compared to GSM

## Cost



A carrier who deploys CDMA instead of GSM will have a lower capital cost

## Clarity



CDMA with PureVoice provides wireline clarity

## Choice



CDMA offers the choice of simultaneous voice, async and packet data, FAX, and SMS.

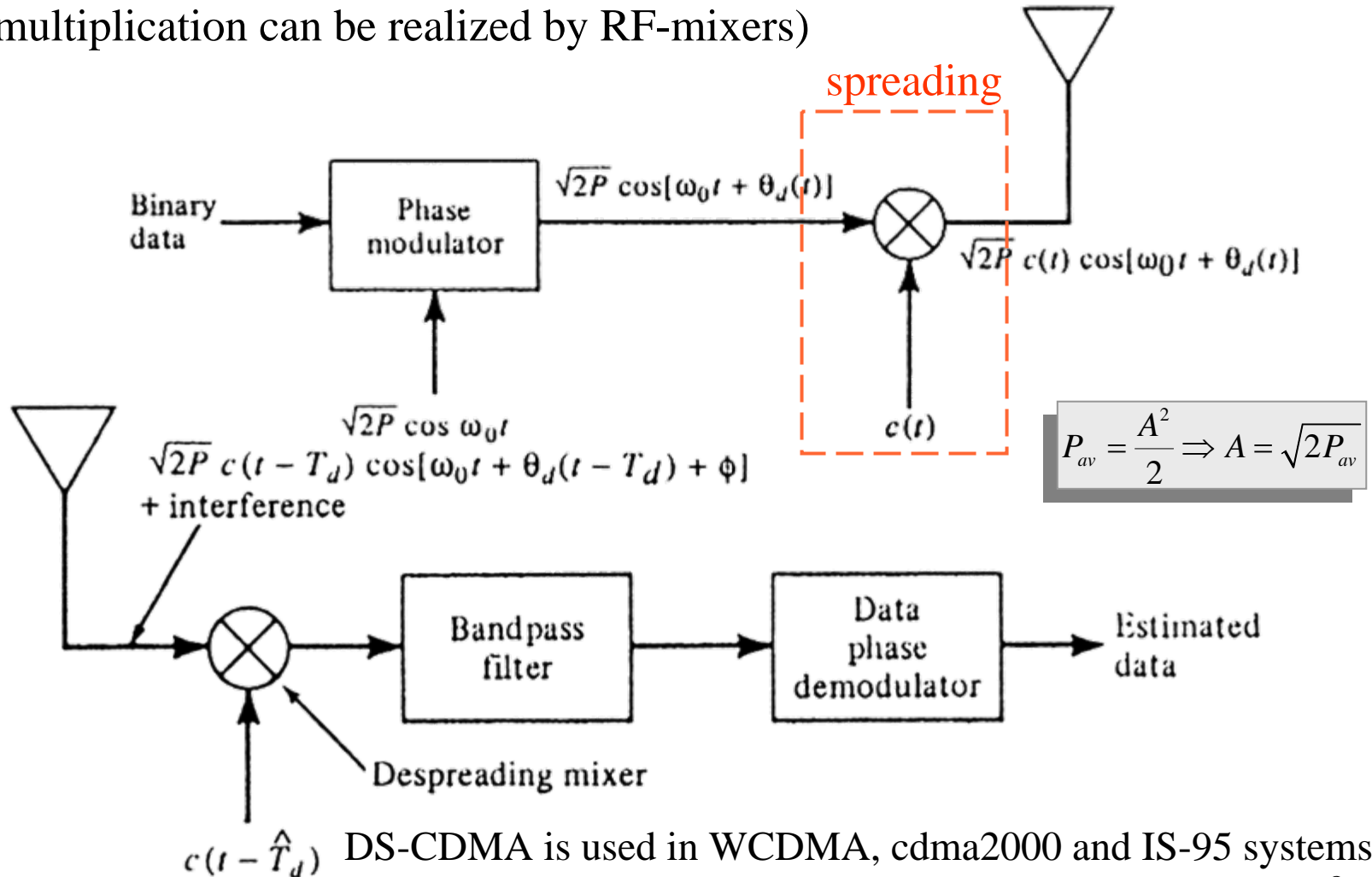
## Customer satisfaction



The Most solid foundation for attracting and retaining subscriber is based on CDMA

# Direct Sequence Spread Spectrum (DS-SS)

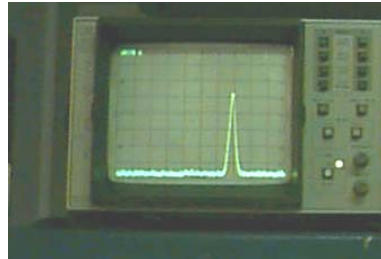
- ◆ This figure shows BPSK-DS transmitter and receiver (multiplication can be realized by RF-mixers)





# Characteristics of Spread Spectrum

- ◆ Bandwidth of the transmitted signal  $W$  is much greater than the original message bandwidth (or the signaling rate  $R$ )
- ◆ Transmission bandwidth is independent of the message. Applied code is known both to the transmitter and receiver



Narrow band signal  
(data)



Wideband signal  
(transmitted SS signal)

- ◆ Interference and noise immunity of SS system is larger, the larger the **processing gain**  $L_c = W / R = T_b / T_c$
- ◆ Multiple SS systems can co-exist in the same band (=CDMA). Increased user independence (decreased interference) for (1) **higher processing gain** and higher (2) **code orthogonality**
- ◆ Spreading sequence can be very long -> enables low transmitted PSD-> **low probability of interception** (especially in military communications)

# Characteristics of Spread Spectrum (cont.)

## ◆ Processing gain, in general

$$L_c = W / R = (1/T_c)/(1/T_b) = T_b / T_c, L_{c,dB} = 10 \log_{10}(L_c)$$

- Large  $L_c$  improves noise immunity, but requires a larger transmission bandwidth
- Note that DS-spread spectrum is a repetition FEC-coded systems

## ◆ Jamming margin

$$M_j = L_c - [L_{sys} + (SNR)_{desp}]$$

- Tells the magnitude of additional interference and noise that can be injected to the channel without hazarding system operation.

Example:

$L_c = 30$  dB, available processing gain

$L_{sys} = 2$  dB, margin for system losses

$SNR_{desp} = 10$  dB, required SNR after despreading (at the RX)

$\Rightarrow M_j = 18$  dB, additional interference and noise can deteriorate received SNR by this amount

# Characteristics of Spread Spectrum (cont.)

- ◆ **Spectral efficiency  $E_{eff}$** : Describes how compactly TX signal fits into the transmission band. For instance for BPSK **with some pre-filtering**:

$$E_{eff} = R_b / B_T = R_b / B_{RF} \quad L_c = T_b / T_c \Rightarrow L_c / T_b = 1 / T_c$$

$$B_{RF} \approx \frac{B_{RF, filt}}{k} \approx \frac{1/T_c}{\log_2 M} = \frac{L_c}{T_b \log_2 M}$$

$$\Rightarrow E_{eff} = \frac{R_b}{B_{RF}} \approx \frac{1}{L_c} \frac{T_b \log_2 M}{L_c} = \frac{\log_2 M}{L_c} \quad (M = 2^k \Rightarrow k = \log_2 M)$$

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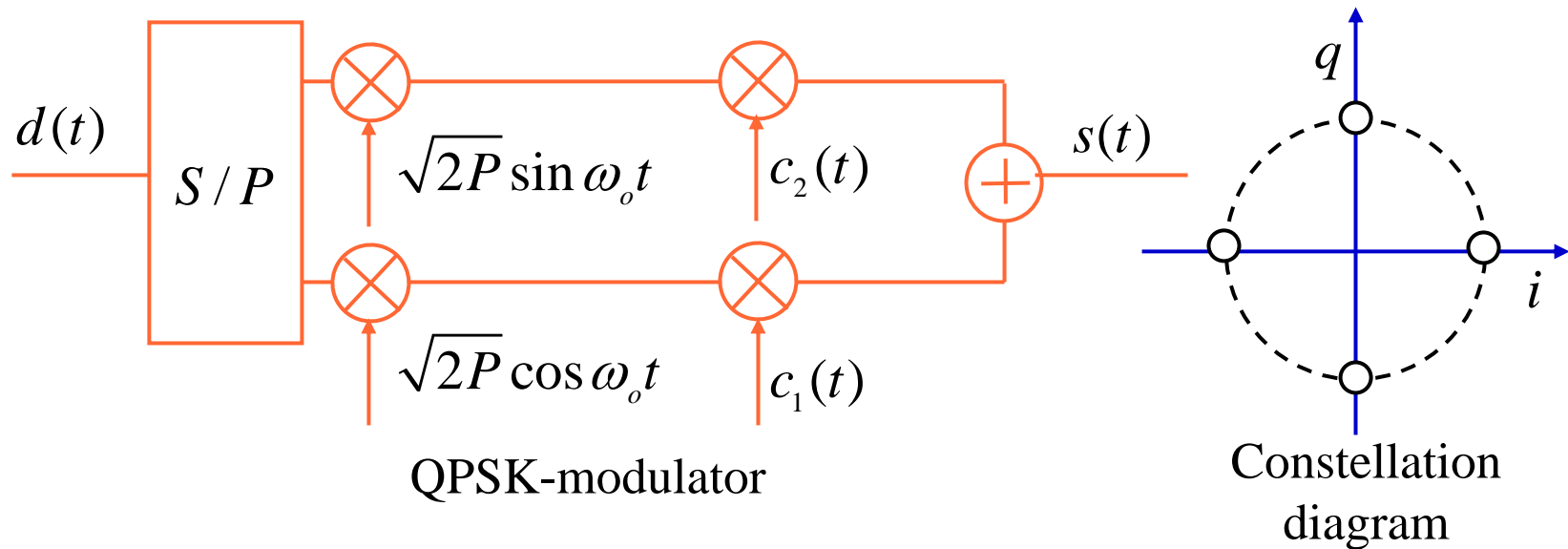
$B_{RF, filt}$  : bandwidth for polar mod.  
 $M$ : number of levels  
 $k$ : number of bits

- ◆ **Energy efficiency (reception sensitivity)**: The value of  $\gamma_b = E_b / N_0$  to obtain a specified error rate (often  $10^{-9}$ ). For BPSK the error rate is

$$p_e = Q(\sqrt{2\gamma_b}), Q(k) = \frac{1}{\sqrt{2\pi}} \int_k^{\infty} \exp(-\lambda^2 / 2) d\lambda$$

- ◆ QPSK-modulation can fit twice the data rate of BPSK in the same bandwidth. Therefore it is more energy efficient than BPSK.

# A QPSK-DS Modulator



- ◆ After serial-parallel conversion (S/P) data modulates the orthogonal carriers  $\sqrt{2P} \cos(\omega_o t)$  and  $\sqrt{2P} \sin(\omega_o t)$
- ◆ Modulation on orthogonal carriers spreaded by codes  $c_1$  and  $c_2$
- ◆ Spreading codes  $c_1$  and  $c_2$  may or may not be orthogonal (System performance is independent of their orthogonality, why?)
- ◆ What kind of circuit can make the demodulation (despreading)?

# DS-CDMA (BPSK) Spectra (Tone Jamming)

- Assume DS - BPSK transmission, with a single tone jamming (jamming power  $J [W]$ ). The received signal is

$$r(t) = \sqrt{2P}c_1(t - T_d)\cos(\omega_0 t + \theta_d(t)) + \sqrt{2J}\cos(\omega_0 t + \varphi')$$

- The respective PSD of the received **chip-rate signal** is

$$S_r(f) = \frac{1}{2}PT_c \operatorname{sinc}^2[(f - f_0)T_c] + \frac{1}{2}PT_c \operatorname{sinc}^2[(f + f_0)T_c]$$

$$+ \frac{1}{2}J\{\delta(f - f_0) + \delta(f + f_0)\}$$

Spreading of jammer power

- At the receiver  $r(t)$  is multiplied with the local code  $c(t)$  (=despreading)

$$d(t) = \sqrt{2P}c_1(t - T_d)c(t - \hat{T}_d)\cos(\omega_0 t + \theta_d(t)) + \sqrt{2J}c(t - \hat{T}_d)\cos(\omega_0 t + \varphi')$$

data

- The received signal and the local code are phase-aligned:

$$c_1(t - \hat{T}_d)c(t - T_d) = 1 \Rightarrow S_d(f) = \frac{1}{2}PT_b \operatorname{sinc}^2[(f - f_0)T_b] + \frac{1}{2}PT_b \operatorname{sinc}^2[(f + f_0)T_b]$$

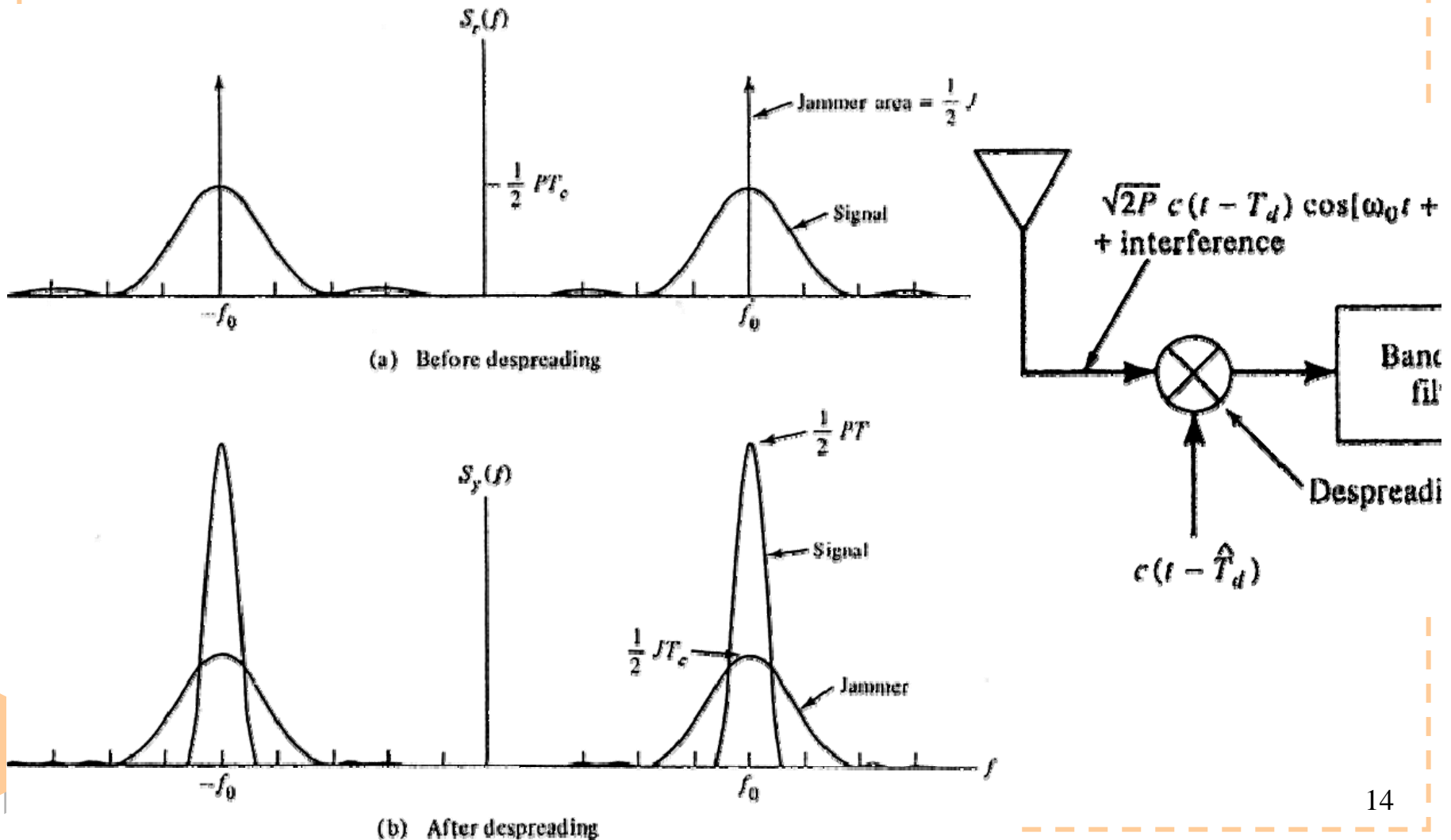
Data spectra after phase modulator

$$+ \frac{1}{2}JT_c \operatorname{sinc}^2[(f - f_0)T_c] + \frac{1}{2}JT_c \operatorname{sinc}^2[(f + f_0)T_c]$$

$$\mathbb{F}\{\sqrt{2J}c(t - \hat{T}_d)\cos(\omega_0 t + \varphi')\}$$

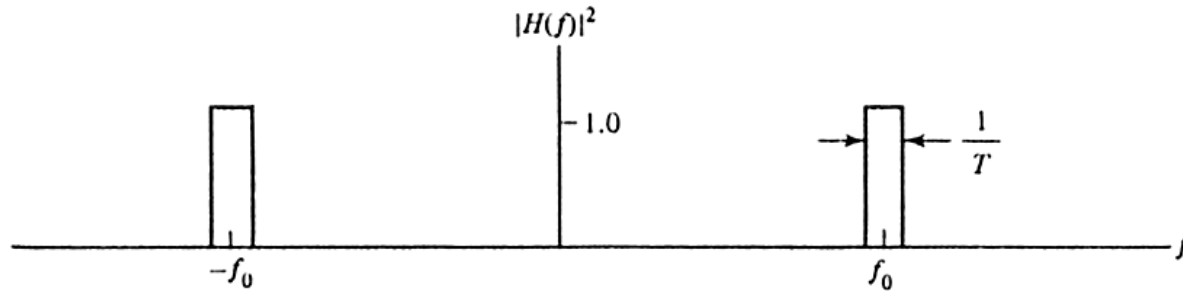
# Tone Jamming (cont.)

- ◆ Despreading spreads the jammer power and despreads the signal power:

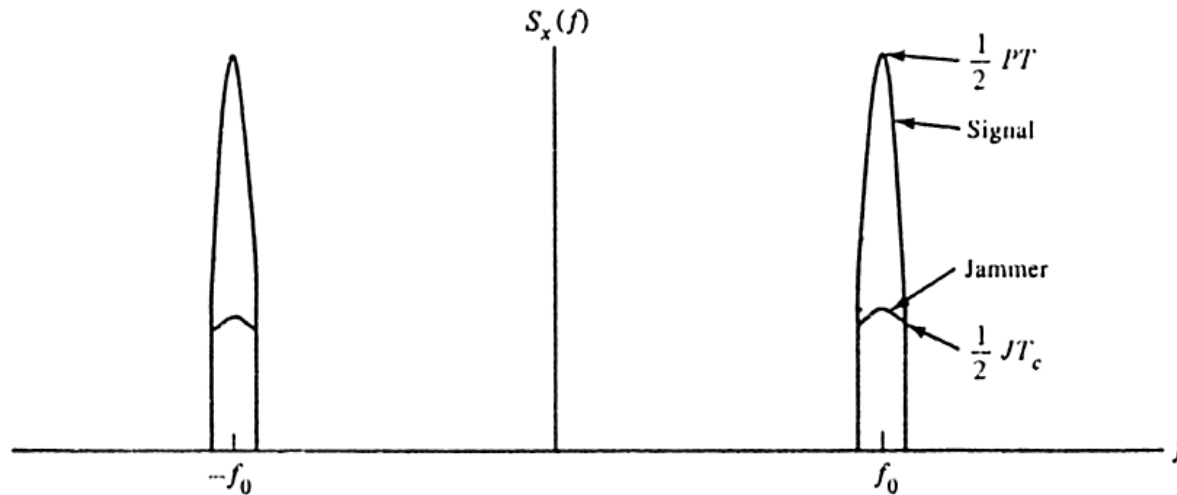


# Tone Jamming (cont.)

- ◆ Filtering (at the BW of the phase modulator) after despreading suppresses the jammer power:



(c) IF filter power transfer function



(d) Output of IF filter

# Error Rate of BPSK-DS System\*

- ◆ DS system is a form of coding, therefore code weight determines, from its own part, error rate
- ◆ Assuming that the chips are uncorrelated, prob. of code word error for a binary-block coded BPSK-DS system with code weight  $w$  is therefore

$$P_e = Q\left(\sqrt{\frac{2E_b}{N_0} R_c w_m}\right), R_c = k/n \text{ (= code rate, } n > k)$$

- ◆ This can be expressed in terms of processing gain  $L_c$  by denoting the average signal and noise power by  $P_{av}, N_{av}$ , respectively, yielding

$$E_b = P_{av} T_b, N_0 = N_{av} T_c \Rightarrow$$

$$P_e = Q\left(\sqrt{\frac{2P_{av} T_b}{N_{av} T_c} R_c w_m}\right) = Q\left(\sqrt{\frac{2P_{av}}{N_{av}} L_c R_c w_m}\right)$$

- ◆ Note that the symbol error rate is upper bounded due to repetition code nature of the DS by

$$P_{es} \leq \sum_{m=t+1}^n \binom{n}{m} p^m (1-p)^{n-m}, t = \left\lfloor \frac{1}{2} (d_{\min} - 1) \right\rfloor$$

where  $t$  denotes the number of erroneous bits that can be corrected in the coded word,  $d_{\min} = n$  (rep. coding)



# Example: Error Rate of Uncoded Binary BPSK-DS

- ◆ For uncoded DS  $w=n$  (repetition coding), thus  $R_c w = (1/n)n = 1$  and

$$P_e = Q\left(\sqrt{\frac{2E_b}{N_0} R_c w_m}\right) = Q\left(\sqrt{\frac{2E_b}{N_0}}\right)$$

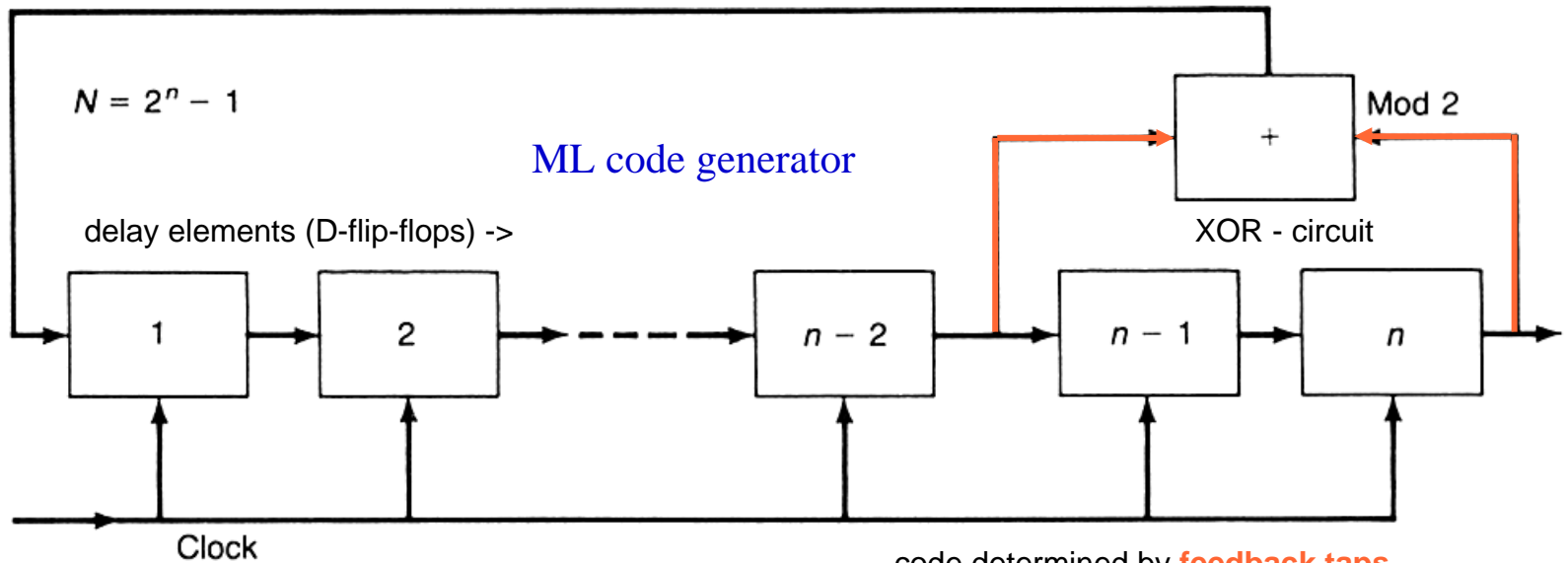
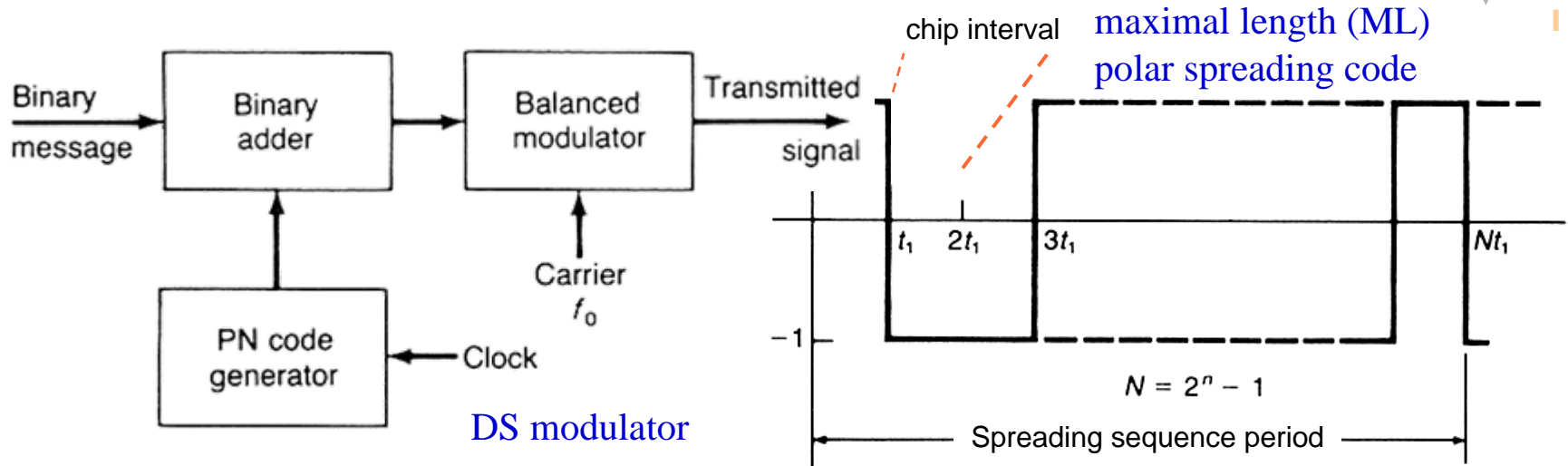
- ◆ We note that  $E_b = P_{av} T_b = P_{av} / R_b$  and  $N_0 = P_N / W$  [W/Hz] yielding

$$\frac{E_b}{N_0} = \frac{P_{av} / R}{P_N / W} = \frac{W / R}{P_N / P_{av}}$$

$$\Rightarrow P_e = Q\left(\sqrt{\frac{2W / R}{P_N / P_{av}}}\right)$$

- ◆ Therefore, we note that by increasing system processing gain  $W/R$  or transmitted signal power  $P_{av}$ , error rate can be improved

# Code Generation in DS-SS

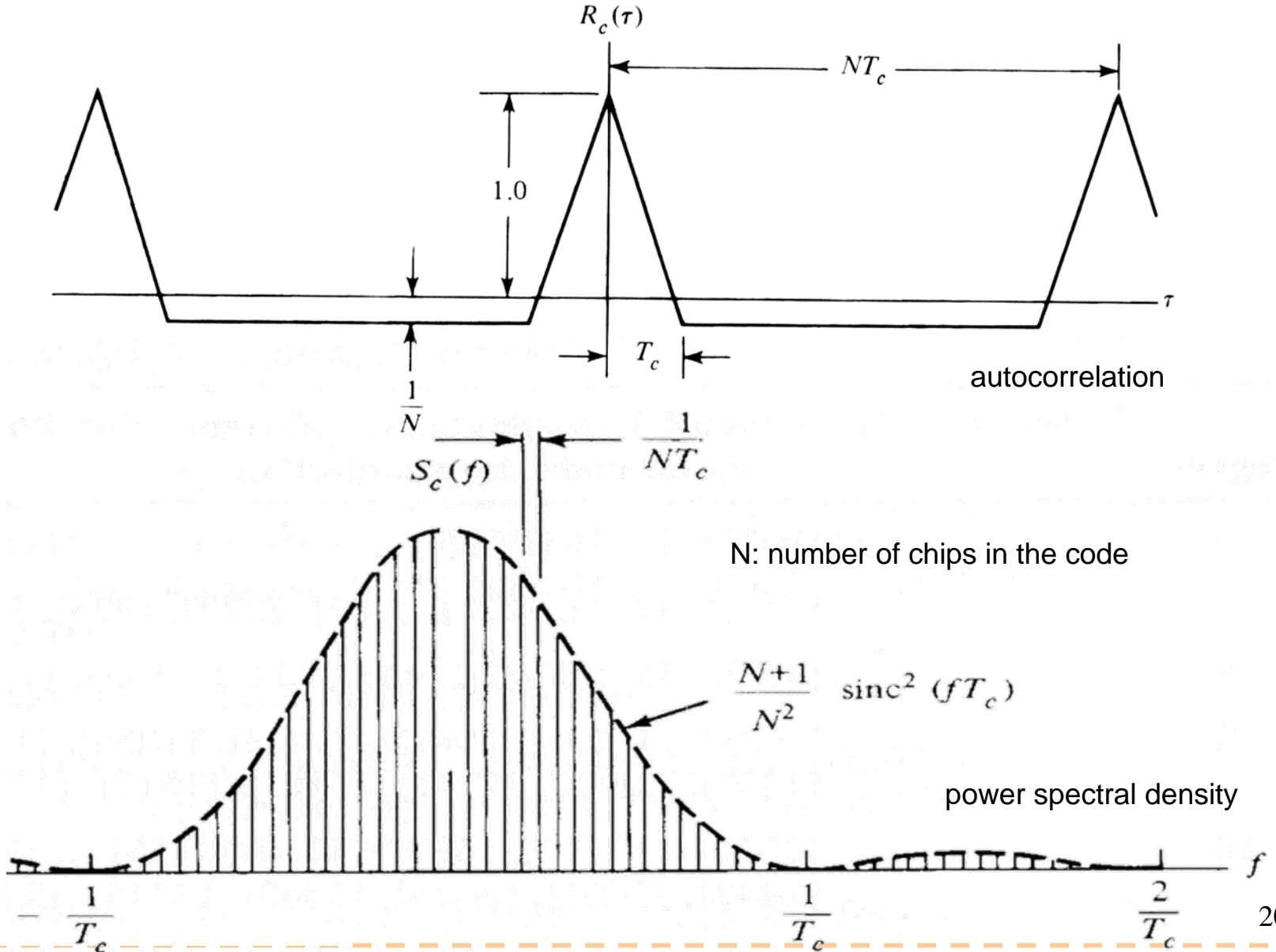


- code determined by **feedback taps**
- code rate determined by clock rate

# Some Cyclic Block Codes

- ◆  $(n,1)$  **Repetition codes**. High coding gain, but low rate
- ◆  $(n,k)$  **Hamming codes**. Minimum distance always 3. Thus can detect 2 errors and correct one error.  $n=2^m-1$ ,  $k = n - m$ ,  $m \geq 3$
- ◆ **Maximum-length codes**. For every integer  $k \geq 3$  there exists a maximum length code  $(n,k)$  with  $n = 2^k - 1$ ,  $d_{\min} = 2^{k-1}$ . Hamming codes are dual<sup>1</sup> of maximal codes.
- ◆ **BCH-codes**. For every integer  $m \geq 3$  there exists a code with  $n = 2^m - 1$ ,  $k \geq n - mt$  and  $d_{\min} \geq 2t + 1$  where  $t$  is the error correction capability
- ◆  $(n,k)$  **Reed-Solomon (RS) codes**. Works with  $k$  **symbols** that consist of  $m$  bits that are encoded to yield code words of  $n$  **symbols**. For these codes  $n = 2^m - 1$ , number of check symbols  $n - k = 2t$  and  $d_{\min} = 2t + 1$
- ◆ Nowadays BCH and RS are very popular due to large  $d_{\min}$ , large number of codes, and easy generation

# Maximal Length Codes



# Maximal Length Codes (cont.)

- ◆ Have **very good autocorrelation** but cross correlation not granted
- ◆ Are **linear, cyclic block codes** - generated by feedbacked shift registers
- ◆ Number of available codes\* depends on the number of shift register stages: 5 stages->6 codes, 10 stages ->60 codes, 25 stages ->1.3x10<sup>6</sup> codes
- ◆ Code generator design based on tables showing tap feedbacks:

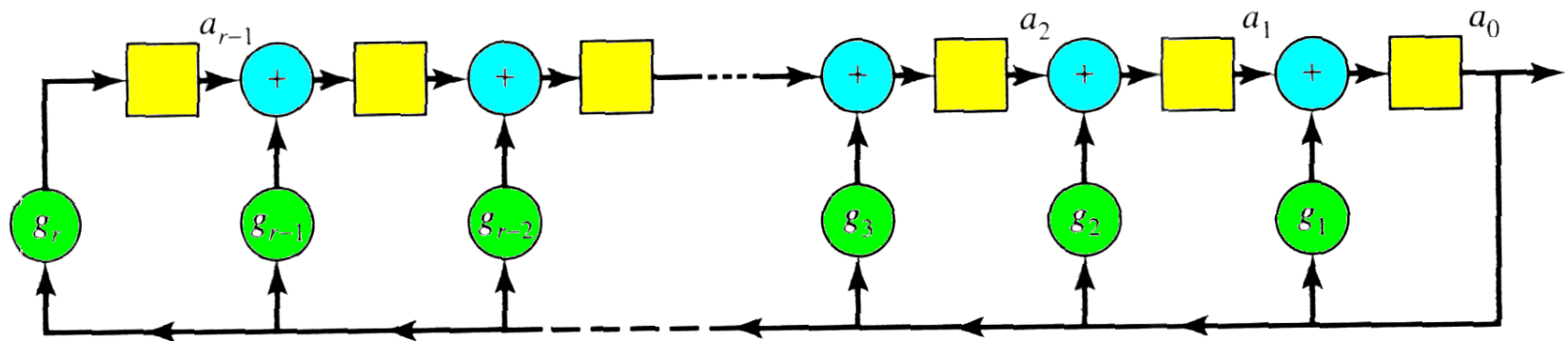
**TABLE 3-5.** Primitive Polynomials Having Degree  $r \leq 34$  (continued)

Degree	Octal Representation of Generator Polynomial ( $g_0$ on right to $g_r$ on left)
6	[103]*, [147], [155]
7	[211]*, [217], [235], <u>[367]</u> , [277], [325], [203]*, [313], [345]
8	[435], [551], [747], [453], [545], [537], [703], [543]
9	[1021]*, [1131], [1461], [1423], [1055], [1167], [1541], [1333], [1605], [1751], [1743], [1617], [1553], [1157]
10	[2011]*, [2415], [3771], [2157], [3515], [2773], [2033], [2443], [2461], [3023], [3543], [2745], [2431], [3177]
11	[4005]*, [4445], [4215], [4055], [6015], [7413], [4143], [4563], [4053], [5023], [5623], [4577], [6233], [6673]

# Design of Maximal Length Generators by a Table Entry

- Feedback connections can be written directly from the table:

3			6			7			octal
0	1	1	1	1	0	1	1	1	binary
↓	↓	↓	↓	↓	↓	↓	↓	↓	coefficient
$g_7$	$g_6$	$g_6$	$g_5$	$g_4$	$g_3$	$g_2$	$g_1$	$g_0$	



# Other Spreading Codes

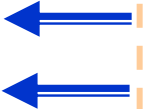
- ◆ **Walsh codes:** Orthogonal, used in *synchronous systems*, also in WCDMA downlink

- ◆ Generation recursively:  $H_0 = [0] \quad H_n = \begin{bmatrix} H_{n-1} & H_{n-1} \\ H_{n-1} & \overline{H_{n-1}} \end{bmatrix}$

- ◆ All rows and columns of the matrix are orthogonal:

$$\Rightarrow (-1)(-1) + (-1)1 + 1(-1) + 1 \cdot 1 = 0$$

$$H_2 = \begin{bmatrix} 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 1 \\ 0 & 0 & 1 & 1 \\ 0 & 1 & 1 & 0 \end{bmatrix}$$



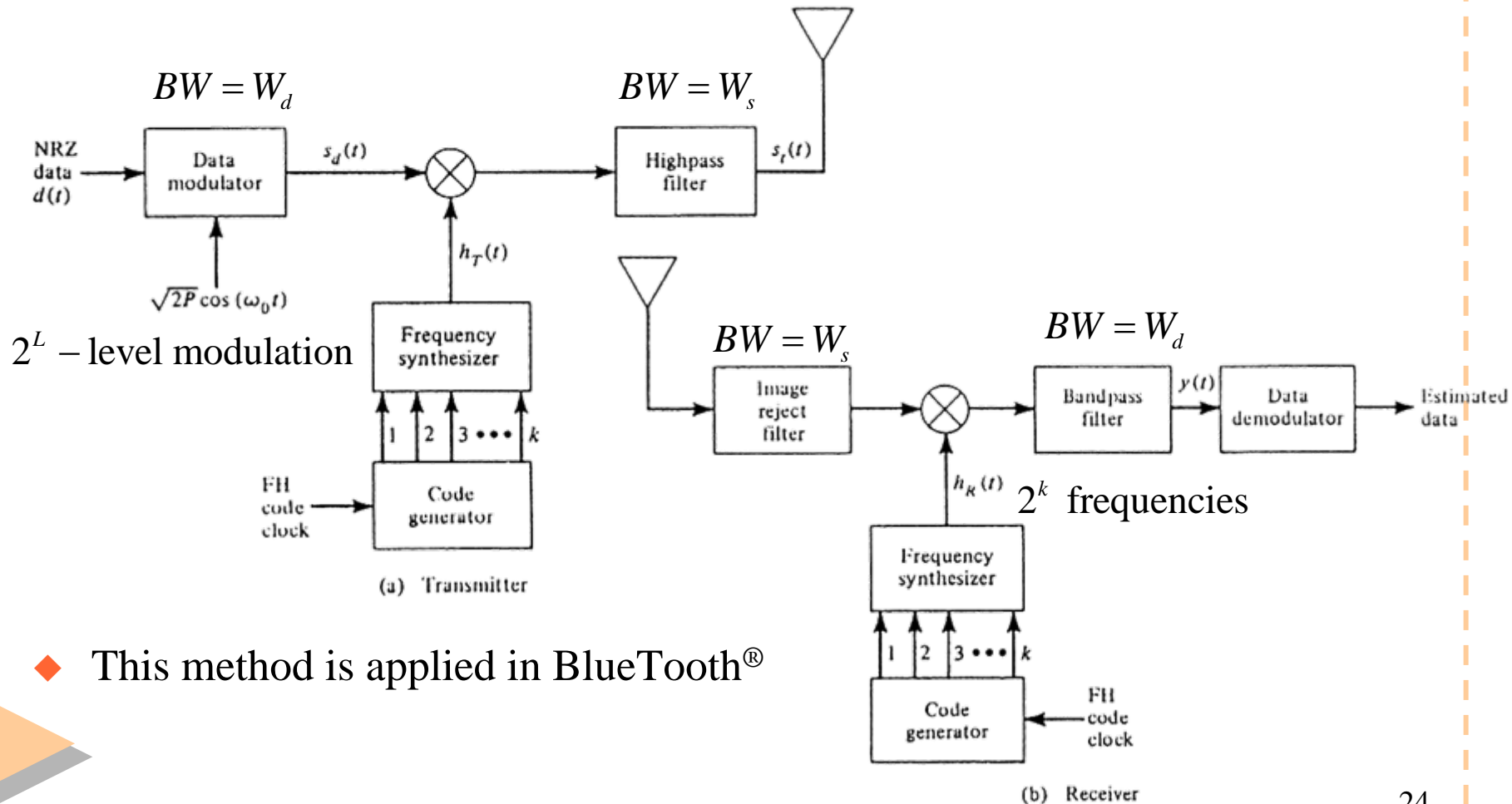
- ◆ **Gold codes:** Generated by summing *preferred pairs* of maximal length codes. Have a guarantee 3-level crosscorrelation:  $\{-t(n)/N, 1/N, (t(n)-2)/N\}$
- ◆ For  $N$ -length code there exists  $N + 2$  codes in a code family and

$$N = 2^n - 1 \text{ and } t(n) = \begin{cases} 1 + 2^{(n+1)/2}, & \text{for } n \text{ odd} \\ 1 + 2^{(n+2)/2}, & \text{for } n \text{ even} \end{cases} \quad (n: \text{number of stages in the shift register})$$

- ◆ Walsh and Gold codes are used especially in multiple access systems
- ◆ Gold codes are used in *asynchronous communications* because their crosscorrelation is quite good as formulated above

# Frequency Hopping Transmitter and Receiver

- ◆ In FH-SS hopping frequencies are determined by the code and the message (bits) are usually non-coherently FSK-modulated

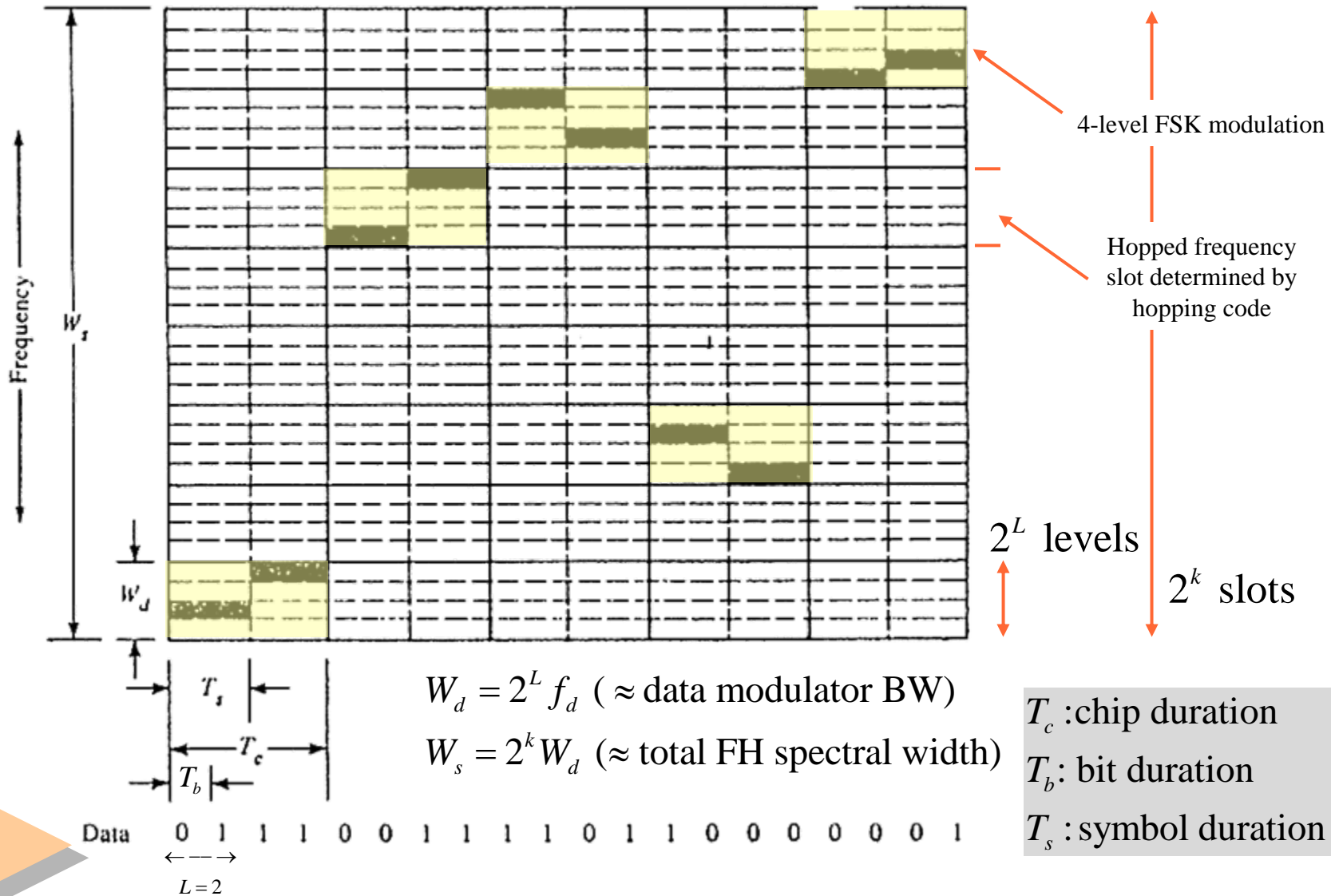


- ◆ This method is applied in BlueTooth®



# Frequency Hopping Spread Spectrum (FH-SS)

(example: transmission of two symbols/chip)



# Error Rate in Frequency Hopping

- ◆ If there are multiple hops/symbol (symbol is distributed to different frequencies) we have a fast-hopping system. If there is a single hop/symbol (or below (multiple symbols/frequency)), we have a slow-hopping system.

- ◆ For slow-hopping non-coherent FSK-system, binary error rate is

$$P_e = \frac{1}{2} \exp(-\gamma_b / 2), \gamma_b = E_b / N_0$$

and the respective symbol error rate is (hard-decisions)

$$P_{es} = \frac{1}{2} \exp(-\gamma_b R_c / 2), R_c = k / n < 1$$

- ◆ A fast-hopping FSK system is a diversity-gain system. Assuming non-coherent, square-law combining of respective output signals from matched filters yields the binary error rate (with  $L$  hops/symbol)

$$P_e = \exp(-\gamma_b / 2) \underbrace{\frac{1}{2^{2L-1}} \sum_{i=0}^{L-1} K_i (\gamma_b / 2)^i}_{\text{diversity gain - component}}, \gamma_b = L\gamma_c = LR_c E_b / N_0$$

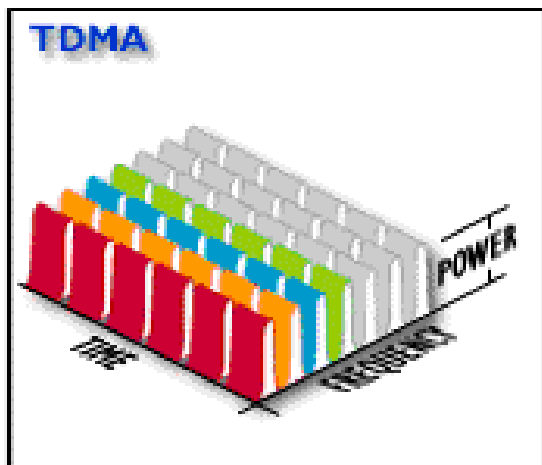
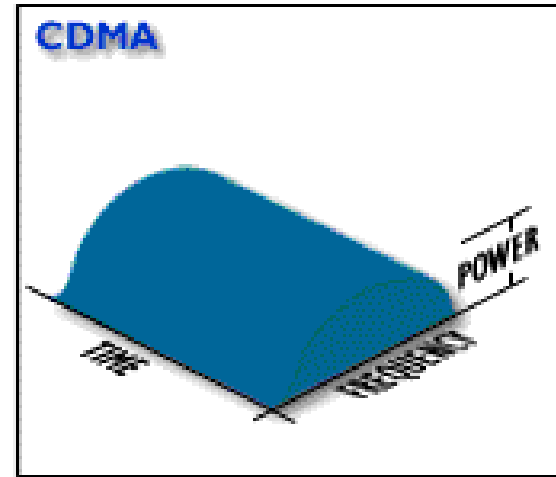
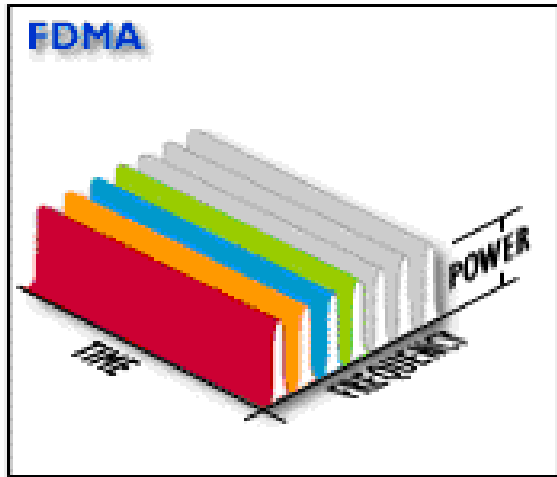
$$K_i = \frac{1}{i!} \sum_{r=0}^{L-1-i} \binom{2L-1}{\gamma}$$

(For further details, see J.G.Proakis: Digital Communications (IV Ed), Section 13.3)

# DS and FH compared

- ◆ FH is applicable in environments where there exist **tone jammers** that can be overcome by avoiding hopping on those frequencies
- ◆ DS is applicable for *multiple access* because it allows *statistical multiplexing* (**resource reallocation**) to other users (power control)
- ◆ FH applies usually **non-coherent modulation** due to carrier synchronization difficulties -> modulation method degrades performance
- ◆ Both methods were first used in *military communications*,  $L_c \rightarrow 10^2 \dots 10^7$ 
  - FH can be advantageous because the **hopping span** can be very large (makes *eavesdropping* difficult)
  - DS can be advantageous because **spectral density** can be much smaller than background noise density (transmission is unnoticed)
- ◆ FH is an **avoidance system**: does not suffer *near-far effect*!
- ◆ By using **hybrid systems** some benefits can be combined: The system can have a low probability of interception and negligible near-far effect at the same time. (*Differentially coherent modulation* is applicable)

# Multiple access: FDMA, TDMA and CDMA

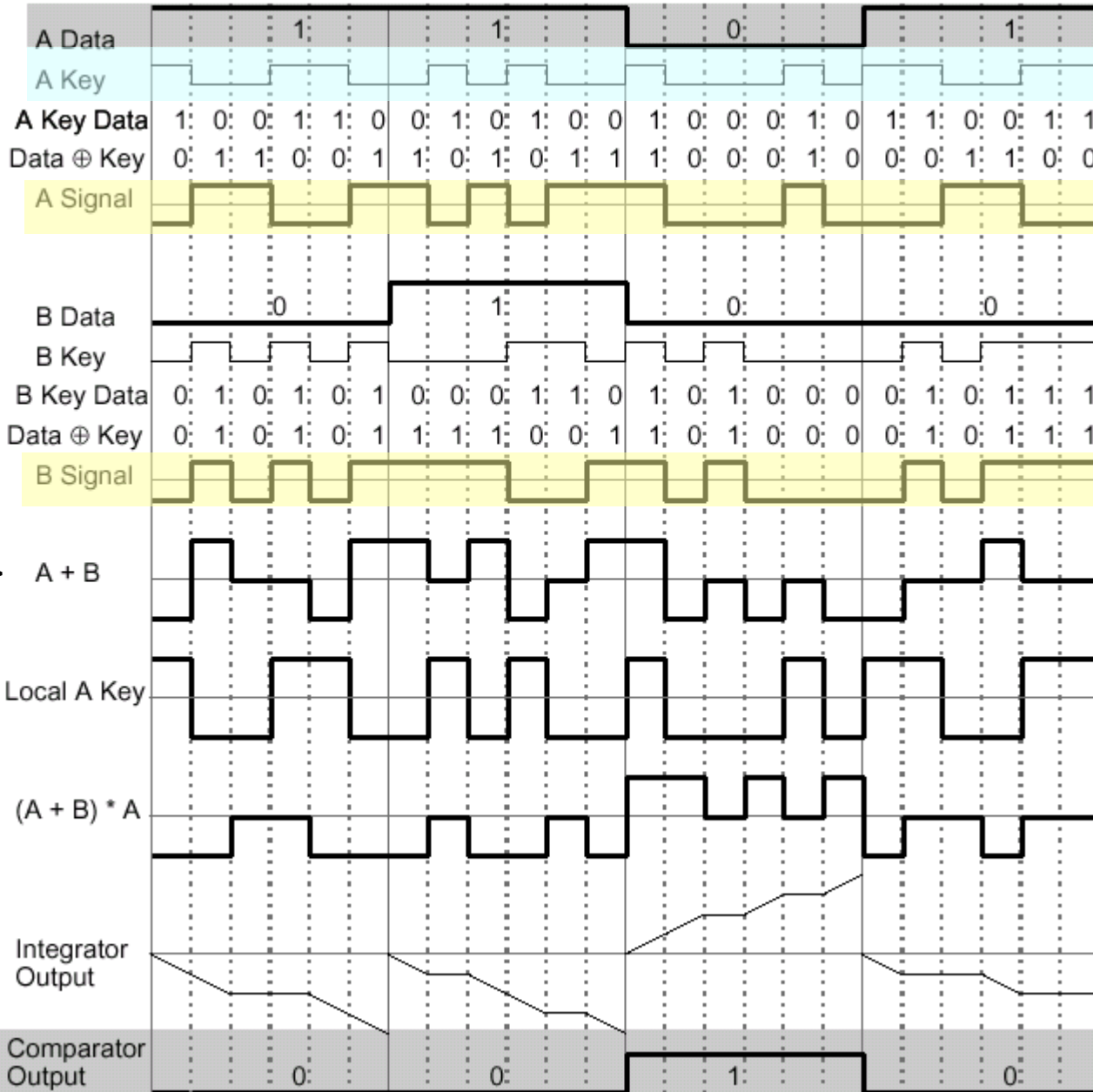


- FDMA, TDMA and CDMA yield conceptually the same capacity
- However, in **wireless communications** CDMA has improved capacity due to
  - statistical multiplexing
  - graceful degradation
- Performance can still be improved by **adaptive antennas, multiuser detection, FEC, and multi-rate encoding**

# FDMA, TDMA and CDMA compared

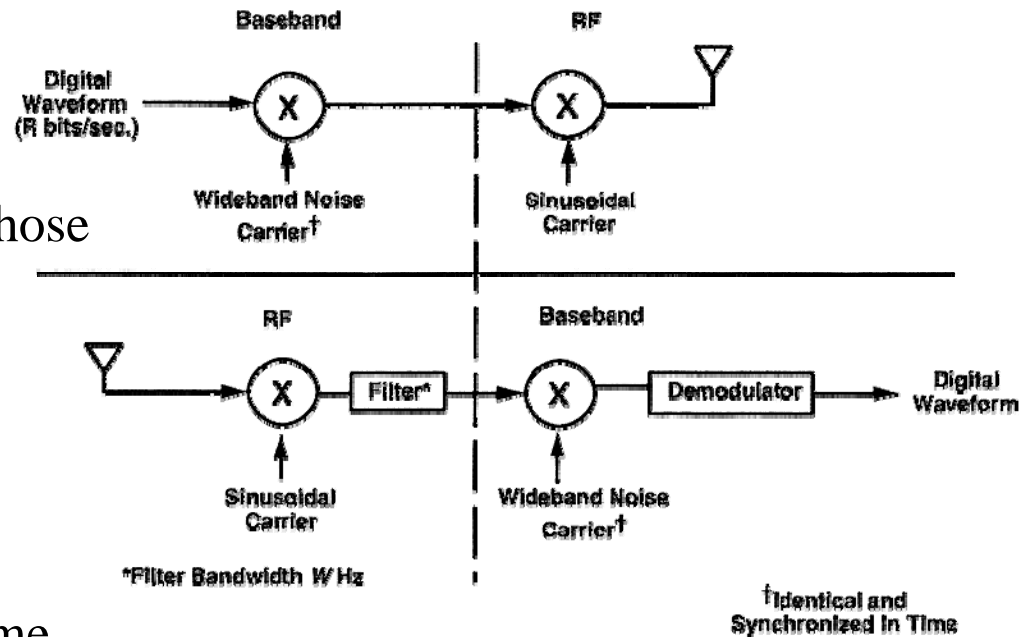
- ◆ TDMA and FDMA principle:
  - TDMA allocates **a time instant** for a user
  - FDMA allocates **a frequency band** for a user
  - CDMA allocates **a code** for user
- ◆ CDMA-system can be *synchronous* or *asynchronous*:
  - Synchronous CDMA difficult to apply in multipath channels that destroy code orthogonality
  - Therefore, in wireless CDMA-systems as in IS-95, cdma2000, WCDMA and IEEE 802.11 users are asynchronous
- ◆ Code classification:
  - **Orthogonal**, as Walsh-codes for orthogonal or near-orthogonal systems
  - **Near-orthogonal and non-orthogonal codes**:
    - ◆ Gold-codes, for asynchronous systems
    - ◆ Maximal length codes for asynchronous systems

# Example of DS multiple access waveforms



# Capacity of a cellular CDMA system

- ◆ Consider uplink (MS->BS)
- ◆ Each user transmits Gaussian noise (SS-signal) whose deterministic characteristics are stored in RX and TX
- ◆ Reception and transmission are simple multiplications
- ◆ Perfect power control: each user's power at the BS the same
- ◆ Each user receives multiple copies of power  $P_r$  that is other user's interference power, therefore each user receives the interference power



$$I_k = (U - 1)P_r \quad (1)$$

where  $U$  is the number of equal power users

# Capacity of a cellular CDMA system (cont.)

- ◆ Each user applies a demodulator/decoder characterized by a certain *reception sensitivity*  $E_b/I_o$  (3 - 9 dB depending on channel coding, channel, modulation method etc.)
- ◆ Each user is exposed to the interference power density (assumed to be produced by other users only)  $I_o = I_k / B_T$  [W/Hz] (2)  
where  $B_T$  is the spreading (and RX) bandwidth

- ◆ Received signal energy / bit at the signaling rate  $R$  is

$$E_b = P_r / R \quad [J] = [W][s] \quad (3)$$

- ◆ Combining (1)-(3) yields the number of users

$$I_k = (U - 1)P_r \Rightarrow U - 1 = \frac{I_k}{P_r} = \frac{I_o B_T}{E_b R} = \frac{(1/R) B_T}{E_b (1/I_o)} = \boxed{\frac{W/R}{E_b/I_o}} \quad (4)$$

- ◆ This can still be increased by using **voice activity coefficient**  $G_v = 2.67$  (only about 37% of speech time effectively used), **directional antennas**, for instance for a 3-way antenna  $G_A = 2.5$ .



# Capacity of a cellular CDMA system (cont.)

- ◆ In cellular system neighboring cells introduce interference that decreases capacity. It has been found out experimentally that this reduces the number of users by the factor

$$1 + f \approx 1.6$$

- ◆ Hence asynchronous CDMA system capacity can be approximated by

$$U = \frac{W / R}{E_b / I_o} \frac{G_v G_A}{1 + f}$$

yielding with the given values  $G_v=2.67$ ,  $G_A=2.4$ ,  $1+f=1.6$ ,

$$U = \frac{4W / R}{E_b / I_o}$$

- ◆ Assuming efficient error correction algorithms, dual diversity antennas, and RAKE receiver, it is possible to obtain  $E_b/I_o=6 \text{ dB} = 4$ , and then

$$U \approx \frac{W}{R}$$

This is of order of magnitude larger value than with the conventional (GSM;TDMA) systems!

# Lessons Learned

- ◆ You understand what is meant by code gain, jamming margin, and spectral efficiency and what is their meaning in SS systems
- ◆ You understand how spreading and despreading works
- ◆ You understand the basic principles of DS and FH systems and know their error rates by using BPSK and FSK modulations (if required, formulas will be given in exam)
- ◆ You know the bases of code selection for SS system. (What kind of codes can be applied in SS systems and when they should be applied.)
- ◆ You understand how the capacity of asynchronous CDMA system can be determined