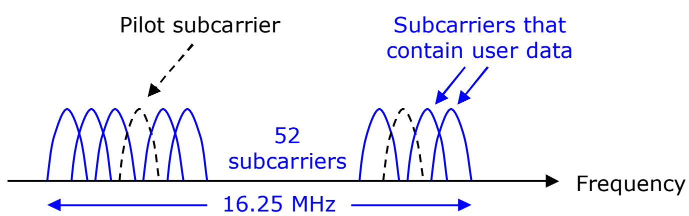
Usage of OFDM in a wideband fading channel

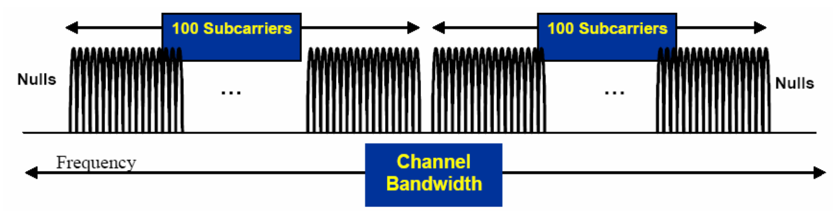
- OFDM signal structure
- Subcarrier modulation and coding
- Signals in frequency and time domain
- Inter-carrier interference
- Purpose of pilot subcarriers

OFDM example 1: IEEE 802.11a&g (WLAN)



48 data subcarriers + 4 pilot subcarriers. There is a ”null” at the center carrier. Around each data subcarrier is centered a subchannel carrying a low bitrate data signal (low bitrate => no intersymbol interference).

OFDM example 2: IEEE 802.16a (WiMAX)



Only 200 of 256 subcarriers are used: 192 data subcarriers + 8 pilot subcarriers. There are 56 ”nulls” (center carrier, 28 lower frequency and 27 higher frequency guard carriers).

Usage of OFDM

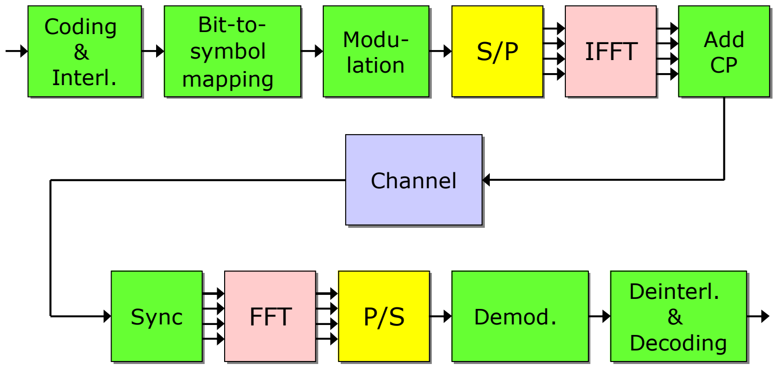
OFDM is used (among others) in the following systems:

- IEEE 802.11a&g (WLAN) systems
- IEEE 802.16a (WiMAX) systems
- ADSL (DMT = Discrete MultiTone) systems
- DAB (Digital Audio Broadcasting)
- DVB-T (Digital Video Broadcasting)

OFDM is spectral efficient, but not power efficient (due to linearity requirements of power amplifier).

OFDM is primarily a modulation method; OFDMA is the corresponding multiple access scheme.

OFDM system block diagram



Subcarrier modulation (IEEE 802.11a&g)

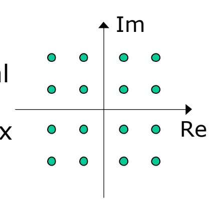
Modulation	Bit rate
BPSK	6 Mbit/s
BPSK	9 Mbit/s
QPSK	12 Mbit/s
QPSK	18 Mbit/s
16-QAM	24 Mbit/s
16-QAM	36 Mbit/s
64-QAM	48 Mbit/s
64-QAM	54 Mbit/s

BPSK = Binary Phase Shift Keying (PSK)

QPSK = Quaternary PSK

QAM = Quadrature Amplitude Modulation

16-QAM signal constellation in the complex plane



Why (for instance) 54 Mbit/s ?

Symbol duration = 4 μs

Data-carrying subcarriers = 48

Bits / subchannel = 6 (64-QAM)

Bits / OFDM symbol = 6 x 48 = 288

Channel coding: number reduced to 3/4 x 288 = 216 bits/symbol

=> Bit rate = 216 bits / 4 μs = 54 Mbit/s

Subcarrier modulation and coding

N data subcarriers or subchannels carry *N* data symbols in parallel (= transmitted at the same time). A symbol carries 1 bit (BPSK), 2 bits (4-PSK), 4 bits (16-QAM), or 6 bits of user data (64-QAM). *N* data symbols in parallel form one OFDM symbol.

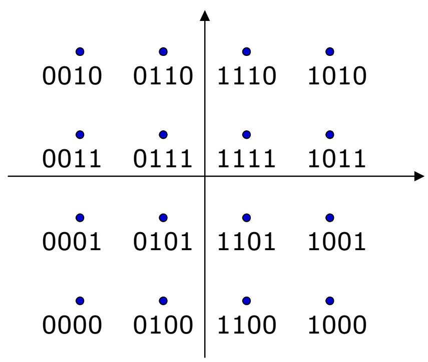
For each modulation method, there are several coding options for FEC (Forward Error Control). They must be taken into account when calculating user data rates, as shown on the previous slide. Typical coding options: 1/2 (convolutional encoding), 2/3 and 3/4 (puncturing) coding rates.

Gray bit-to-symbol mapping in QAM

Gray bit-to-symbol mapping is usually used in QAM systems.

The reason: it is optimal in the sense that a symbol error (involving two adjacent symbols in the QAM signal constellation) results in a single bit error.

Example for 16-QAM



BER performance of QAM (1)

A rectangular *M*-ary QAM constellation, where $M = 2^k$ and k is even, is equivalent to two PAM (Pulse Amplitude Modulation) signals on quadrature carriers, each having $\sqrt{M}$ signal points and symbol error probability $P_{\sqrt{M}}$.

Probability of correct symbol decision for *M*-ary QAM:

$$P_c = \left(1 - P_{\sqrt{M}}\right)^2$$

Probability of symbol error for *M*-ary QAM:

$$P_M = 1 - \left(1 - P_{\sqrt{M}}\right)^2 \approx 2P_{\sqrt{M}}$$

Proakis, 3rd Ed. 5-2-9

BER performance of QAM (2)

Probability of symbol error for $\sqrt{M}$-ary PAM:

$$P_{\sqrt{M}} = 2\left(1 - \frac{1}{\sqrt{M}}\right) Q\left(\sqrt{\frac{3}{M-1}\frac{E_{av}}{N_0}}\right)$$

Proakis, 3rd Ed. 5-2-6

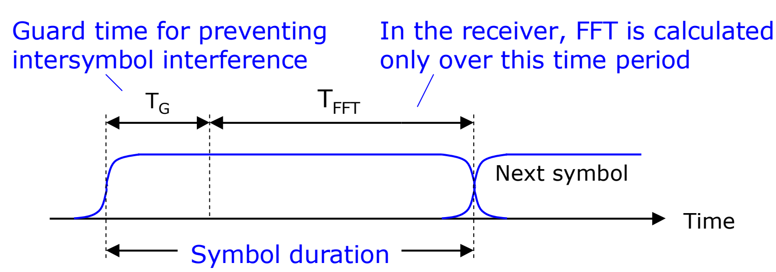
where E_{av}/N_0 is the average SNR per symbol.

Finally, the bit error probability for *M*-ary QAM:

$$P_b = \frac{P_M}{\log_2 M} = P_M / k$$

(Gray mapping assumed)

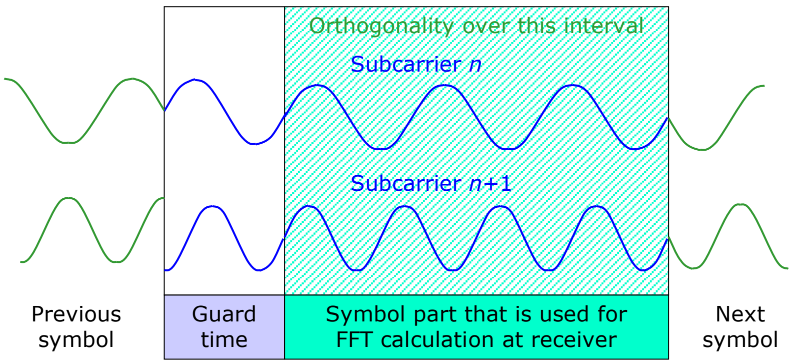
Subcarrier signal in time domain



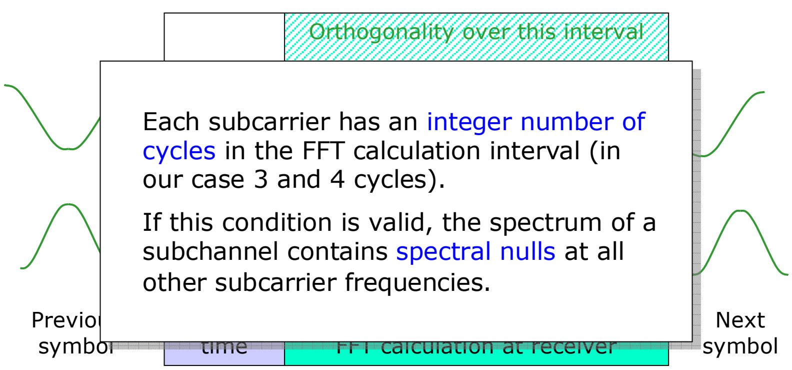
IEEE 802.11a&g: T_G = 0.8 μs, T_{FFT} = 3.2 μs

IEEE 802.16a offers flexible bandwidth allocation (i.e. different symbol lengths) and T_G choice: T_G/T_{FFT} = 1/4, 1/8, 1/16 or 1/32

Orthogonality between subcarriers (1)



Orthogonality between subcarriers (2)



Each subcarrier has an integer number of cycles in the FFT calculation interval (in our case 3 and 4 cycles).

If this condition is valid, the spectrum of a subchannel contains spectral nulls at all other subcarrier frequencies.

Orthogonality between subcarriers (2)

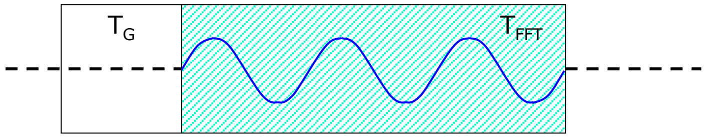
Orthogonality over the FFT interval:

$$\int_0^{T_{FFT}} \cos\left(2\pi mt/T_{FFT}\right)\cos\left(2\pi nt/T_{FFT}\right)dt = \begin{cases} T_{FFT}/2 & m=n \\ 0 & m\neq n \end{cases}$$

Phase shift in either subcarrier - orthogonality over the FFT interval is still retained:

$$\int_0^{T_{FFT}} \cos\left(2\pi mt/T_{FFT}+\phi\right)\cos\left(2\pi nt/T_{FFT}\right)dt = 0 \qquad m\neq n$$

Time vs. frequency domain



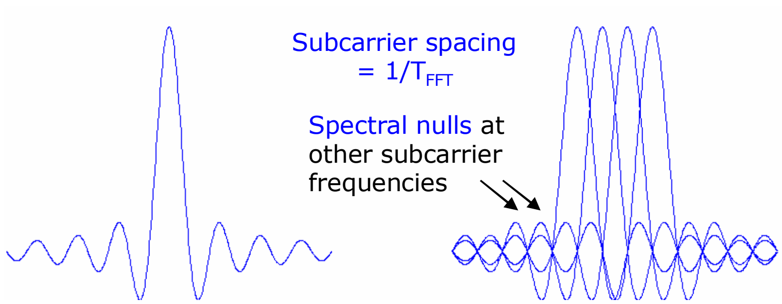
Square-windowed sinusoid in time domain

=>

"sinc" shaped subchannel spectrum in frequency domain

$$\text{sinc}\left(fT_{FFT}\right)=\left[\sin\left(\pi fT_{FFT}\right)\right]/\left(\pi fT_{FFT}\right)$$

Subchannels in frequency domain



Presentation of OFDM signal

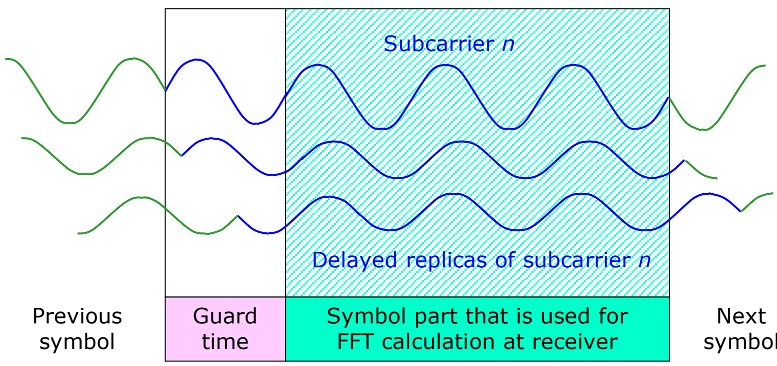
$$s(t) = \sum_{k=-\infty}^{\infty} g_k(t - kT)$$ Sequence of OFDM symbols

The k:th OFDM symbol (in complex LPE form) is

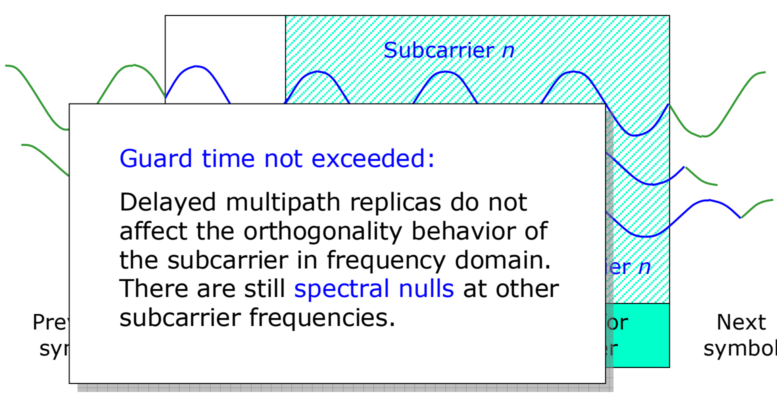
$$g_k(t) = \sum_{\substack{n=-N/2 \\ n \neq 0}}^{N/2} a_{n,k} \exp\left(j2\pi \frac{n}{T_{FFT}} t \right) \qquad (k-1)T < t < kT$$

where N = number of subcarriers, $T = T_G + T_{FFT}$ = symbol period, and $a_{n,k}$ is the complex data symbol modulating the n:th subcarrier during the k:th symbol period.

Multipath effect on subcarrier *n* (1)



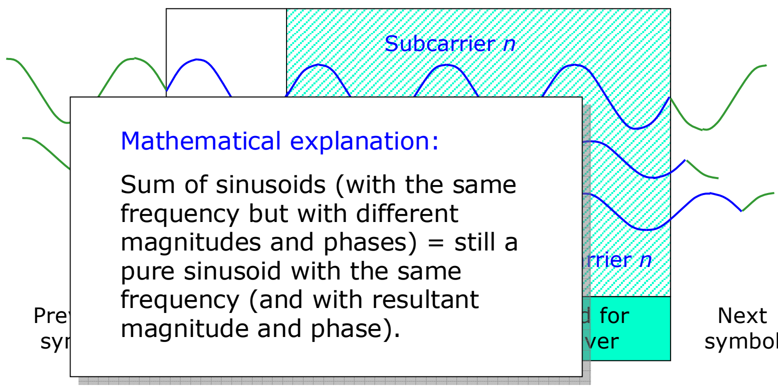
Multipath effect on subcarrier *n* (2)



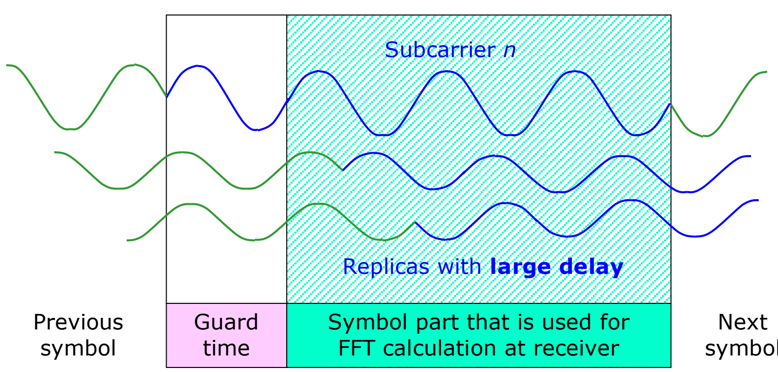
Guard time not exceeded:

Delayed multipath replicas do not affect the orthogonality behavior of the subcarrier in frequency domain. There are still spectral nulls at other subcarrier frequencies.

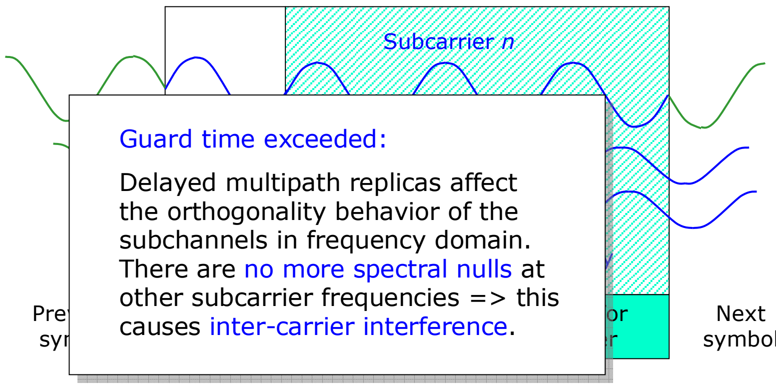
Multipath effect on subcarrier *n* (3)



Multipath effect on subcarrier *n* (4)



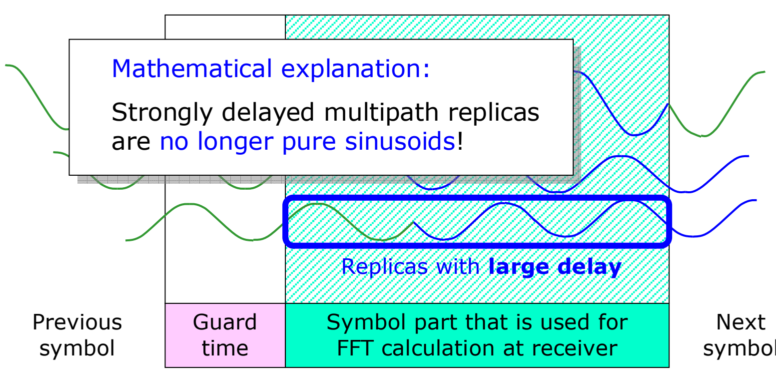
Multipath effect on subcarrier *n* (5)



Guard time exceeded:

Delayed multipath replicas affect the orthogonality behavior of the subchannels in frequency domain. There are no more spectral nulls at other subcarrier frequencies => this causes inter-carrier interference.

Multipath effect on subcarrier *n* (6)



Mathematical explanation:

Strongly delayed multipath replicas are no longer pure sinusoids!

Discrete multitone (DMT) modulation

DMT is a special case of OFDM where the different signal-to-noise ratio values of different subcarriers are utilised constructively in the following way:

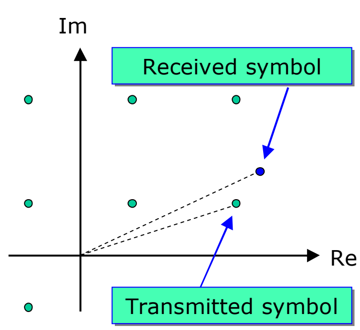
- Subcarriers with high S/N carry more bits (for instance by using a modulation scheme with more bits/symbol or by using a less heavy FEC scheme)
- Subcarriers with low S/N (due to frequency selective fading) carry less bits.

Note the requirement of a feedback channel.

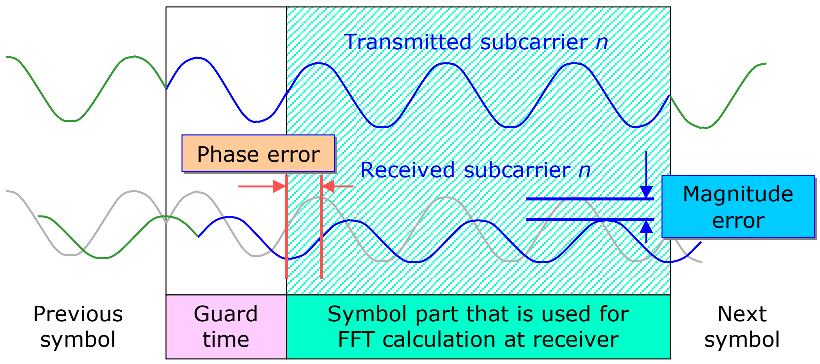
Task of pilot subcarriers

Pilot subcarriers contain signal values that are known in the receiver.

These pilot signals are used in the receiver for correcting the magnitude (important in QAM) and phase shift offsets of the received symbols (see signal constellation example on the right).

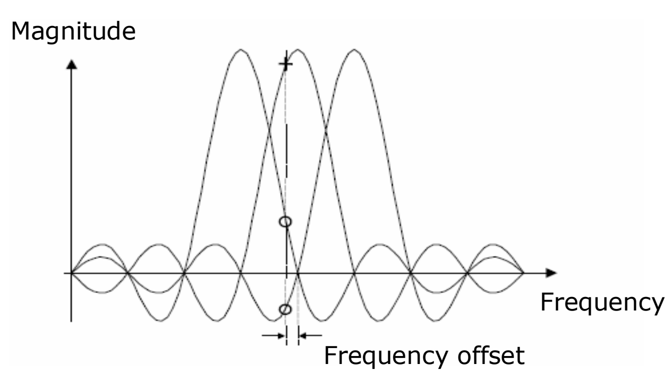


Transmitted and received subcarrier *n*



Frequency offset at receiver

Frequency offset causes inter-carrier interference (ICI)



Summary: Inter-carrier interference

Inter-carrier interference (ICI) means that the orthogonality between different subchannels in the OFDM signal is destroyed.

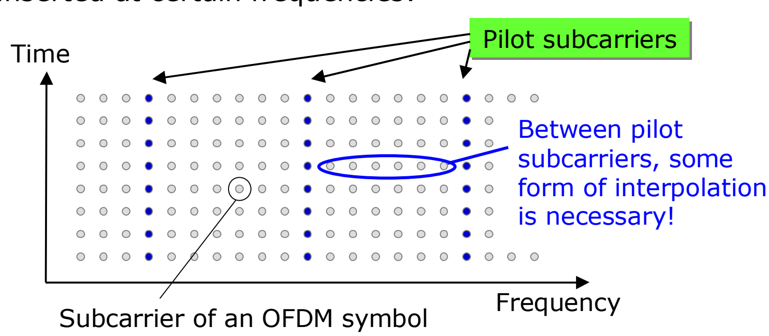
There are two causes of inter-carrier interference:

Delay spread of radio channel exceeds guard interval

Frequency offset at the receiver

Pilot allocation example 1 (1)

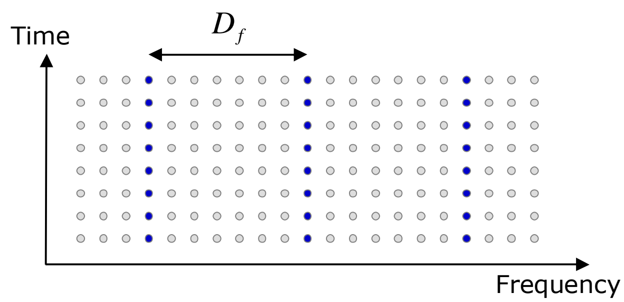
To be able to equalize the frequency response of a frequency selective channel, pilot subcarriers must be inserted at certain frequencies:



Pilot allocation example 1 (2)

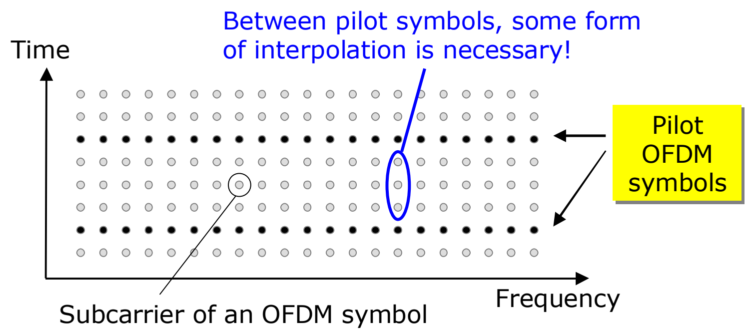
The Shannon sampling theorem must be satisfied, otherwise error-free interpolation is not possible:

$D_f \leq 1/2T_\mathrm{m}$ $\quad T_m$ = maximum delay spread



Pilot allocation example 2 (1)

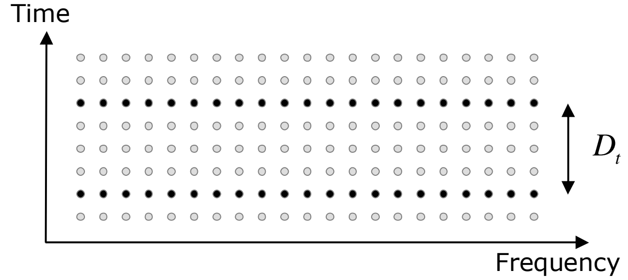
An alternative pilot scheme for equalizing the frequency response of a frequency selective channel:



Pilot allocation example 2 (2)

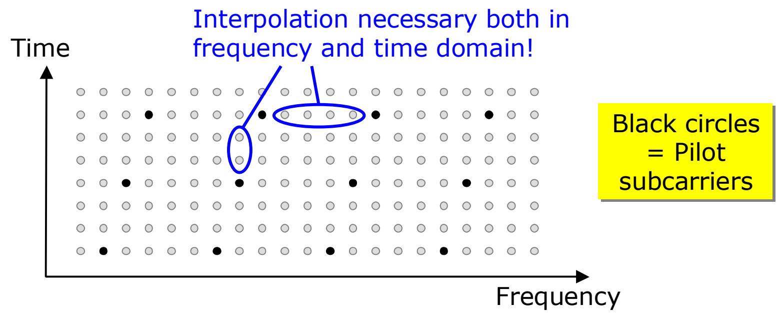
The Shannon sampling theorem must again be satisfied, otherwise error-free interpolation is not possible:

$$D_t \leq 1/B_D \qquad B_D = \text{ maximum p-p Doppler spread}$$
$$\leq 1/2\nu_{\max} \qquad \nu_{\max} = \text{ maximum Doppler frequency}$$



Pilot allocation example 3

An efficient pilot scheme (used in DVB-T) makes use of interpolation both in frequency and time domain:



Summary: OFDM features

In summary, OFDM offers the following features:

Multipath propagation (fading) does not cause intersymbol or intercarrier interference if the guard interval is sufficiently large and there is no frequency offset at the receiver.

Multipath fading, however, causes frequency selectivity in the transmission bandwidth. Pilot signals are employed for correcting (equalizing) the magnitude and phase of the received subcarriers at the pilot subcarrier frequencies.

Some form of interpolation is necessary for equalization at other than pilot subcarrier frequencies. Many pilot allocation schemes have been proposed in the literature, see e.g.

www.s3.kth.se/signal/grad/OFDM/URSIOFDM9808.htm