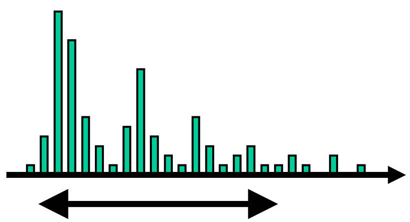
Digital transmission over a fading channel

- Narrowband system (introduction)
- BER vs. SNR in a narrowband system
- Wideband TDMA (introduction)
- Wideband DS-CDMA (introduction)
- Rake receiver (structure & analysis)
- Diversity techniques

Three kinds of systems (1)

Narrowband system:

Flat fading channel, single-tap channel model, performance enhancement through diversity (future lecture).



$$\boxed{T >> T_m}$$

$$T \propto W$$

System bandwidth

T_m = delay spread of multipath channel

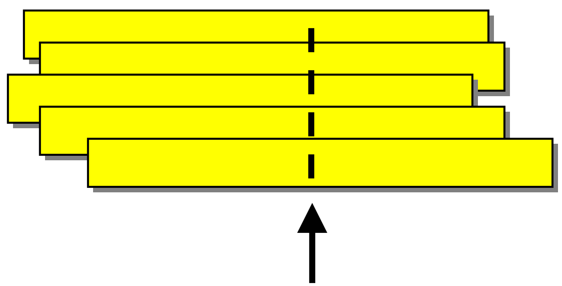
bit or symbol

T = bit or symbol duration

No intersymbol interference (ISI)

Narrowband system:

Adjacent symbols (bits) do not affect the decision process (in other words there is no intersymbol interference).



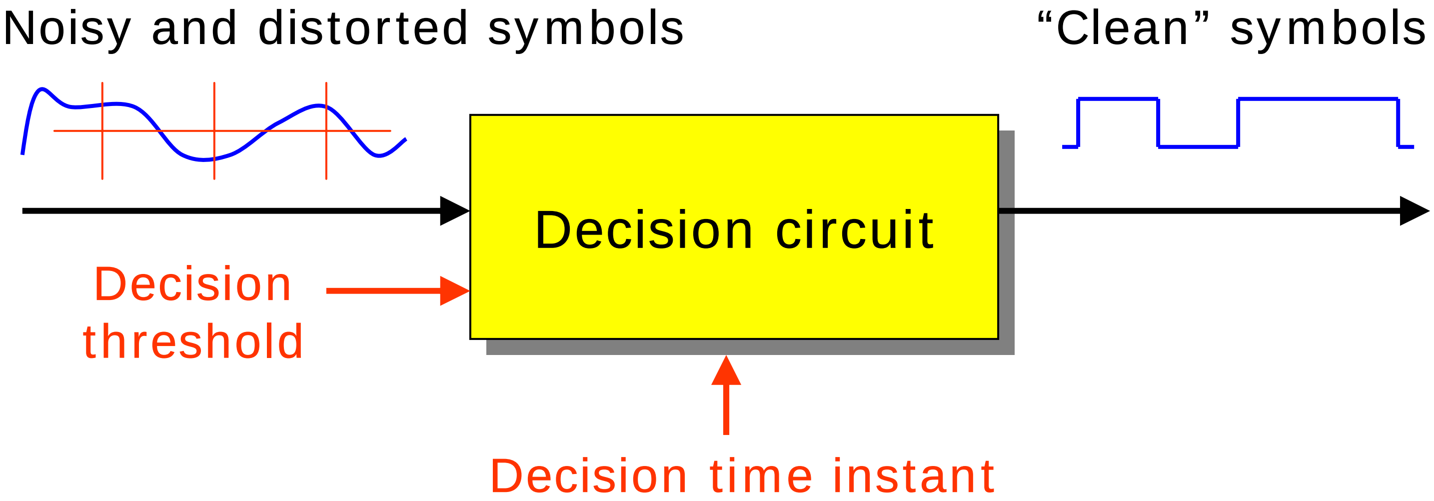
Received replicas of same symbol overlap in multipath channel

No intersymbol interference at decision time instant

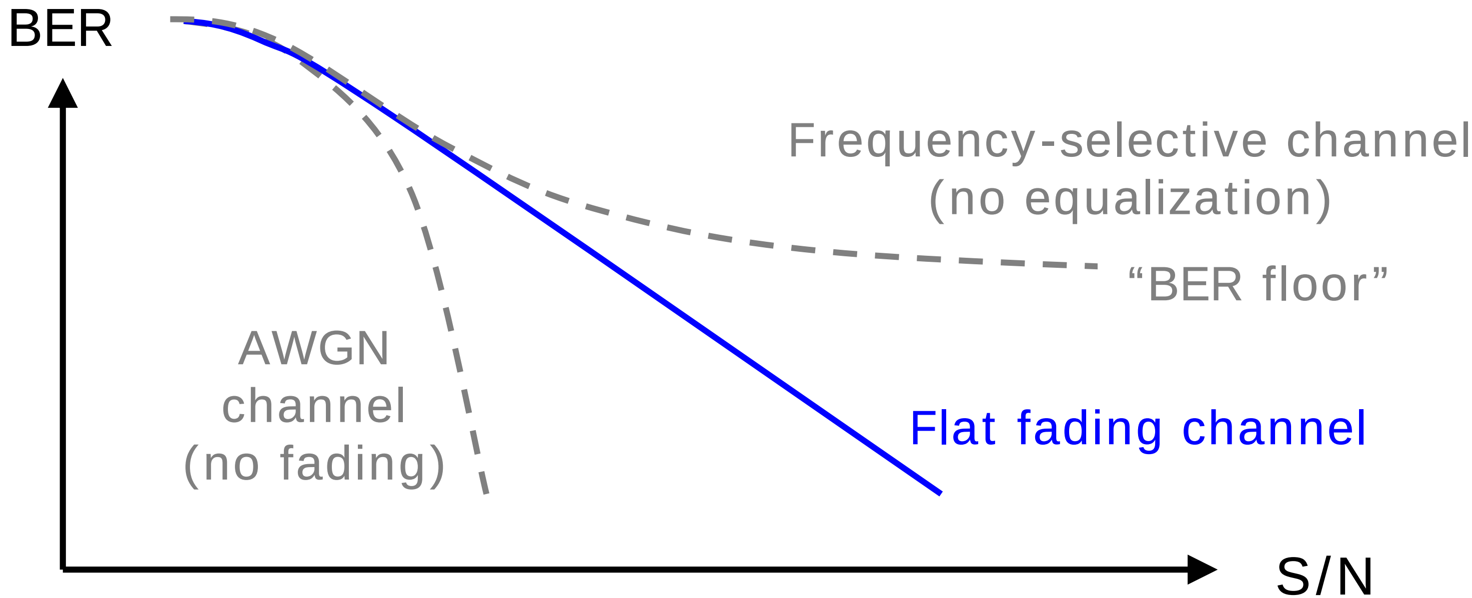
However: Fading (destructive interference) is still possible

Decision circuit

In the binary case, the decision circuit compares the received signal with a threshold at specific time instants (usually somewhere in the middle of each bit period):



Narrowband system: BER performance



BER vs. SNR in a flat fading channel

Proakis, 3rd Ed. 14-3

In a flat fading channel (or narrowband system), the CIR (channel impulse response) reduces to a single impulse scaled by a time-varying complex coefficient.

The received (equivalent lowpass) signal is of the form

$$r(t) = a(t)e^{j\phi(t)}s(t) + n(t)$$

We assume that the phase changes "slowly" and can be perfectly tracked

=> important for coherent detection

BER vs. SNR (cont.)

We assume:

- the time-variant complex channel coefficient changes slowly (=> constant during a symbol interval)
- the channel coefficient magnitude (= attenuation factor) a is a Rayleigh distributed random variable
- coherent detection of a binary PSK signal (assuming ideal phase synchronization)

Let us define instantaneous SNR and average SNR:

$$\gamma = a^2 E_b / N_0 \qquad\qquad \gamma_0 = E\left\{a^2\right\} \cdot E_b / N_0$$

BER vs. SNR (cont.)

Since

$$p(a) = \frac{2a}{E\{a^2\}} e^{-a^2 / E\{a^2\}} \qquad a \geq 0,$$

Rayleigh distribution

using

$$p(\gamma) = \frac{p(a)}{|d\gamma / da|}$$

Exponential distribution

we get

$$p(\gamma) = \frac{1}{\gamma_0} e^{-\gamma/\gamma_0} \qquad \gamma \geq 0.$$

BER vs. SNR (cont.)

The average bit error probability is

$$P_e = \int_0^\infty P_e(\gamma)\, p(\gamma)\, d\gamma$$

Important formula for obtaining statistical average

where the bit error probability for a certain value of a is

$$P_e(\gamma) = Q\left(\sqrt{2a^2 E_b / N_0}\right) = Q\left(\sqrt{2\gamma}\right).$$

2-PSK

We thus get

$$P_e = \int_0^\infty Q\left(\sqrt{2\gamma}\right) \frac{1}{\gamma_0} e^{-\gamma/\gamma_0} d\gamma = \frac{1}{2}\left(1 - \sqrt{\frac{\gamma_0}{1+\gamma_0}}\right).$$

BER vs. SNR (cont.)

Approximation for large values of average SNR is obtained in the following way. First, we write

$$P_e = \frac{1}{2}\left(1 - \sqrt{\frac{\gamma_0}{1+\gamma_0}}\right) = \frac{1}{2}\left(1 - \sqrt{1 + \frac{-1}{1+\gamma_0}}\right)$$

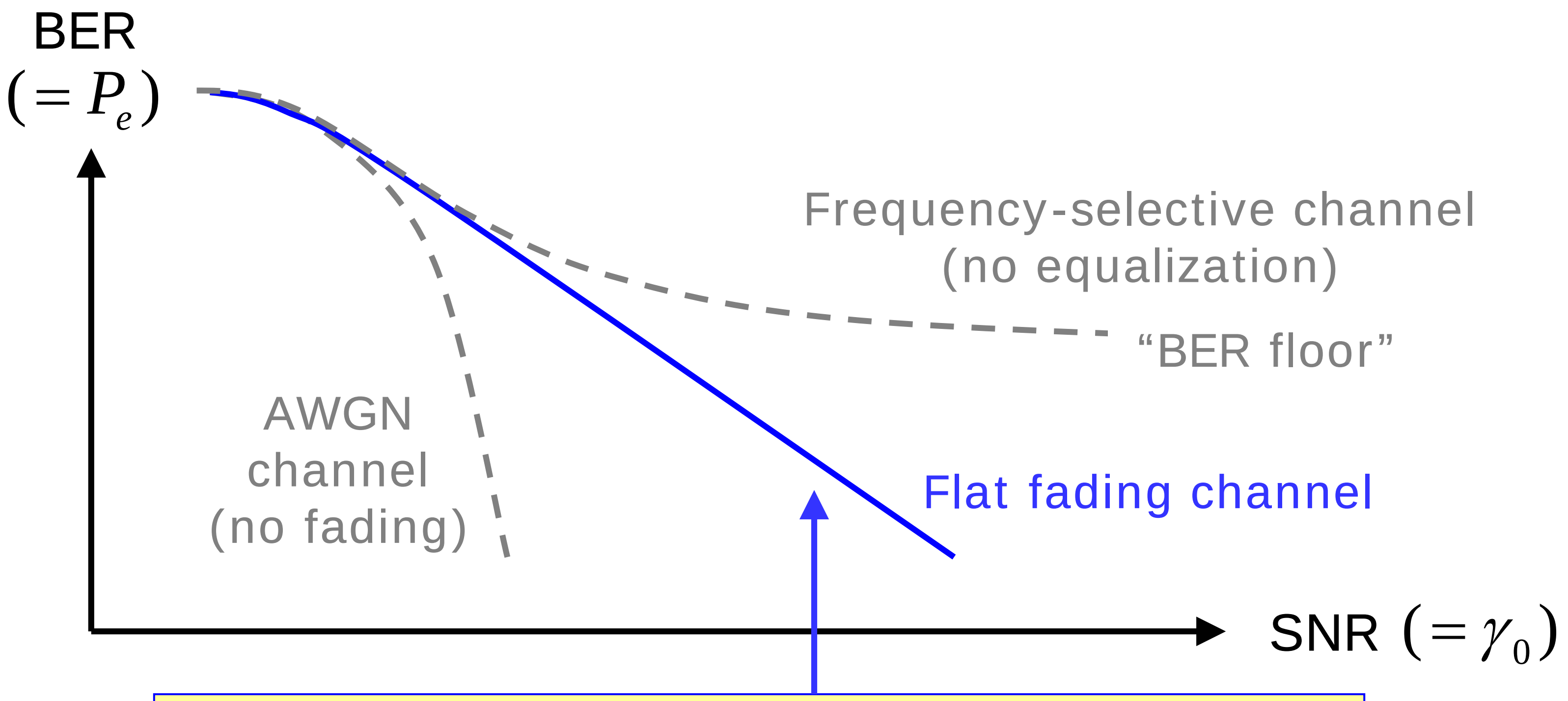
Then, we use

$$\sqrt{1+x} = 1 + x/2 + \ldots$$

which leads to

$$\boxed{P_e \approx 1/4\gamma_0}$$ for large γ_0 .

BER vs. SNR (cont.)



$P_e \approx 1/4\gamma_0$ means a straight line in log/log scale

BER vs. SNR, summary

Modulation	$P_e(\gamma)$	P_e	P_e (for large γ_0)
2-PSK	$Q\left(\sqrt{2\gamma}\right)$	$\frac{1}{2}\left(1-\sqrt{\frac{\gamma_0}{1+\gamma_0}}\right)$	$1/4\gamma_0$
DPSK	$e^{-\gamma}/2$	$1/\left(2\gamma_0+2\right)$	$1/2\gamma_0$
2-FSK (coh.)	$Q\left(\sqrt{\gamma}\right)$	$\frac{1}{2}\left(1-\sqrt{\frac{\gamma_0}{2+\gamma_0}}\right)$	$1/2\gamma_0$
2-FSK (non-c.)	$e^{-\gamma/2}/2$	$1/\left(\gamma_0+2\right)$	$1/\gamma_0$

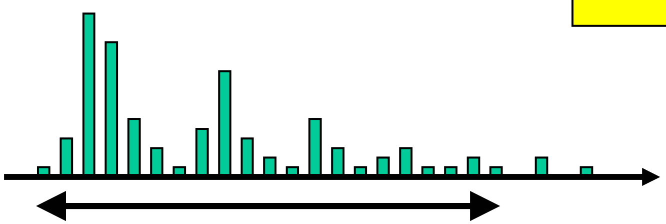
Three kinds of systems (2)

Wideband system (TDM, TDMA):

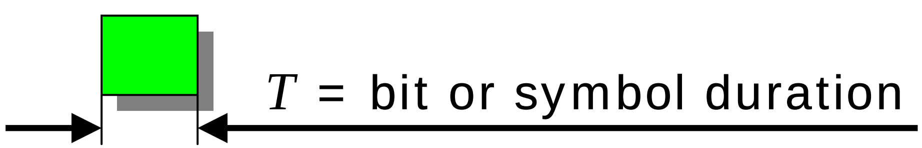
Selective fading channel, transversal filter channel model, good performance possible through adaptive equalization.

$$\boxed{T \approx T_m \quad \ldots \quad T << T_m}$$

Intersymbol interference causes signal distortion



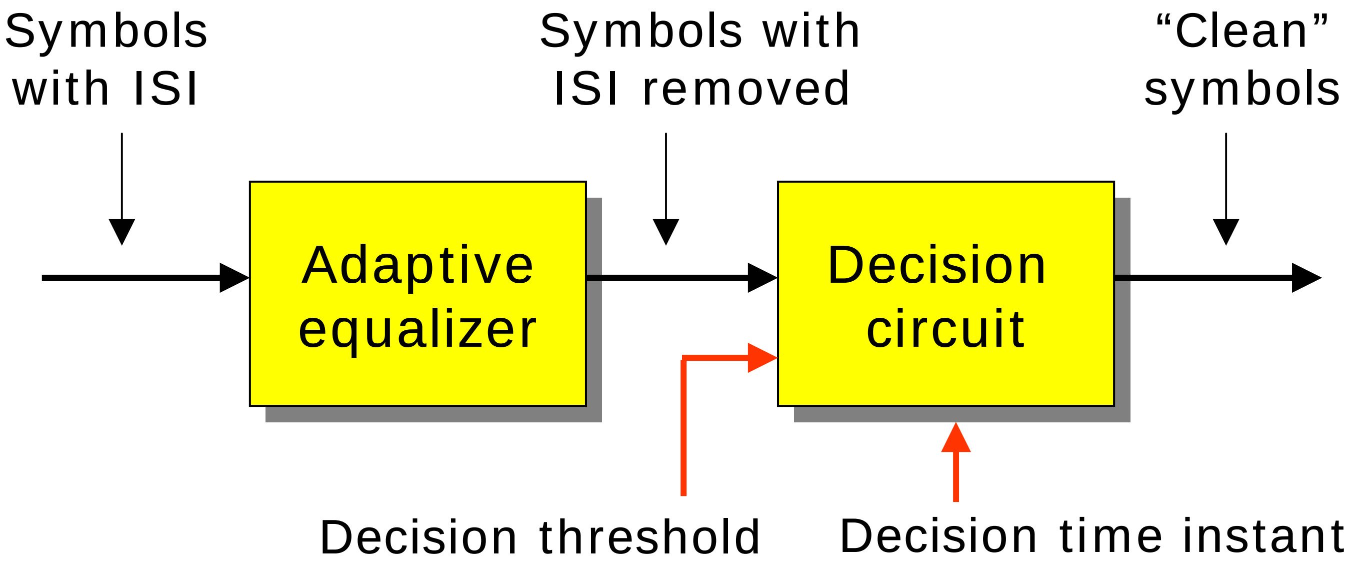
T_m = delay spread of multipath channel



T = bit or symbol duration

Receiver structure

The intersymbol interference of received symbols (bits) must be removed before decision making (the case is illustrated below for a binary signal, where symbol = bit):



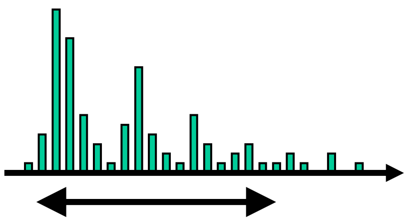
Three kinds of systems (3)

Wideband system (DS-CDMA):

Selective fading channel, transversal filter channel model, good performance possible through use of Rake receiver.

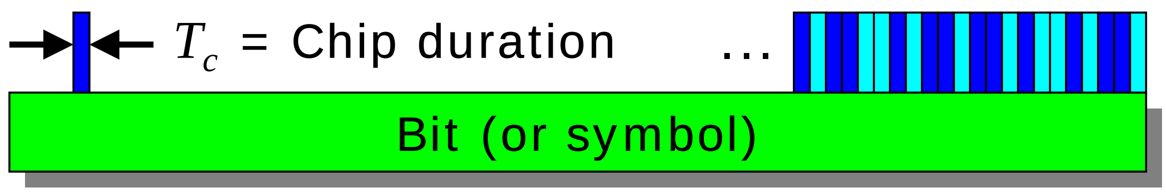
$$T >> T_m >> T_c$$

$$T_c \propto W$$



T_m = delay spread of multipath channel

T_c = Chip duration ...



T = bit (or symbol) duration

Rake receiver structure and operation

- Rake receiver <=> a signal processing example that illustrates some important concepts
- Rake receiver is used in DS-CDMA (Direct Sequence Code Division Multiple Access) systems
- Rake “fingers” synchronize to signal components that are received via a wideband multipath channel
- Important task of Rake receiver is channel estimation
- Output signals from Rake fingers are combined, for instance using Maximum Ratio Combining (MRC)

To start with: multipath channel

Suppose a signal $s(t)$ is transmitted. A multipath channel with M physical paths can be presented (in equivalent low-pass signal domain) in form of its Channel Impulse Response (CIR)

$$h(t) = \sum_{m=0}^{M-1} a_m e^{j\phi_m} \delta\left(t - \tau_m\right)$$

in which case the received (equivalent low-pass) signal is of the form

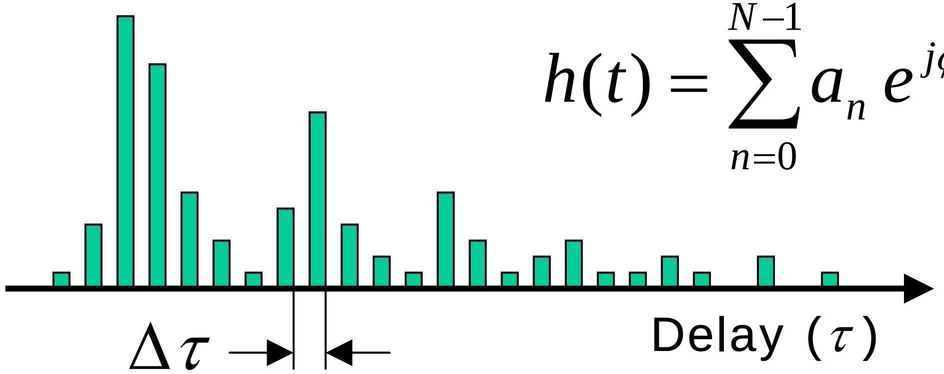
$$r(t) = s(t) * h(t) = \sum_{m=0}^{M-1} a_m e^{j\phi_m} s\left(t - \tau_m\right).$$

Sampled channel impulse response

Sampled Channel Impulse Response (CIR)

The CIR can also be presented in sampled form using N complex-valued samples uniformly spaced at most $1/W$ apart, where W is the RF system bandwidth:

$$h(t) = \sum_{n=0}^{N-1} a_n \, e^{j\phi_n} \, \delta\left(t - n \cdot \Delta\tau\right)$$



Uniformly spaced channel samples

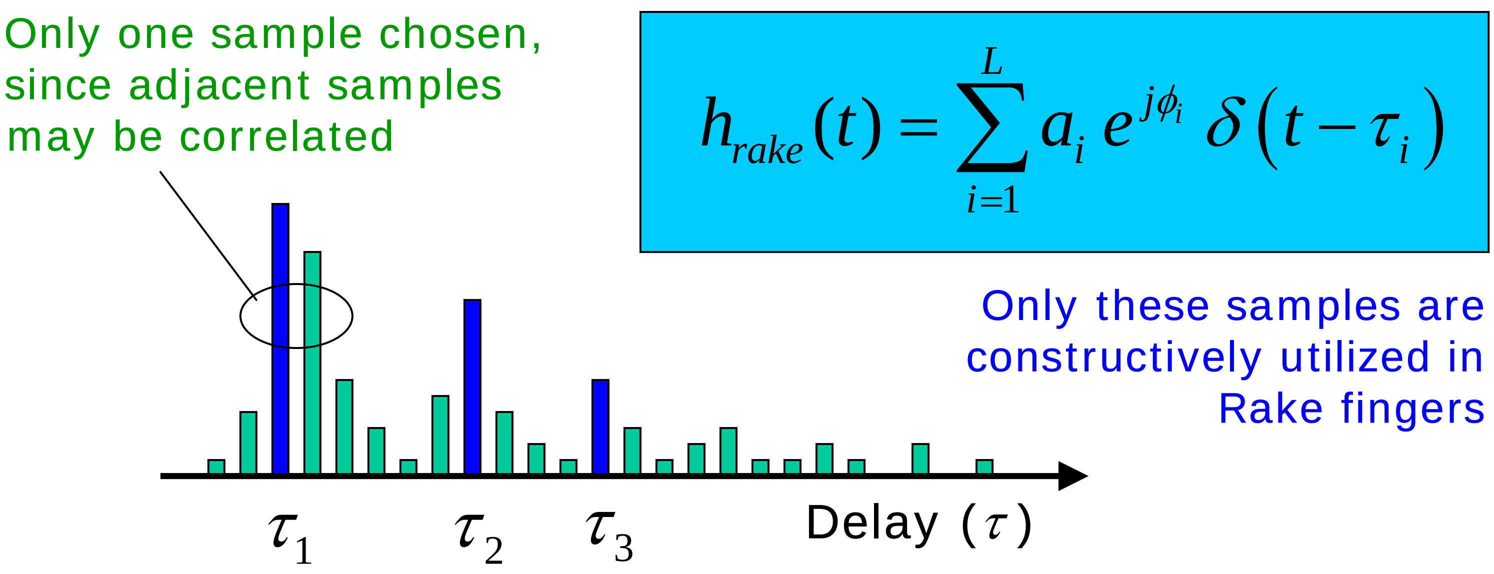
$$\Delta\tau \le 1/W$$

CIR sampling rate = for instance sampling rate used in receiver during A/D conversion.

Rake finger selection

Channel estimation circuit of Rake receiver selects L strongest samples (components) to be processed in L Rake fingers:

$$h_{rake}(t) = \sum_{i=1}^{L} a_i \, e^{j\phi_i} \, \delta\left(t - \tau_i\right)$$

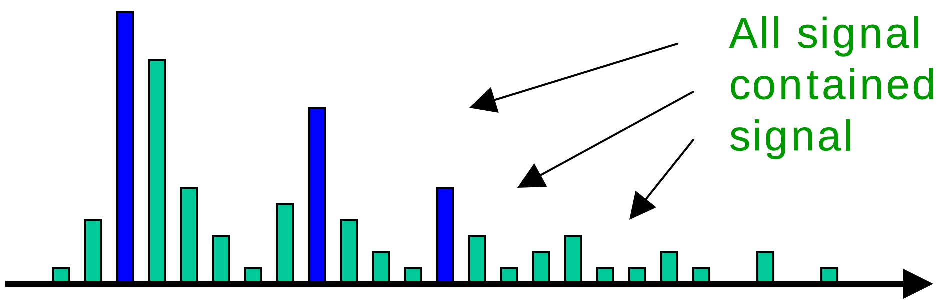


In the Rake receiver example to follow, we assume L = 3.

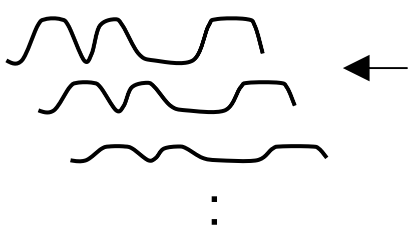
Received multipath signal

Received signal consists of a sum of delayed (and weighted) replicas of transmitted signal.

Blue samples indicate signal replicas processed in Rake fingers
Green samples only cause interference



All signal replicas are contained in received signal

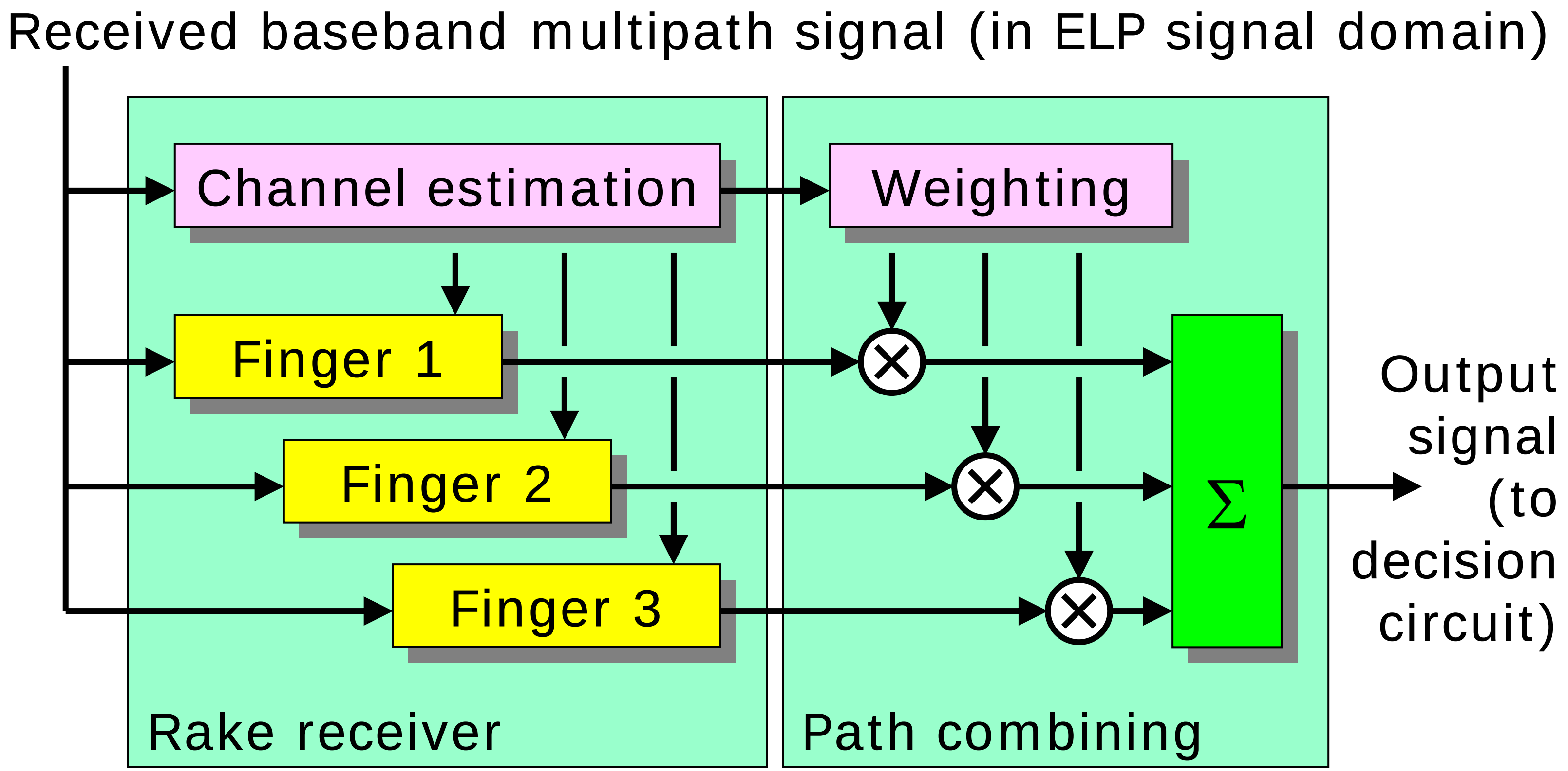


Signal replicas: same signal at different delays, with different amplitudes and phases

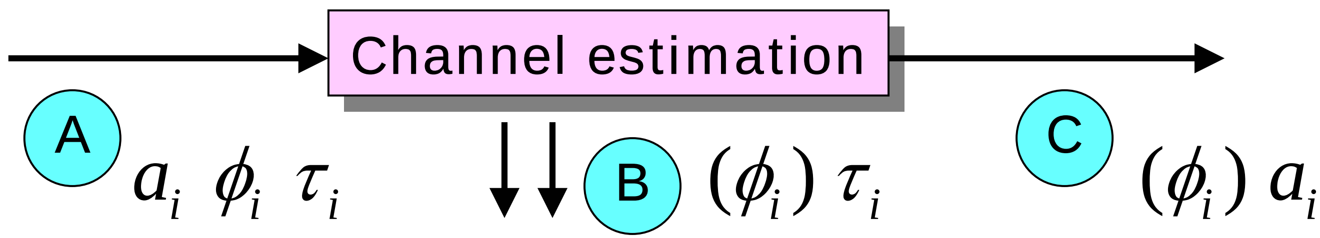
Summation in channel <=> “smeared” end result

Rake receiver

(Generic structure, assuming 3 fingers)



Channel estimation



A Amplitude, phase and delay of signal components detected in Rake fingers must be estimated.

B Each Rake finger requires delay (and often also phase) information of the signal component it is processing.

C Maximum Ratio Combining (MRC) requires amplitude (and phase if this is not utilized in Rake fingers) of components processed in Rake fingers.

Rake finger processing

Case 1: same code in I and Q branches

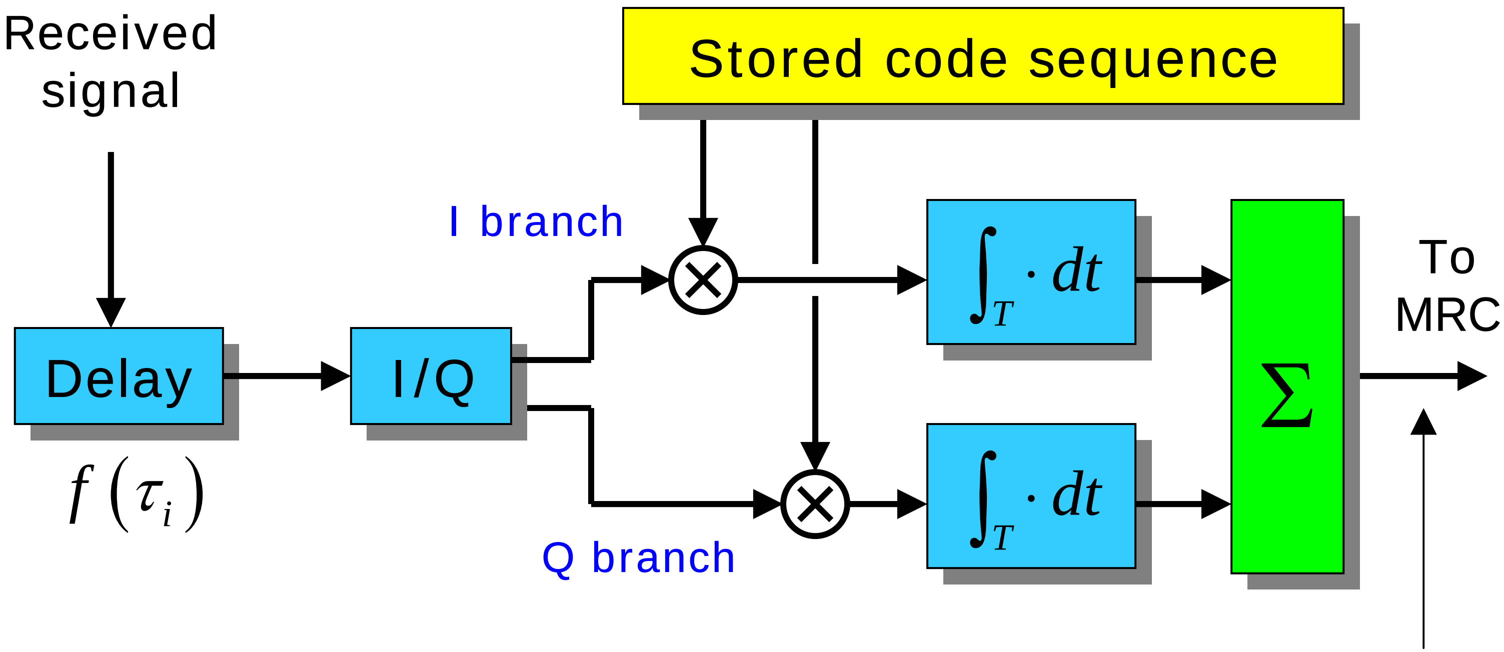
- for purpose of easy demonstration only
- no phase synchronization in Rake fingers

Case 2: different codes in I and Q branches

- the real case e.g. in IS-95 and WCDMA
- phase synchronization in Rake fingers

Rake finger processing

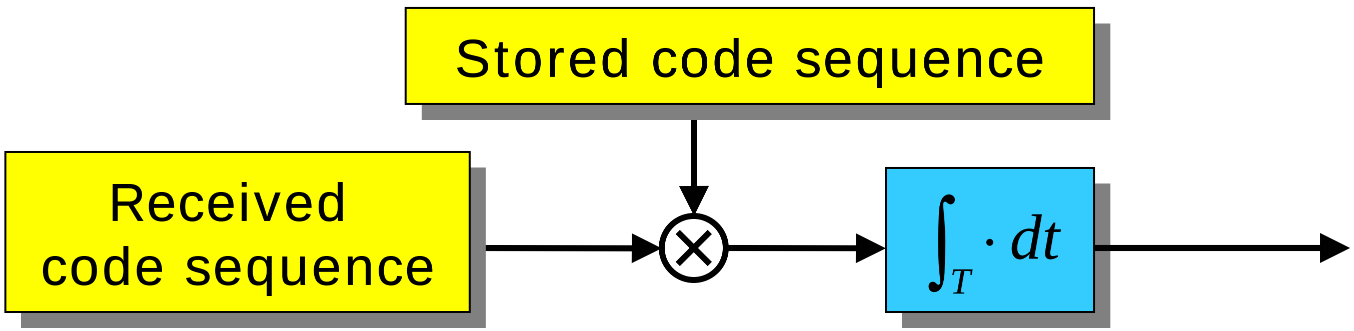
(Case 1: same code in I and Q branches)



Output of finger: a complex signal value for each detected bit

Correlation vs. matched filtering

Basic idea of correlation:



Same result through matched filtering and sampling:

Same end result (in theory)

Rake finger processing

Correlation with stored code sequence has different impact on different parts of the received signal

$$r(t) = z(t) + v(t) + w(t)$$

$$= a_i\, e^{j\phi_i} s(t-\tau_i) + \sum_{\substack{n=1 \\ n \neq i}}^{N} a_n\, e^{j\phi_n} s(t-\tau_n) + w(t)$$

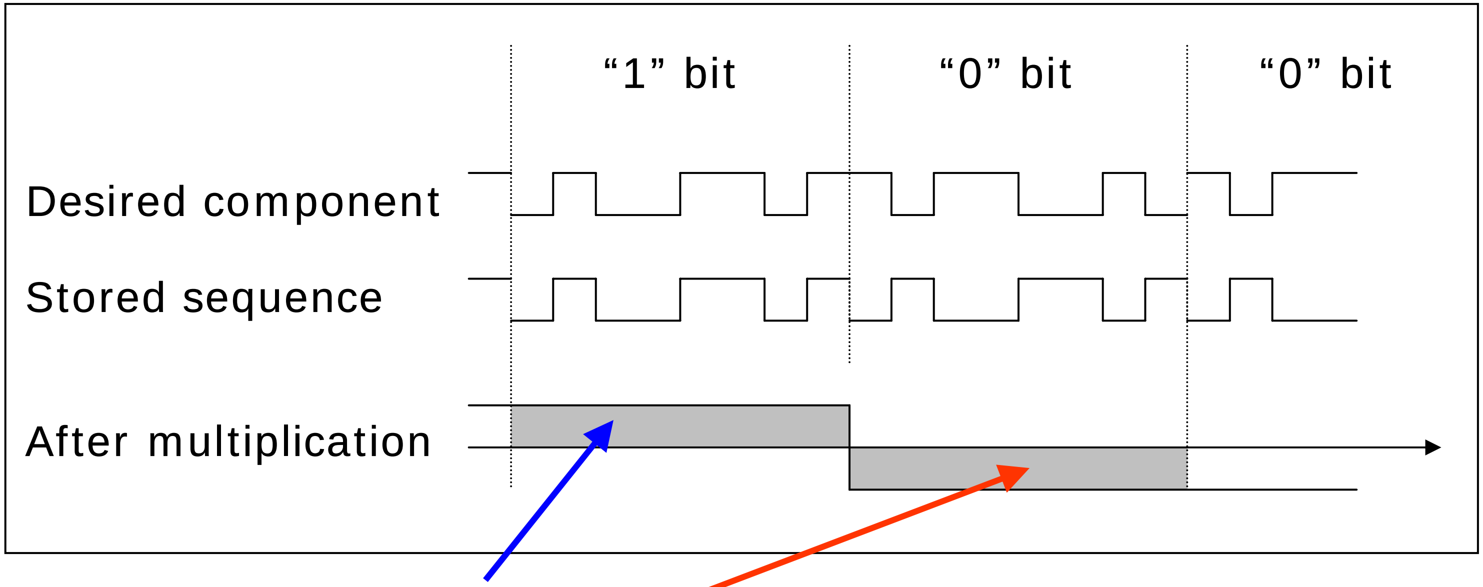
$z(t)$ = desired signal component detected in *i*:th Rake finger

$v(t)$ = other signal components causing interference

$w(t)$ = other codes causing interference (+ noise ...)

Rake finger processing

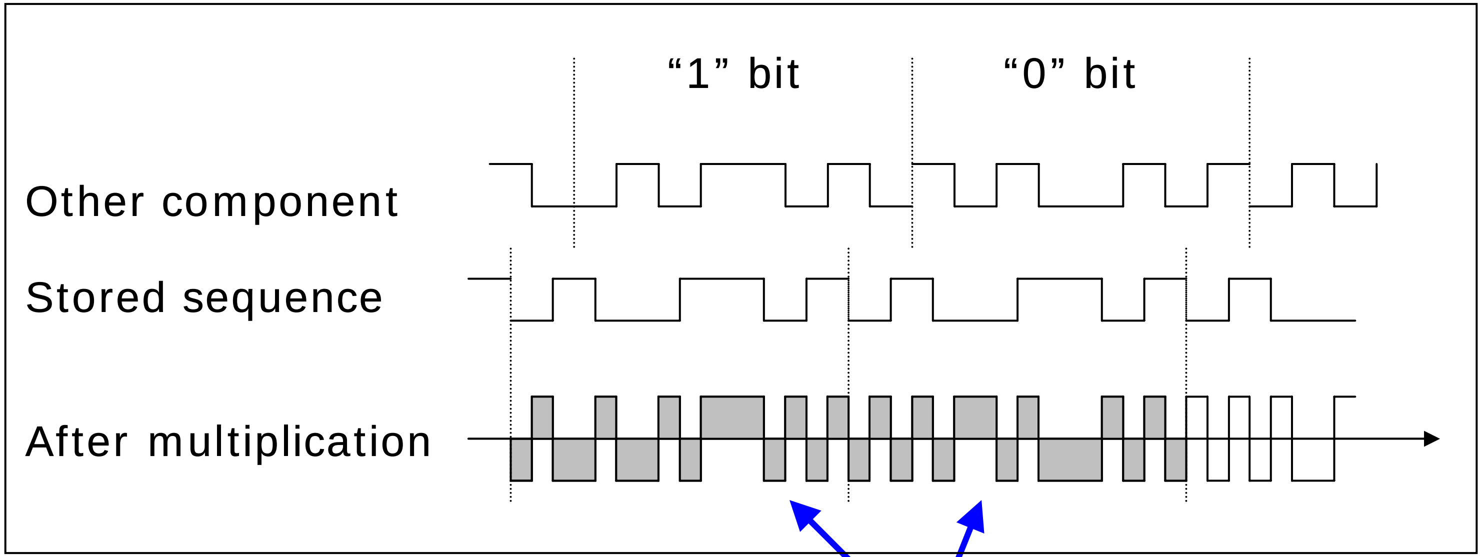
Illustration of correlation (in one quadrature branch) with desired signal component (i.e. correctly aligned code sequence)



Strong positive/negative “correlation result” after integration

Rake finger processing

Illustration of correlation (in one quadrature branch) with some other signal component (i.e. non-aligned code sequence)



Weak "correlation result" after integration

Rake finger processing

Mathematically:

Correlation result for bit between $\left(0,T\right)$

$$C_i = \int_0^T \left[z(t) + v(t) + w(t) \right] s(t)\, dt$$

$$= a_i\, e^{j\phi_i} \int_0^T s^2(t)\, dt$$

Desired signal

Interference from same signal

$$+ \sum_{\substack{n=1 \\ n \neq i}}^{N} a_n\, e^{j\phi_n} \int_0^T s(t - \tau_n + \tau_i)\, s(t)\, dt + \int_0^T w(t)\, s(t)\, dt$$

Interference from other signals

Rake finger processing

Set of codes must have both:

- good autocorrelation properties (same code sequence)
- good cross-correlation properties (different sequences)

$$C_i = \boxed{a_i e^{j\phi_i} \int_0^T s^2(t)\,dt}$$

Large

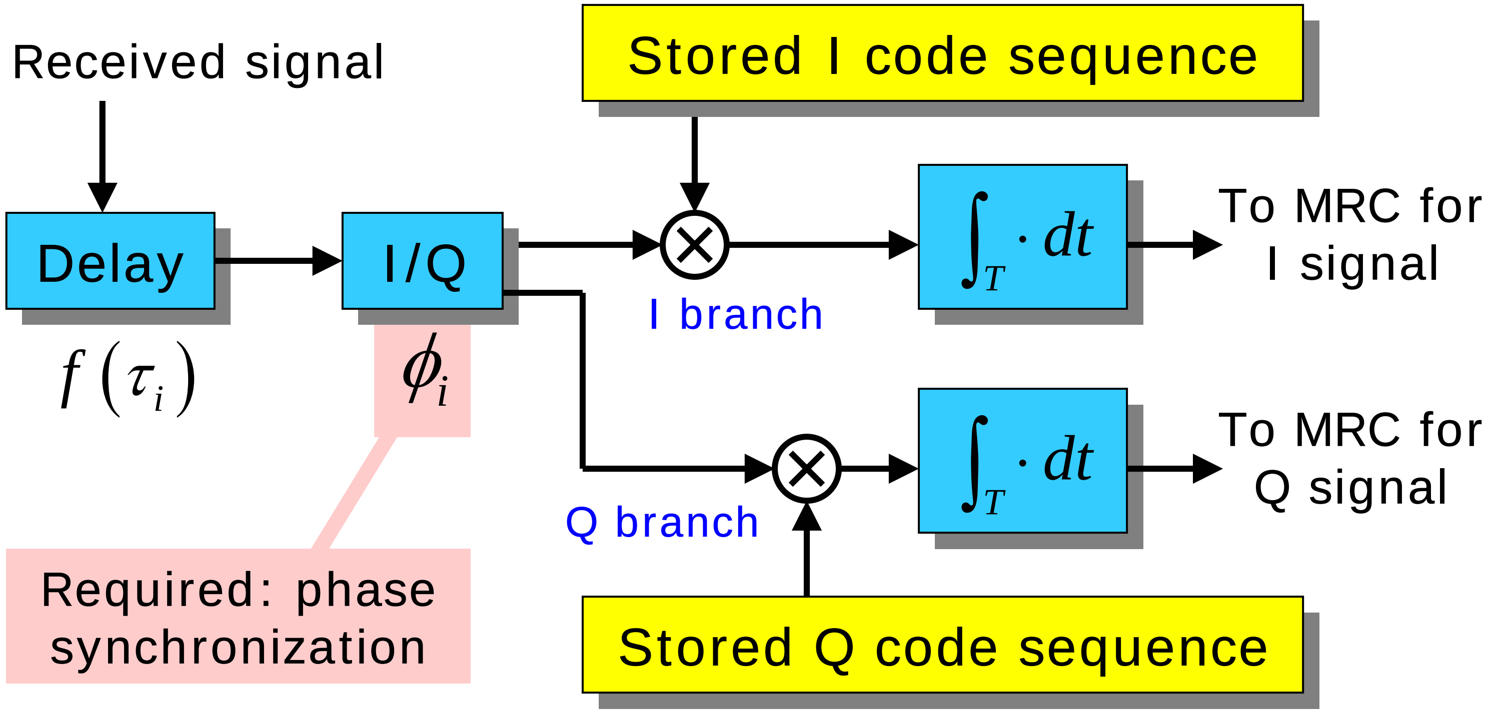
Small

Small

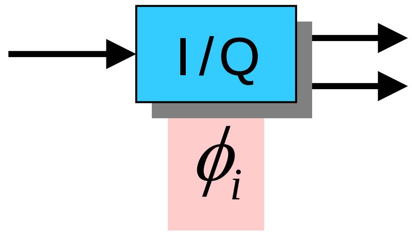
$$+ \boxed{\sum_{\substack{n=1 \\ n \neq i}}^{N} a_n e^{j\phi_n} \int_0^T s(t - \tau_n + \tau_i)\, s(t)\,dt} + \boxed{\int_0^T w(t)\, s(t)\,dt}$$

Rake finger processing

(Case 2: different codes in I and Q branches)

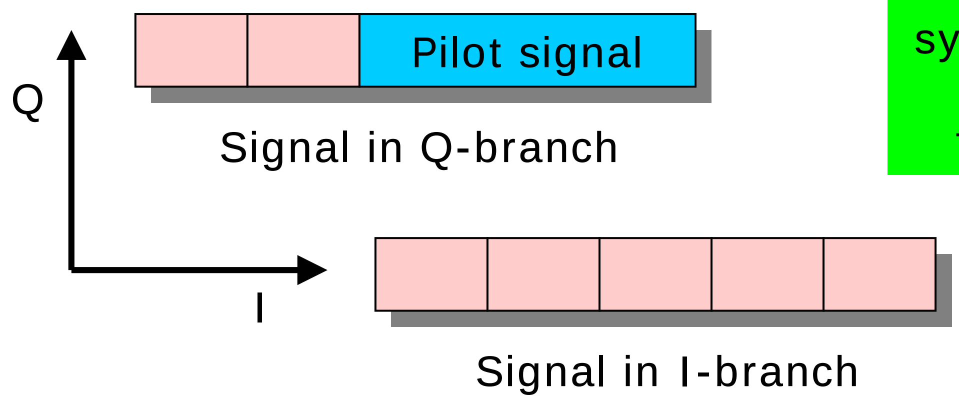


Phase synchronization



When different codes are used in the quadrature branches (as in practical systems such as IS-95 or WCDMA), phase synchronization is necessary.

Phase synchronization is based on information within received signal (pilot signal or pilot channel).



Note: phase synchronization must be done for each finger separately!

Weighting

(Case 1: same code in I and Q branches)

Maximum Ratio Combining (MRC) means weighting each Rake finger output with a complex number after which the weighted components are summed “on the real axis”:

Component is weighted

Phase is aligned

Rake finger output is complex-valued

$$Z = \sum_{i=1}^{3} \underbrace{a_i e^{-j\phi_i} \cdot a_i e^{j\phi_i}}_{\text{real-valued}}$$

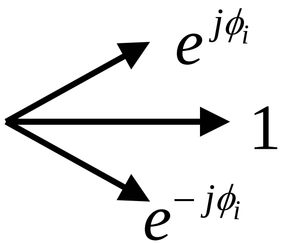
real-valued

Instead of phase alignment: take absolute value of finger outputs ...

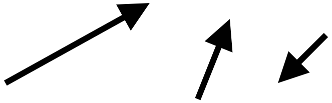
Phase alignment

The complex-valued Rake finger outputs are phase-aligned using the following simple operation:

$$\boxed{e^{-j\phi_i} \cdot e^{j\phi_i} = 1}$$

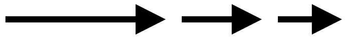


Before phase alignment:



Phasors representing complex-valued Rake finger outputs

After phase alignment:



Maximum Ratio Combining

(Case 1: same code in I and Q branches)

The signal value after Maximum Ratio Combining is:

$$\boxed{Z = a_1^{\,2} + a_2^{\,2} + a_3^{\,2}}$$

The idea of MRC: strong signal components are given more weight than weak signal components.

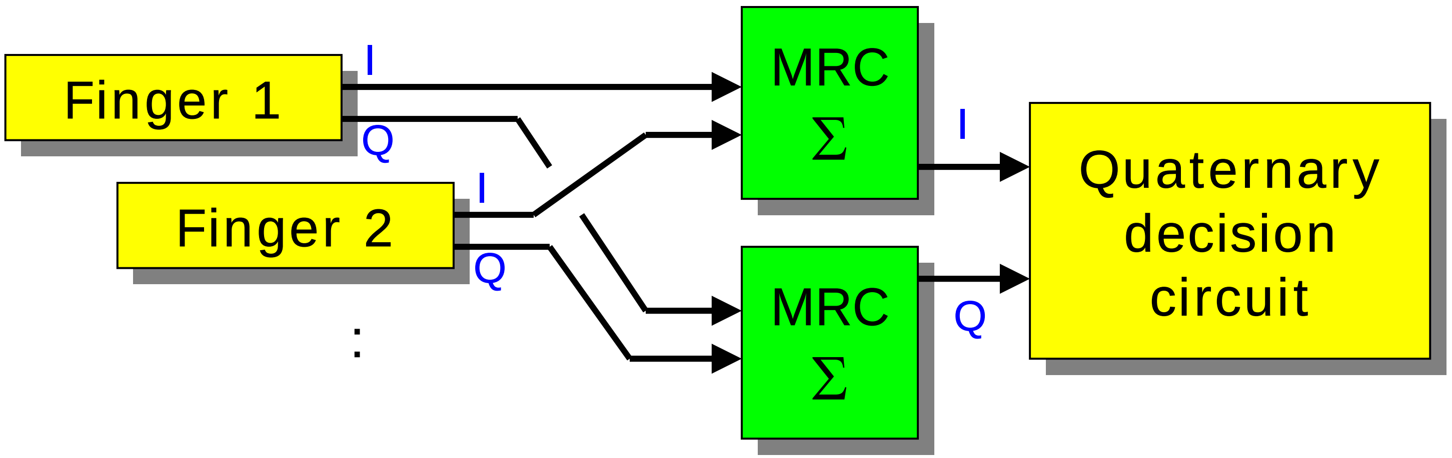
Why is the performance of MRC better than that of Equal Gain Combining (EGC)?

The answer will be given in future lecture (diversity methods).

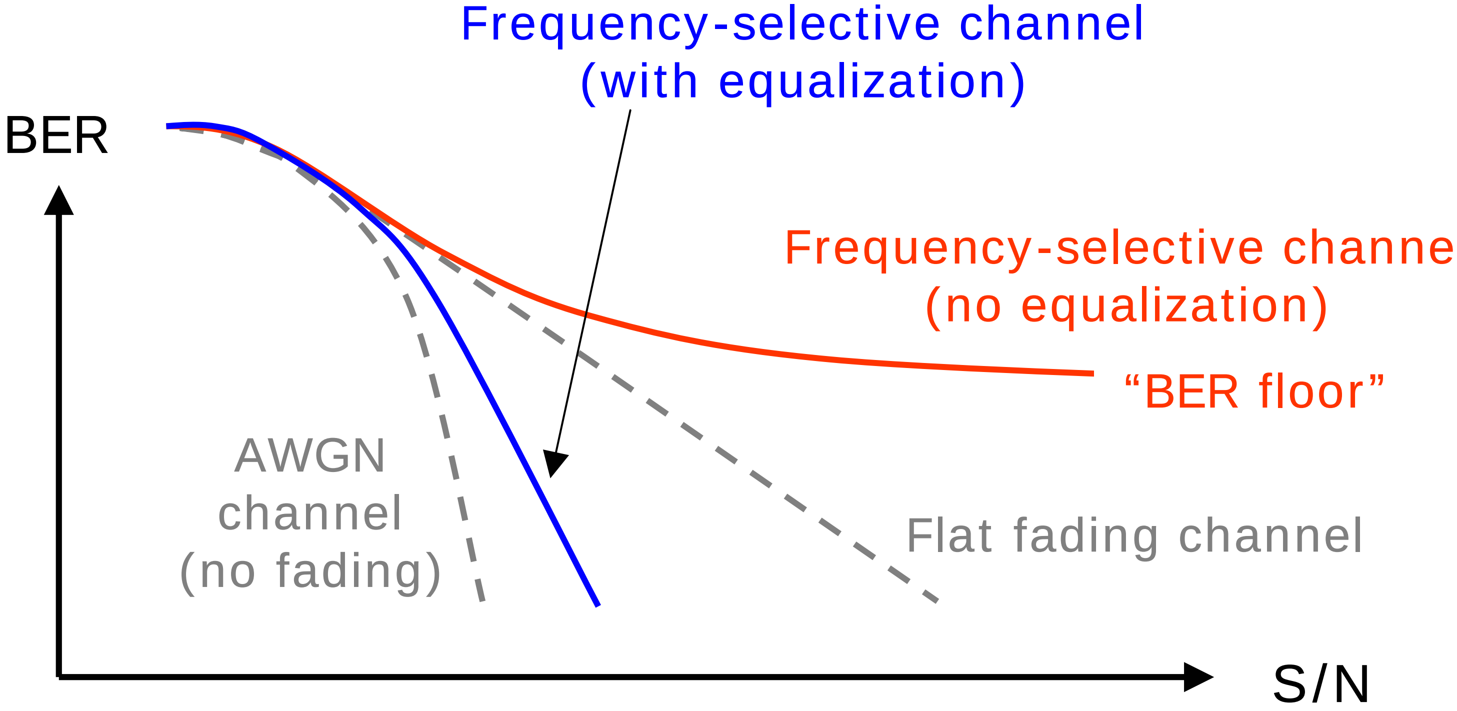
Maximum Ratio Combining
(Case 2: different codes in I and Q branches)

Output signals from the Rake fingers are already phase aligned (this is a benefit of finger-wise phase synchronization).

Consequently, I and Q outputs are fed via separate MRC circuits to the decision circuit (e.g. QPSK demodulator).



Wideband system: BER performance



Better performance through diversity

Diversity ⇔ the receiver is provided with multiple copies of the transmitted signal. The multiple signal copies should experience *uncorrelated fading* in the channel.

In this case the probability that *all* signal copies fade simultaneously is reduced dramatically with respect to the probability that a *single* copy experiences a fade.

As a rough rule:

P_e is proportional to $\dfrac{1}{\gamma_0^{\,L}}$

Diversity of *L*:th order

BER

Average SNR

Different kinds of diversity methods

Space diversity:

Several receiving antennas spaced sufficiently far apart (spatial separation should be sufficently large to reduce correlation between diversity branches, e.g. > 10λ).

Time diversity:

Transmission of same signal sequence at different times (time separation should be larger than the coherence time of the channel).

Frequency diversity:

Transmission of same signal at different frequencies (frequency separation should be larger than the coherence bandwidth of the channel).

Diversity methods (cont.)

Polarization diversity:

Only two diversity branches are available. Not widely used.

Multipath diversity:

- Signal replicas received at different delays (RAKE receiver in CDMA)
- Signal replicas received via different angles of arrival (directional antennas at the receiver)
- Equalization in a TDM/TDMA system provides similar performance as multipath diversity.

Selection diversity vs. signal combining

Selection diversity: Signal with best quality is selected.

Equal Gain Combining (EGC)

Signal copies are combined coherently:

$$Z_{EGC} = \sum_{i=1}^{L} a_i \, e^{j\phi_i} e^{-j\phi_i} = \sum_{i=1}^{L} a_i$$

Maximum Ratio Combining (MRC, best SNR is achieved)

Signal copies are weighted and combined coherently:

$$Z_{MRC} = \sum_{i=1}^{L} a_i \, e^{j\phi_i} \, a_i \, e^{-j\phi_i} = \sum_{i=1}^{L} a_i^{\,2}$$

Selection diversity performance

We assume:

(*a*) uncorrelated fading in diversity branches

(*b*) fading in *i*:th branch is Rayleigh distributed

(*c*) => SNR is exponentially distributed:

$$p(\gamma_i) = \frac{1}{\gamma_0} e^{-\gamma_i/\gamma_0}, \quad \gamma_i \geq 0.$$

PDF

Probability that SNR in branch *i* is less than threshold y :

$$P(\gamma_i < y) = \int_0^y p(\gamma_i)\, d\gamma_i = 1 - e^{-y/\gamma_0} \,.$$

CDF

Selection diversity (cont.)

Probability that SNR in every branch (i.e. all *L* branches) is less than threshold y :

$$P\left(\gamma_1, \gamma_2, \ldots, \gamma_L < y\right) = \left[\int_0^y p\left(\gamma_i\right) d\gamma_i\right]^L = \left[1 - e^{-y/\gamma_0}\right]^L.$$

Note: this is true only if the fading in different branches is independent (and thus uncorrelated) and we can write

$$p\left(\gamma_1, \gamma_2, \ldots, \gamma_L\right) = p\left(\gamma_1\right) p\left(\gamma_2\right) \ldots p\left(\gamma_L\right).$$

Selection diversity (cont.)

Differentiating the cdf (cumulative distribution function) with respect to y gives the pdf

$$p(y) = L\left[1 - e^{-y/\gamma_0}\right]^{L-1} \cdot \frac{e^{-y/\gamma_0}}{\gamma_0}$$

which can be inserted into the expression for average bit error probability

$$P_e = \int_0^\infty P_e(y)\, p(y)\, dy\,.$$

The mathematics is unfortunately quite tedious ...

Selection diversity (cont.)

... but as a general rule, for large γ_0 it can be shown that

$$P_e \text{ is proportional to } \frac{1}{\gamma_0{}^L}$$

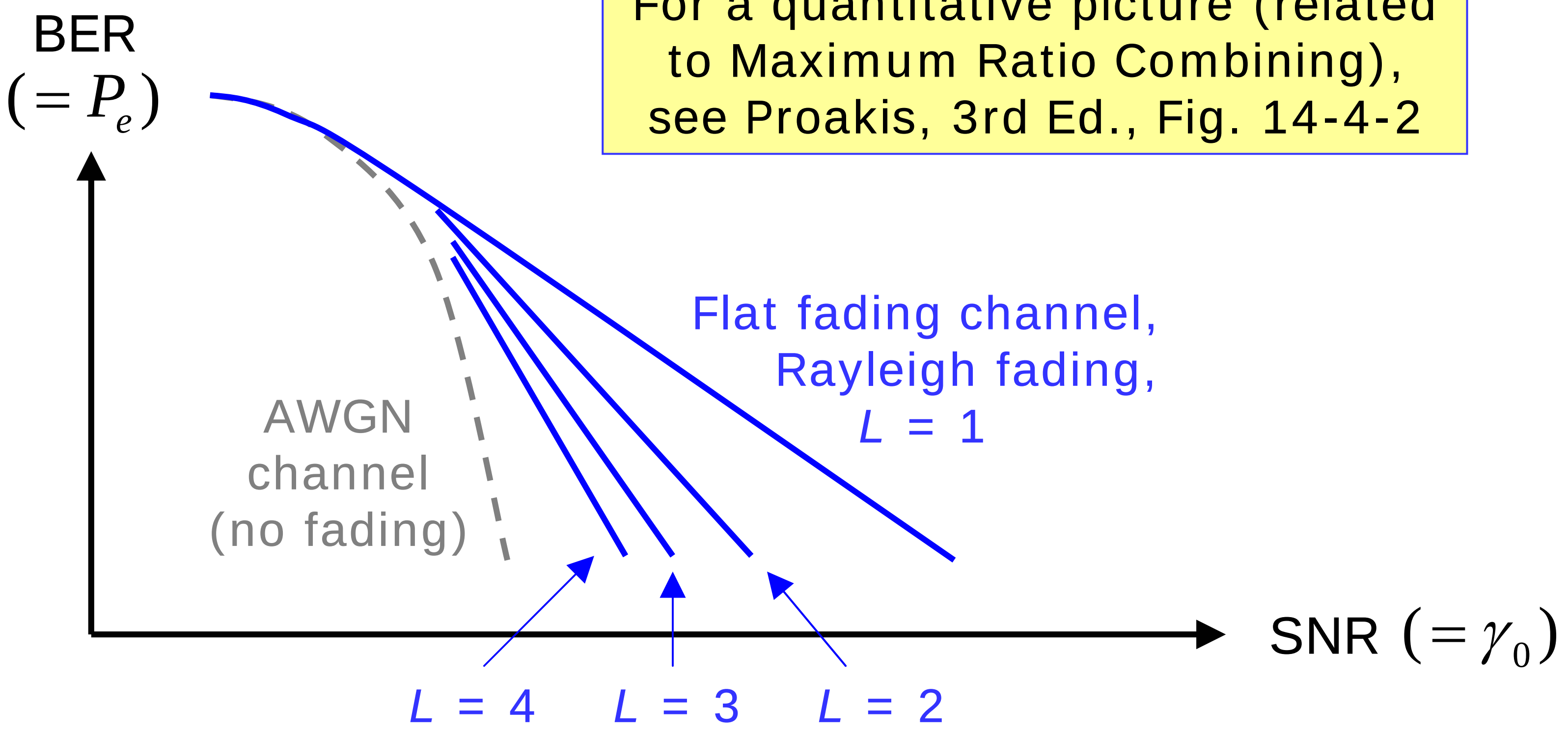
regardless of modulation scheme (2-PSK, DPSK, 2-FSK).

The largest diversity gain is obtained when moving from *L* = 1 to *L* = 2. The relative increase in diversity gain becomes smaller and smaller when *L* is further increased.

This behaviour is typical for all diversity techniques.

BER vs. SNR (diversity effect)

For a quantitative picture (related to Maximum Ratio Combining), see Proakis, 3rd Ed., Fig. 14-4-2



MRC performance

Rayleigh fading => SNR in *i*:th diversity branch is

$$\gamma_i = \frac{E_b}{N_0} a_i^{\,2} = \frac{E_b}{N_0}\left(x_i^{\,2} + y_i^{\,2}\right)$$

Rayleigh distributed magnitude

Gaussian distributed quadrature components

In case of *L* uncorrelated branches with same fading statistics, the MRC output SNR is

$$\gamma = \frac{E_b}{N_0}\left(a_1^{\,2} + a_2^{\,2} \ldots + a_L^{\,2}\right) = \frac{E_b}{N_0}\left(x_1^{\,2} + y_1^{\,2} \ldots + x_L^{\,2} + y_L^{\,2}\right)$$

MRC performance (cont.)

The pdf of γ follows the *chi-square distribution* with 2*L* degrees of freedom

Reduces to exponential pdf when *L* = 1

$$p(\gamma) = \frac{\gamma^{L-1}}{\gamma_0{}^{L}\Gamma(L)} e^{-\gamma/\gamma_o} = \frac{\gamma^{L-1}}{\gamma_0{}^{L}(L-1)!} e^{-\gamma/\gamma_o}$$

Gamma function   Factorial

For 2-PSK, the average BER is

$$P_e = \int_0^\infty P_e(\gamma)\, p(\gamma)\, d\gamma$$

$$P_e = \left(\frac{1-\mu}{2}\right)^L \sum_{k=0}^{L-1} \binom{L-1+k}{k} \left(\frac{1+\mu}{2}\right)^k$$

$$P_e(\gamma) = Q\left(\sqrt{2\gamma}\right)$$

$$\mu = \sqrt{\gamma_0 / (1+\gamma_0)}$$

MRC performance (cont.)

For large values of average SNR this expression can be approximated by

$$P_e = \left(\frac{1}{4\gamma_0} \right)^L \binom{2L-1}{L}$$

Proakis, 3rd Ed. 14-4-1

which again is according to the general rule

P_e is proportional to $\dfrac{1}{\gamma_0{}^L}$.

MRC performance (cont.)

The second term in the BER expression does not increase dramatically with *L*:

$$\binom{2L-1}{L} = \frac{(2L-1)!}{L!\cdot(L-1)!} = 1 \qquad L = 1$$

$$= 3 \qquad L = 2$$

$$= 10 \qquad L = 3$$

$$= 35 \qquad L = 4$$

BER vs. SNR for MRC, summary

For large γ_0 $\Rightarrow$ $P_e = \left(\frac{1}{k\gamma_0}\right)^L \binom{2L-1}{L}$

Proakis 3rd Ed. 14-4-1

Modulation	$P_e(\gamma)$	P_e (for large γ_0)
2-PSK	$Q\left(\sqrt{2\gamma}\right)$	$k = 4$
DPSK		$k = 2$
2-FSK (coh.)	$Q\left(\sqrt{\gamma}\right)$	$k = 2$
2-FSK (non-c.)		$k = 1$