S-72.3320 Advanced Digital Communication

17.2.2006
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Cyclic Codes

1.

For a *systematic* cyclic code, we define the message bit, check bit and codeword polynomial as $M(p)$, $C(p)$ and $X(p)$, respectively. These three polynomials are related to each other as $X(p) = p^q M(p) + C(p)$, where $q = n - k$.

Each codeword corresponds to the polynomial product $X(p) = Q_M(p)G(p)$, in which $Q_M(p)$ represents a block of k message bits. X and M are code and message vectors which correspond to $X(p)$ and $M(p)$, respectively. The two equations above for $X(p)$ require that

$$\frac{p^q M(p)}{G(p)} = Q_M(p) + \frac{C(p)}{G(p)}$$

In the receiver side, every valid received code word $R(p)$ must be a multiple of $G(p)$ otherwise an error has occurred. Therefore dividing the $R(p)/G(p)$ and considering the remainder as a syndrome can reveal if the error has happened. The syndrome is therefore

$$S(p) = rem\left[\frac{R(p)}{G(p)}\right]$$

Consider a systematic (7, 3) cyclic code generated by $G(p) = p^4 + p^3 + p^2 + 0 + 1$.
Find $Q_M(p)$, $C(p)$ and X when M = (1 0 1). Then take received vector, $Y = X'$ and confirm that $S(p) = 0$.

2.

Figure 1 shows is a shift-register circuit that divides an arbitrary *m*th-order polynomial $Z(p)$ by a fixed polynomial $G(p) = p^q + g_{q-1}p^{q-1} + ... + g_1 p + 1$. If the register has been cleared before $Z(p)$ is shifted in, then the output equals the quotient and the remainder appears in the register after *m* shift cycles. Confirm the division operation by constructing a table containing the input message bits, register contents before shift, register content after shift, and the output. Take $Z(p) = p^3 M(p)$, with message bits 1100, and $G(p) = p^3 + 0 + p + 1$.

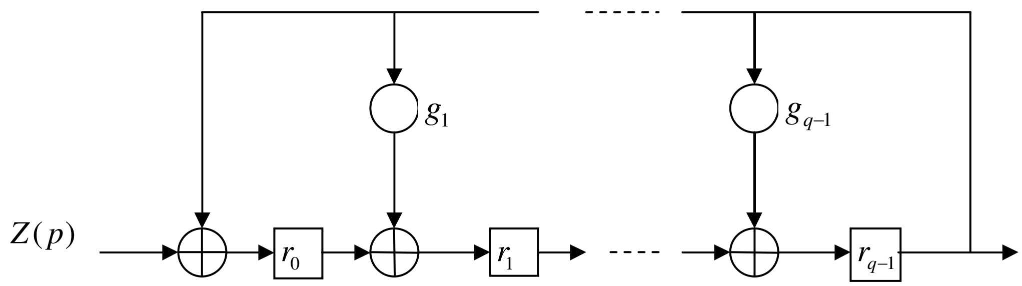

Fig. 1

3.

Let $g(p) = p^8 + p^6 + p^4 + p^2 + 1$ be a polynomial over the binary field.

a. Find the lowest-rate cyclic code whose generator polynomial is *g(p)*. What is the rate of this code?
b. Find the minimum distance of the code found in (a).

OFDM in a wideband fading channel

4.

a. How OFDM solves the multipath problem?
b. How are signals transmitted in parallel without interference in OFDM?
c. What are the main problems of OFDM?

5.

The number of subcarriers N, the bandwidth of each subcarrier $1/NT$, the bandwidth of the system $B \approx 1/T$, and the length of the cyclic prefix Δ are all important parameters in the design of an OFDM system. We have the following restriction on the number of subcarriers

$$\tau B << N << \frac{B}{f_d} \qquad (1)$$

where f_d is the maximum Doppler frequency.

Universal Mobile Telecommunications System (UMTS) has been assigned frequencies in the 2.2 GHz band. Operators expect to be assigned 5 MHz for uplink and 5 MHz for downlink transmission and therefore in the following we assume a sample frequency of 5 MHz. A proper design of a radio interface based on OFDM depends on the characteristics of the radio environment in these bands. For the evaluation of the UMTS, consider the worst values for the Doppler frequencies and channel delay spreads for the vehicular channel model with a mobile speed of 120 km/h and if we want our system to accommodate echoes up to about 10 μsec. Evaluate the range of possible number of subcarriers and the cyclic prefix.