Fading multipath radio channels

- Narrowband channel modelling
- Wideband channel modelling
- Wideband WSSUS channel
  (functions, variables & distributions)

Low-pass equivalent (LPE) signal

RF carrier frequency

$$s(t) = \mathrm{Re}\left\{ z(t) e^{j2\pi f_c t} \right\}$$

Real-valued RF signal

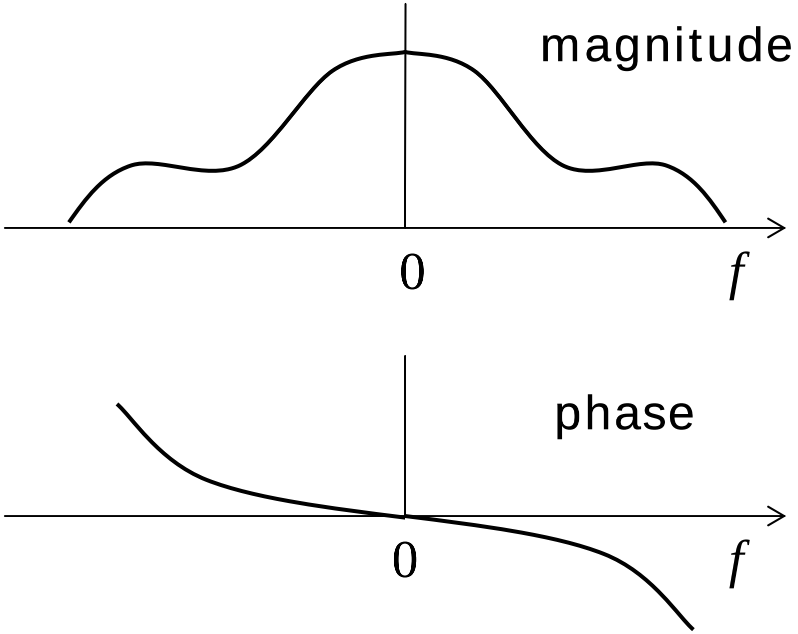
Complex-valued LPE signal

$$z(t) = x(t) + jy(t) = c(t) e^{j\phi(t)}$$

In-phase signal component

Quadrature component

Spectrum characteristics of LPE signal



Real-valued time domain signal (e.g. RF signal)

⟺

Signal spectrum is Hermitian

Complex-valued LPE time domain signal ⟺ Signal spectrum is not Hermitian

Radio channel modelling

Narrowband modelling

Calculation of path loss e.g. taking into account

- free space loss
- reflections
- diffraction
- scattering

Basic problem: signal fading

Wideband modelling

Deterministic models

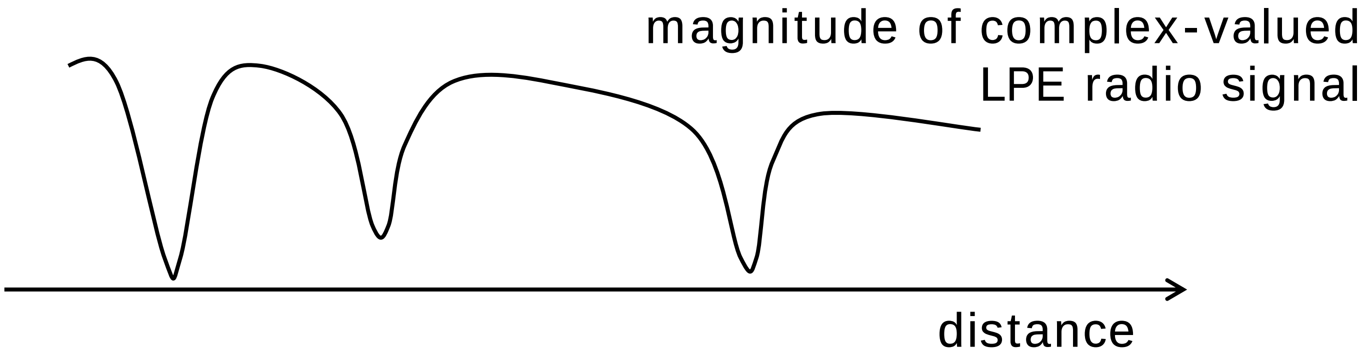
(e.g. ray tracing, playback modelling)

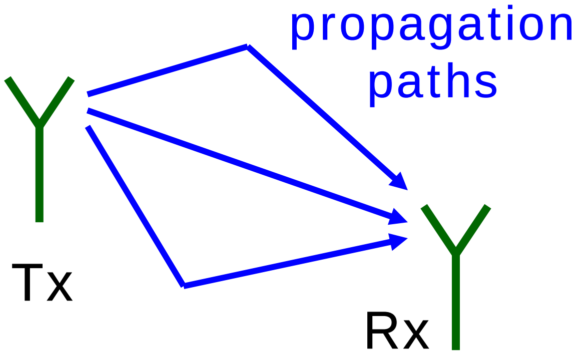
Stochastical models

(e.g. WSSUS)

Basic problem: signal dispersion

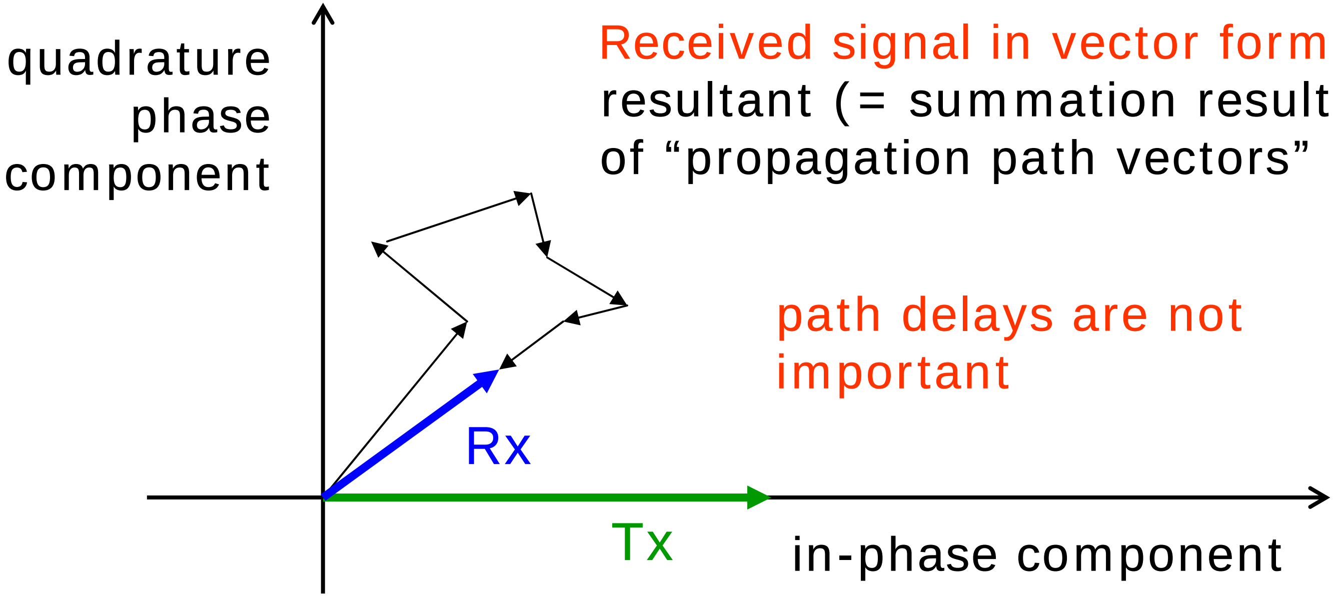
Signal fading in a narrowband channel





fade <=> signal replicas received via different propagation paths cause destructive interference

Fading: illustration in complex plane

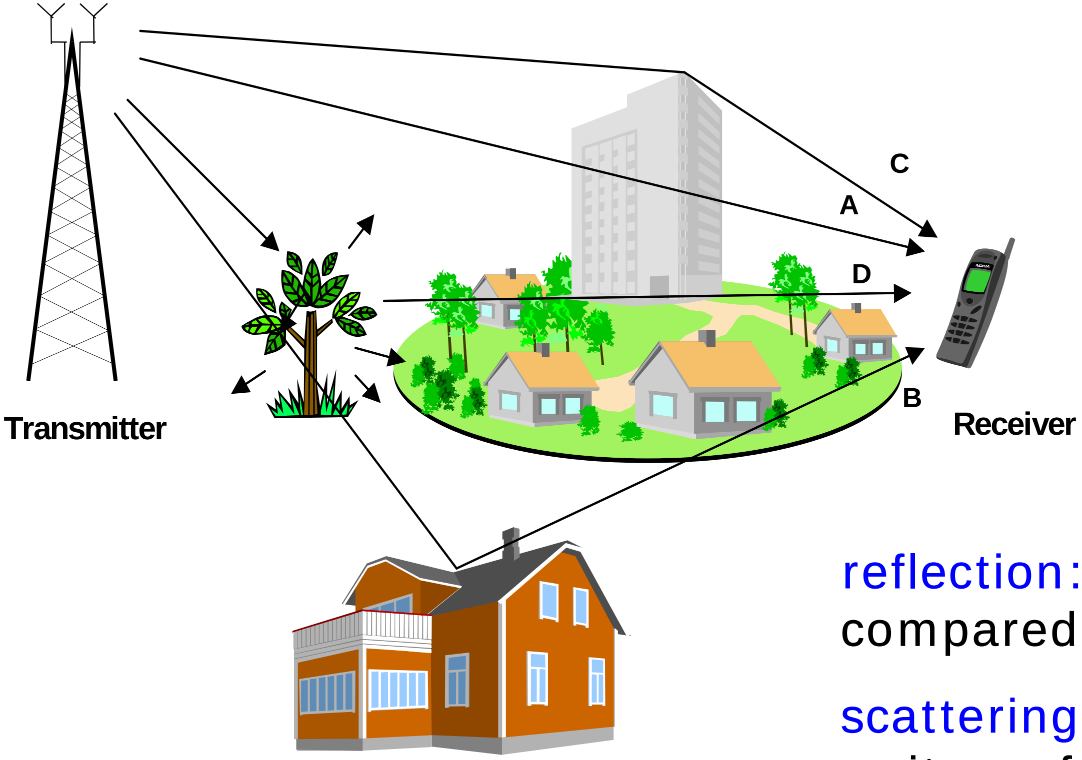


Received signal in vector form: resultant (= summation result) of “propagation path vectors”

path delays are not important

Wideband channel modelling: in addition to magnitudes and phases, also path delays are important.

Propagation mechanisms



A: free space
B: reflection
C: diffraction
D: scattering

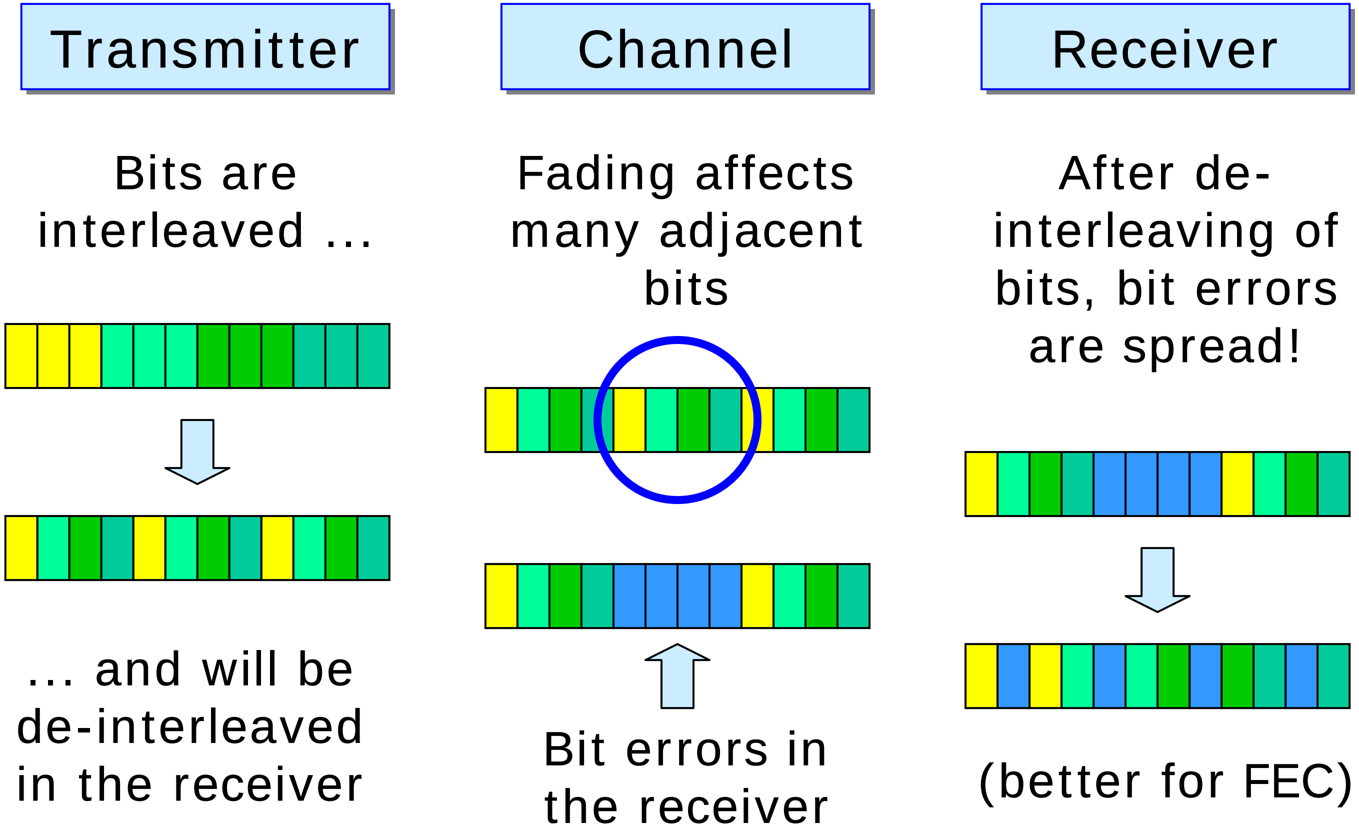
reflection: object is large compared to wavelength

scattering: object is small or its surface irregular

Countermeasures: narrowband fading

- Diversity (transmitting the same signal at different frequencies, at different times, or to/from different antennas)
  - will be investigated in later lectures
  - wideband channels => multipath diversity
- Interleaving (efficient when a fade affects many bits or symbols at a time), frequency hopping
- Forward Error Correction (FEC, uses large overhead)
- Automatic Repeat reQuest schemes (ARQ, cannot be used for transmission of real-time information)

Bit interleaving



Transmitter

Bits are interleaved …

… and will be de-interleaved in the receiver

Channel

Fading affects many adjacent bits

Bit errors in the receiver

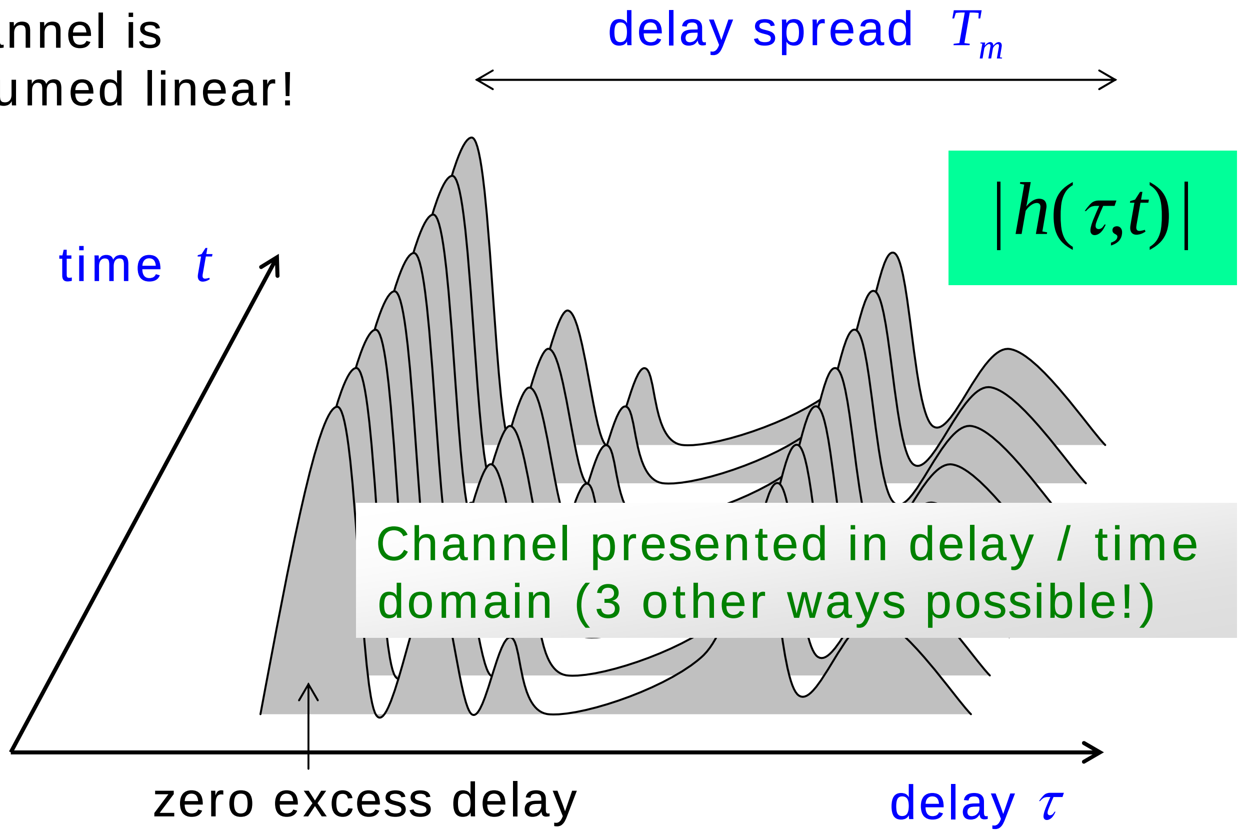
Receiver

After de-interleaving of bits, bit errors are spread!

(better for FEC)

Channel Impulse Response (CIR)

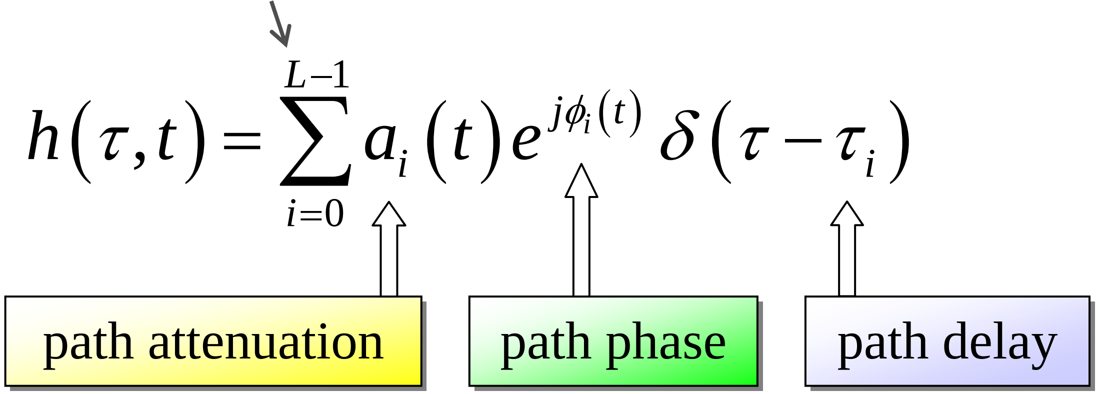
Channel is
assumed linear!



Channel presented in delay / time domain (3 other ways possible!)

CIR of a wideband fading channel

The CIR consists of L resolvable propagation paths

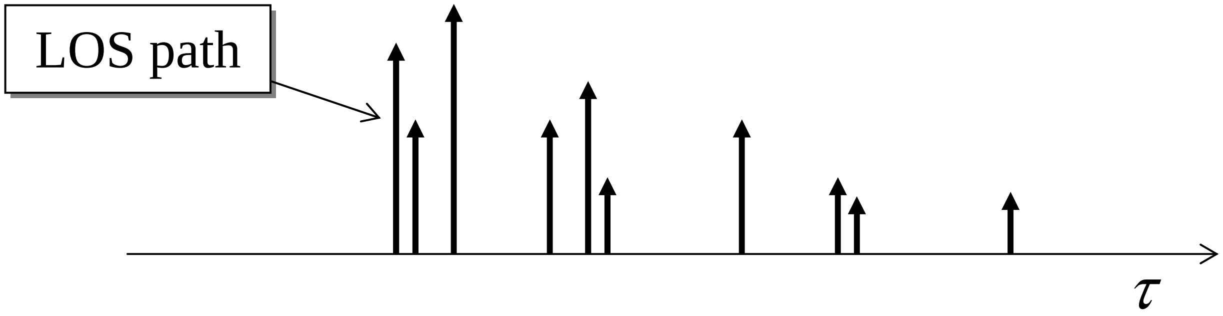


$$h(\tau,t)=\sum_{i=0}^{L-1} a_i(t)\,e^{j\phi_i(t)}\,\delta(\tau-\tau_i)$$

path attenuation

path phase

path delay



Received multipath signal

Transmitted signal:

$$s(t)=\sum_{k=-\infty}^{\infty} \underset{\Uparrow}{b_k}\, \underset{\Uparrow}{p}(t-kT)$$

complex symbol

pulse waveform

Received signal:

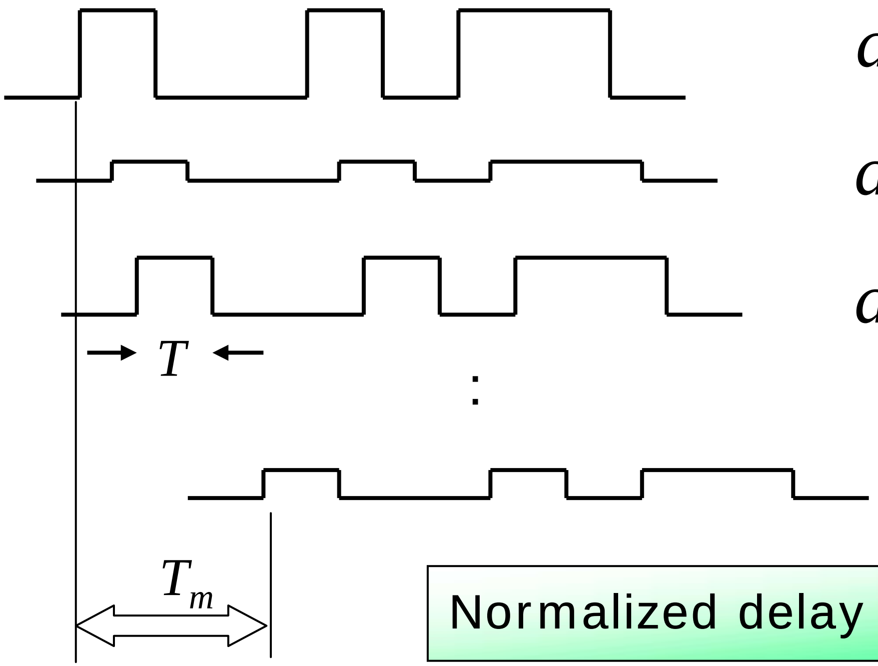
$$r(t)=h(t)*s(t)=\int_{-\infty}^{\infty} h(\tau,t)\,s(t-\tau)\,d\tau$$

$$=\sum_{i=0}^{L-1} a_i(t)\,e^{j\phi_i(t)}\,s(t-\tau_i)$$

$$\boxed{\int f(t)\,\delta(t-t_0)\,dt=f(t_0)}$$

Received multipath signal

The received multipath signal is the sum of L attenuated, phase shifted and delayed replicas of the transmitted signal $s(t)$



$$a_0\, e^{j\phi_0}\, s\left(t-\tau_0\right)$$

$$a_1\, e^{j\phi_1}\, s\left(t-\tau_1\right)$$

$$a_2\, e^{j\phi_2}\, s\left(t-\tau_2\right)$$

Normalized delay spread $D = T_m / T$

Received multipath signal

The normalized delay spread is an important quantity.
When $D << 1$, the channel is

- narrowband
- frequency-nonselective
- flat

and there is no intersymbol interference (ISI).

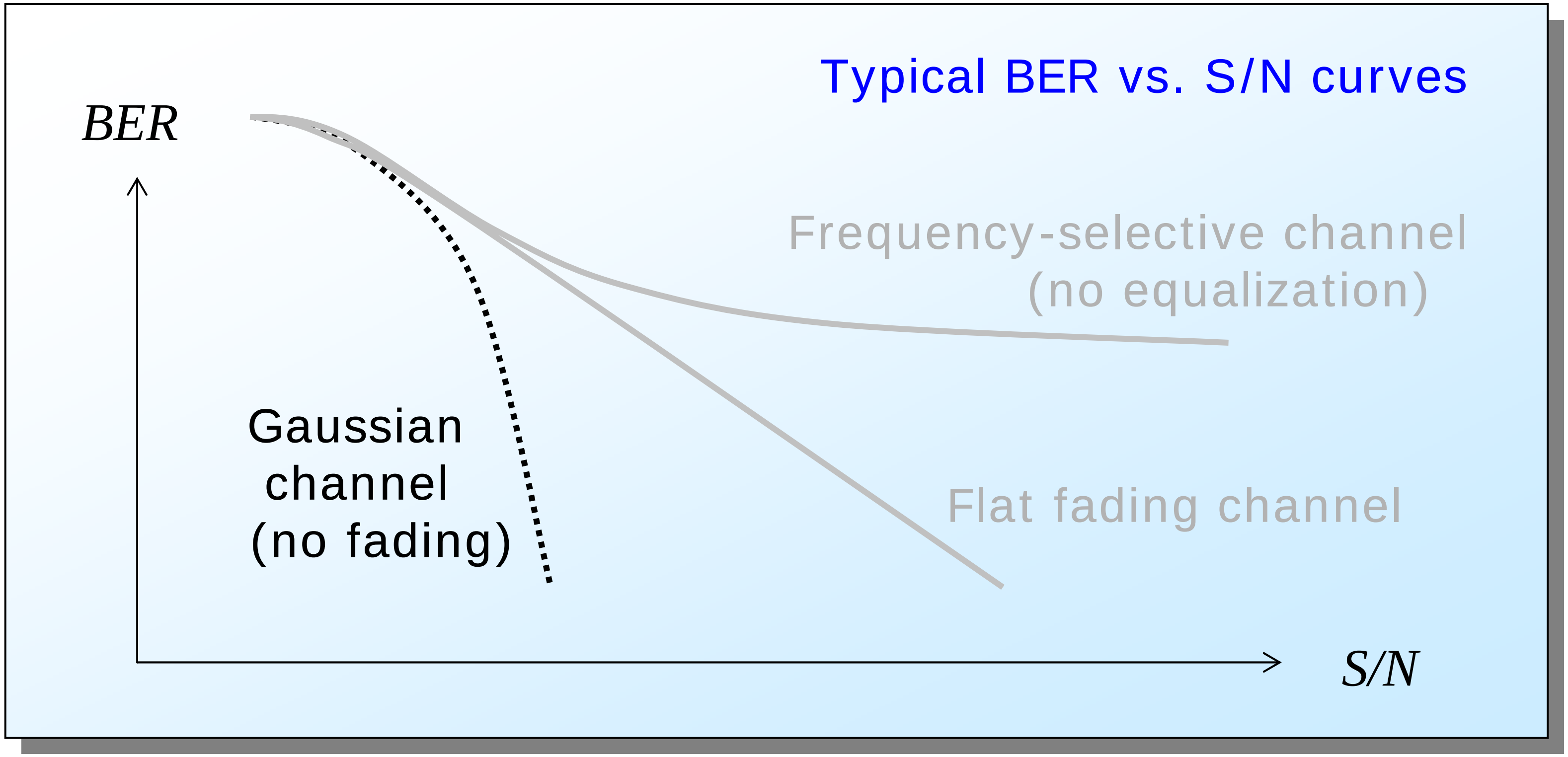
When D approaches or exceeds unity, the channel is

- wideband
- frequency selective
- time dispersive

Important feature
has many names!

BER vs. S/N performance

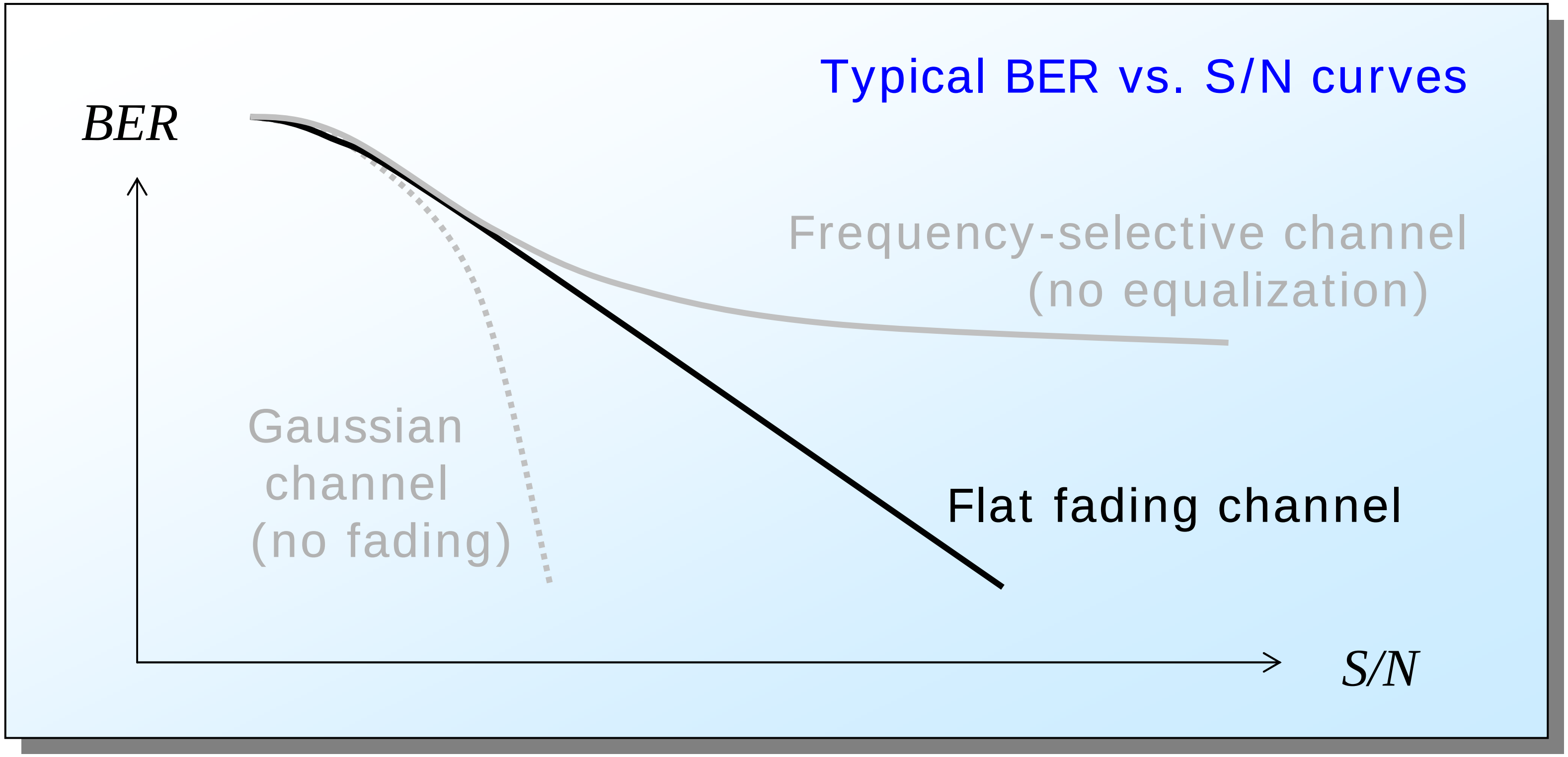
In a Gaussian channel (no fading) $BER <=> Q(S/N)$ erfc(S/N)



BER vs. S/N performance

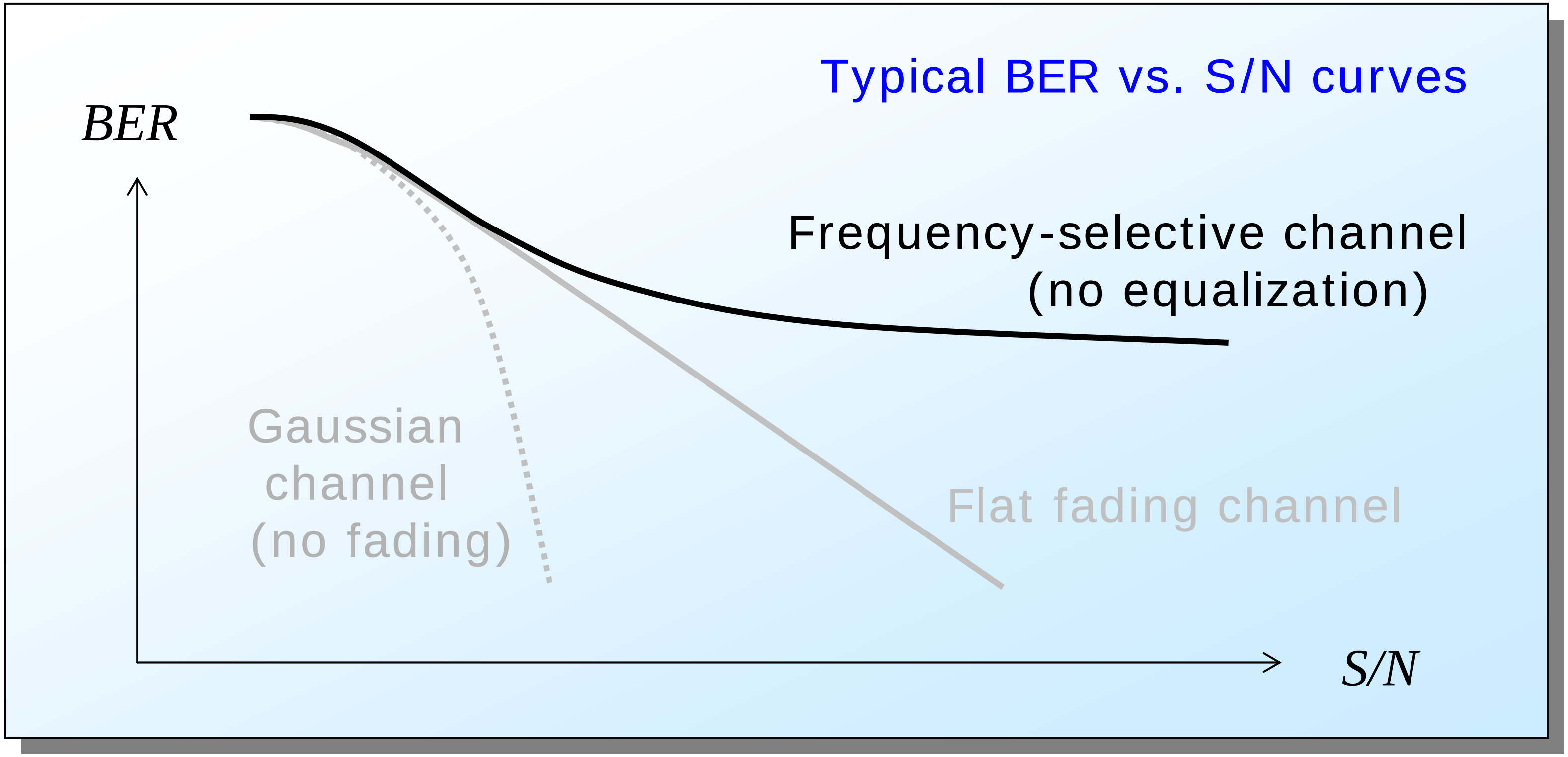
Flat fading (Proakis 7.3): $BER = \int BER\left(S/N \middle| z\right) p\left(z\right) dz$

z = signal power level



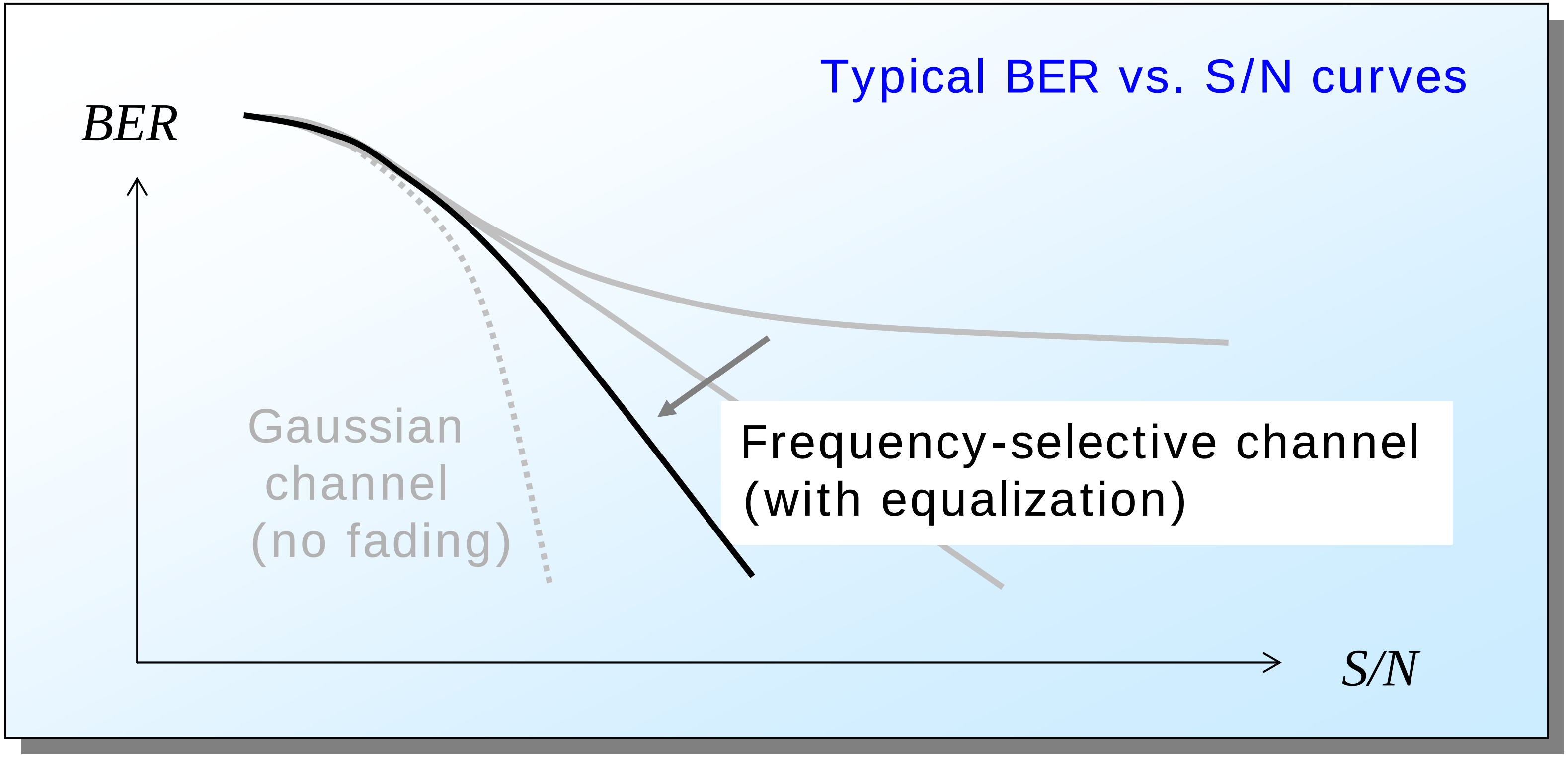
BER vs. S/N performance

Frequency selective fading <=> irreducible BER floor



BER vs. S/N performance

Diversity (e.g. multipath diversity) <=> improved performance



Time-variant transfer function

Time-variant CIR:

$$h(\tau,t)=\sum_{i=0}^{L-1}a_i(t)e^{j\phi_i(t)}\,\delta(\tau-\tau_i)$$

Time-variant transfer function (frequency response):

$$H(f,t)=\int_{-\infty}^{\infty}h(\tau,t)e^{-j2\pi f\tau}d\tau=\sum_{i=0}^{L-1}a_i(t)e^{j\phi_i(t)}e^{-j2\pi f\tau_i}$$

In a narrowband channel
this reduces to:

$$H(f,t)=\sum_{i=0}^{L-1}a_i(t)e^{j\phi_i(t)}$$

Example: two-ray channel (*L* = 2)

$$h(\tau) = a_1 e^{j\phi_1} \delta(\tau - \tau_1) + a_2 e^{j\phi_2} \delta(\tau - \tau_2)$$

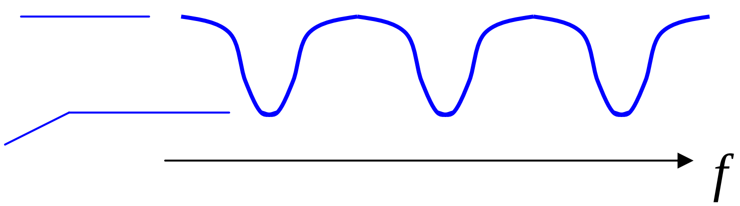
$$\Updownarrow$$

$$H(f) = a_1 e^{j\phi_1} e^{-j2\pi f \tau_1} + a_2 e^{j\phi_2} e^{-j2\pi f \tau_2}$$

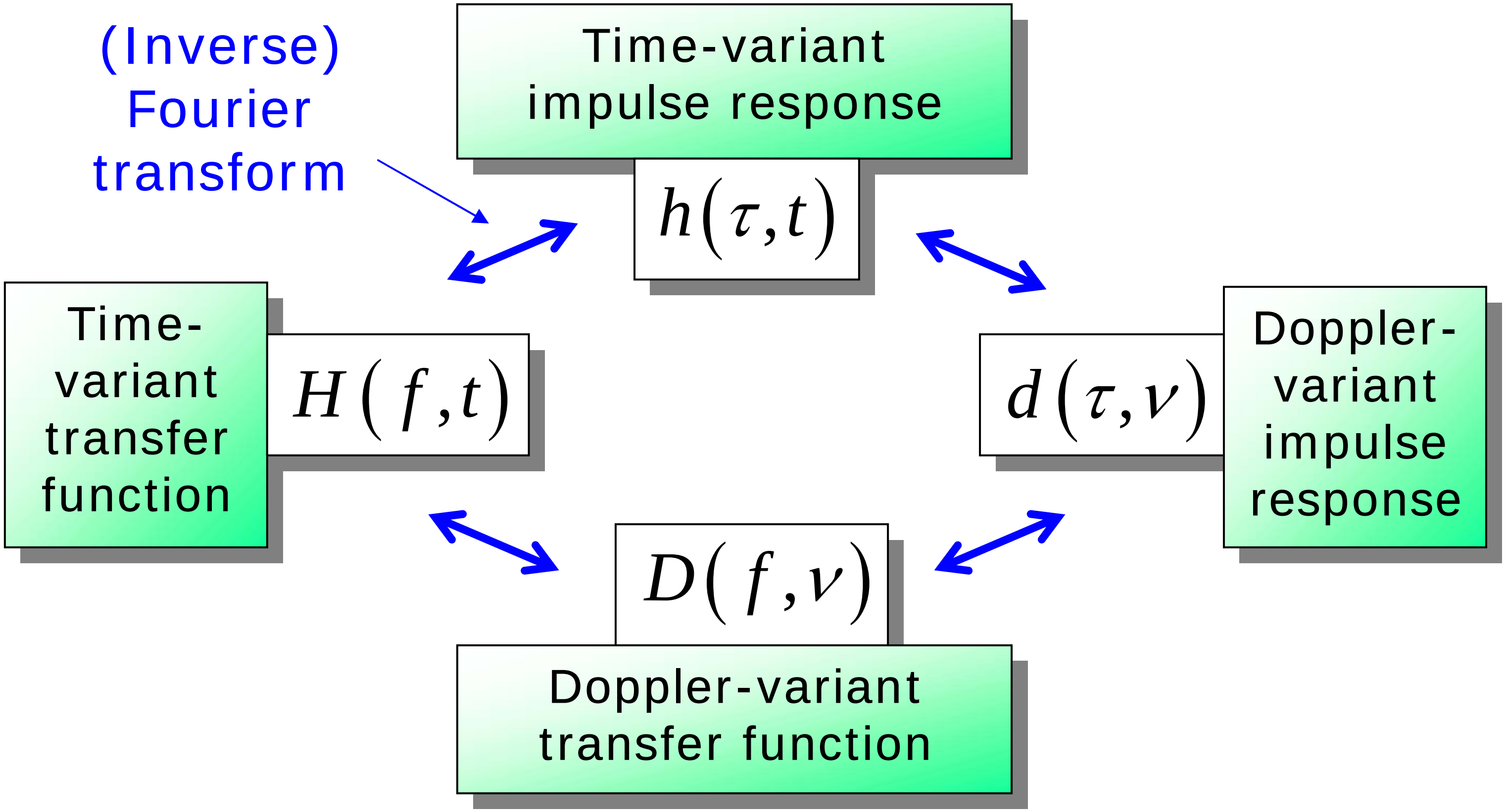
At certain frequencies the two terms add constructively (destructively) and we obtain:

$$\left| H\left(f_{constructive} \right) \right| = a_1 + a_2$$

$$\left| H\left(f_{destructive} \right) \right| = \left| a_1 - a_2 \right|$$



Deterministic channel functions



(Inverse) Fourier transform

Time-variant impulse response

$h(\tau,t)$

Time-variant transfer function

$H(f,t)$

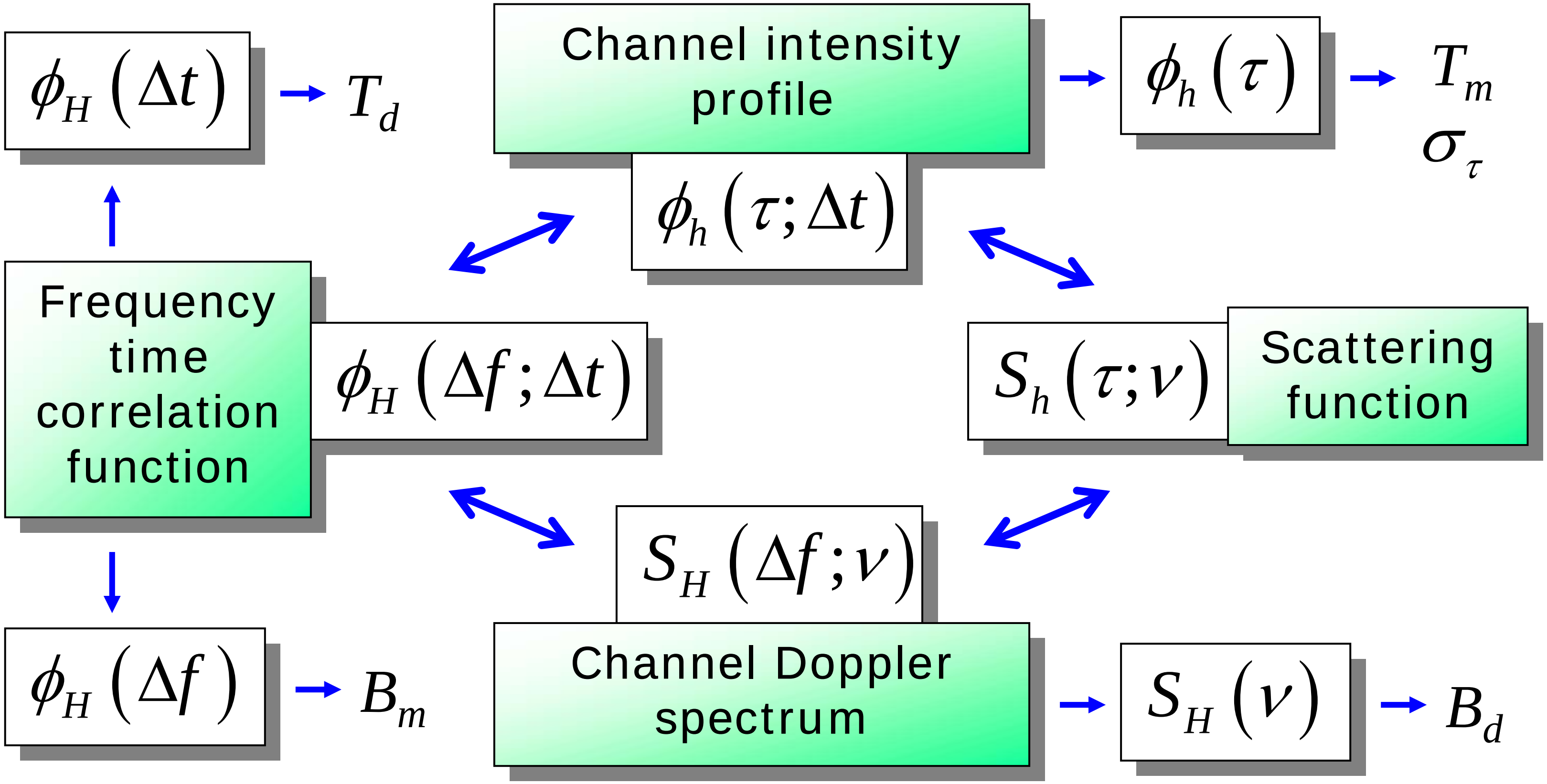
$d(\tau,\nu)$

Doppler-variant impulse response

$D(f,\nu)$

Doppler-variant transfer function

Stochastical (WSSUS) channel functions



$\phi_H(\Delta t) \rightarrow T_d$

Channel intensity profile

$\phi_h(\tau;\Delta t)$

$\rightarrow \phi_h(\tau) \rightarrow T_m$, σ_τ

Frequency time correlation function

$\phi_H(\Delta f;\Delta t)$

$S_h(\tau;\nu)$

Scattering function

$S_H(\Delta f;\nu)$

Channel Doppler spectrum

$\rightarrow S_H(\nu) \rightarrow B_d$

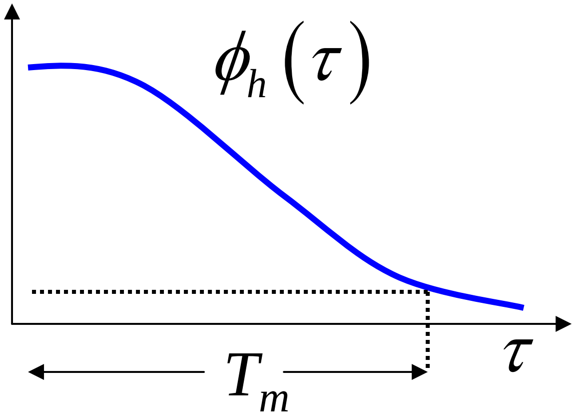
$\phi_H(\Delta f) \rightarrow B_m$

Stochastical (WSSUS) channel variables

Maximum delay spread: T_m

Maximum delay spread may be defined in several ways.

For this reason, the RMS delay spread is often used instead:

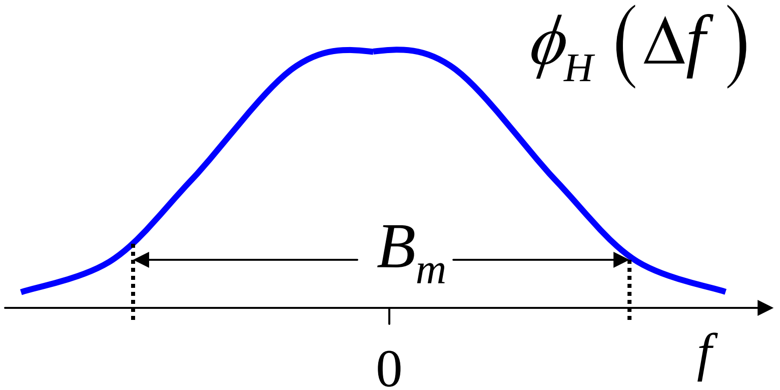


$$\sigma_\tau = \sqrt{\frac{\int \tau^2 \phi_h(\tau) d\tau}{\int \phi_h(\tau) d\tau} - \left[\frac{\int \tau \phi_h(\tau) d\tau}{\int \phi_h(\tau) d\tau}\right]^2}$$

Stochastical (WSSUS) channel variables

Coherence bandwidth of channel:

$$B_m \approx 1/T_m$$



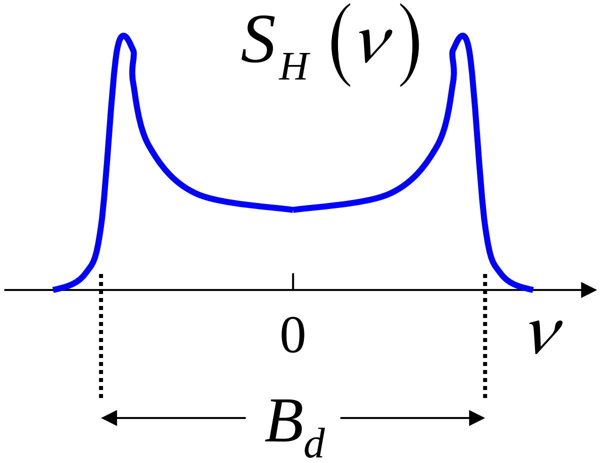
Implication of coherence bandwidth:

If two sinusoids (frequencies) are spaced much less apart than B_m , their fading performance is similar.

If the frequency separation is much larger than B_m , their fading performance is different.

Stochastical (WSSUS) channel variables

Maximum Doppler spread: B_d



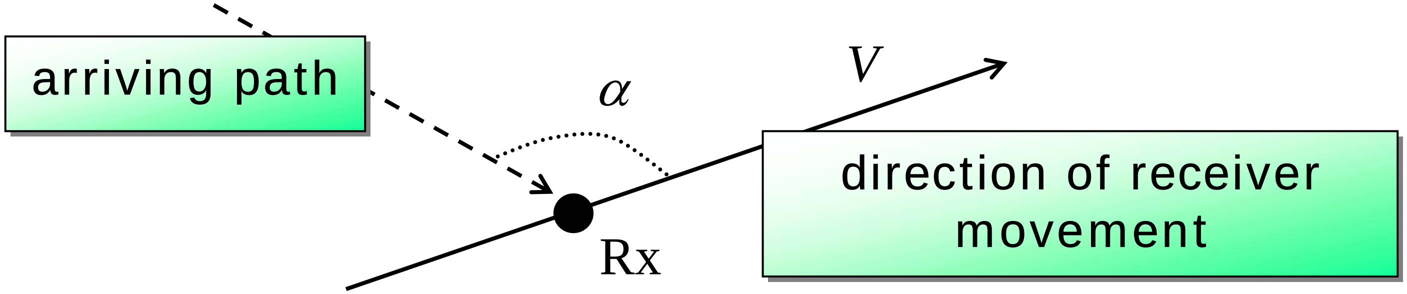
The Doppler spectrum is often U-shaped (like in the figure on the right). The reason for this behaviour is the relationship (see next slide):

$$\nu = \frac{V}{\lambda}\cos\alpha = f_d\cos\alpha$$

$$S_H(\nu) \approx p(\nu)$$

Task: calculate $p(\nu)$ for the case where $p(\alpha) = 1/2\pi$ (angle of arrival is uniformly distributed between 0 and 2π).

Physical interpretation of Doppler shift



arriving path

α

V

Rx

direction of receiver movement

Doppler frequency shift

$$\nu = \frac{V}{\lambda}\cos\alpha = f_d \cos\alpha$$

Maximum Doppler shift

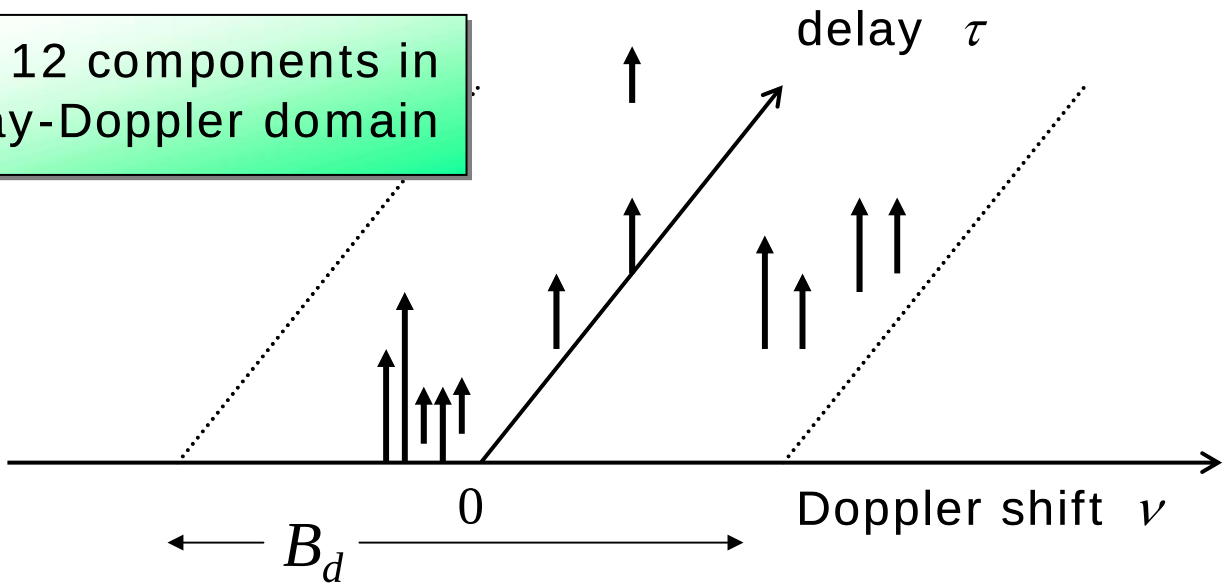
V = speed of receiver

λ = RF wavelength

Angle of arrival of arriving path with respect to direction of movement

Delay - Doppler spread of channel

L = 12 components in delay-Doppler domain



$$h(\tau,t)=\sum_{i=0}^{L-1} a_i(t)e^{j(2\pi\nu_i t+\phi_i)}\,\delta(\tau-\tau_i)$$

Fading distributions (Rayleigh)

In a flat fading channel, the (time-variant) CIR reduces to a (time-variant) complex channel coefficient:

$$c(t) = a(t)e^{j\phi(t)} = x(t) + j\,y(t) = \sum_i a_i(t)e^{j\phi_i(t)}$$

When the quadrature components of the channel coefficient are independently and Gaussian distributed, we get:

$$p(a) = \frac{a}{\sigma^2} e^{-a^2/2\sigma^2} \qquad p(\phi) = \frac{1}{2\pi}$$

Rayleigh distribution

Uniform distribution

Fading distributions (Rice)

In case there is a strong (e.g., LOS) multipath component in addition to the complex Gaussian component, we obtain:

$$c(t) = a_0 + a(t)e^{j\phi(t)} = a_0 + \sum_i a_i(t)e^{j\phi_i(t)}$$

From the joint (magnitude and phase) pdf we can derive:

$$p(a) = \frac{a}{\sigma^2} e^{-\left(a^2 + {a_0}^2\right)/2\sigma^2} I_0\left(\frac{aa_0}{\sigma^2}\right)$$

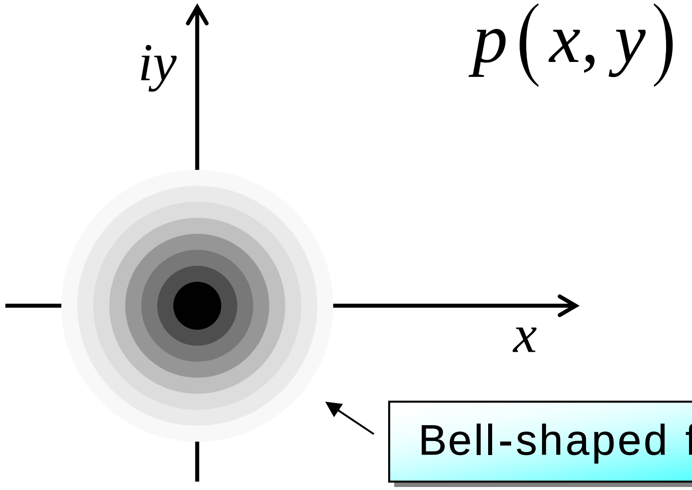
Rice distribution

Modified Bessel function of first kind and order zero

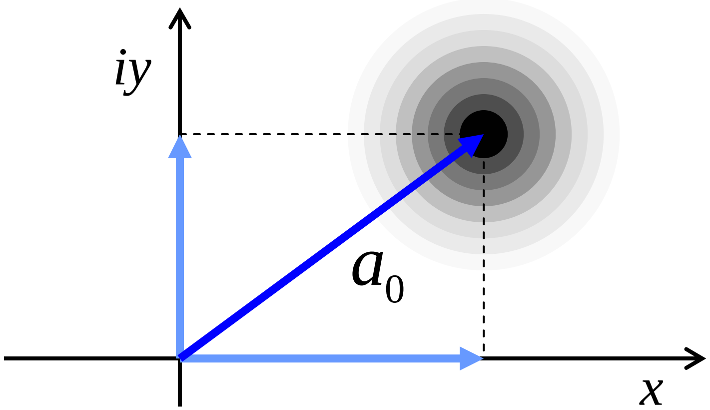
Representation in complex plane

Complex Gaussian distribution is centered at the origin of the complex plane => magnitude is Rayleigh distributed, the probability of a deep fade is larger than in the Rician case

Complex Gaussian distribution is centered around the “strong path” => magnitude is Rice distributed, probability of deep fade is extremely small



Bell-shaped function



Countermeasures: wideband systems

- Equalization (in TDMA systems)
  - linear equalization
  - Decision Feedback Equalization (DFE)
  - Maximum Likelihood Sequence Estimation (MLSE) using Viterbi algorithm
- Rake receiver schemes (in DS-CDMA systems)
- Sufficient number of subcarriers and sufficiently long guard interval (in OFDM or multicarrier systems)
- Interleaving, FEC, ARQ etc. may also be helpful in wideband systems.