S-72.3410 Coding Methods

1. (6p.) Let $\mathbf{G} = \begin{bmatrix} 1 & 2 & 0 & 2 \\ 1 & 1 & 1 & 0 \end{bmatrix}$ be a generator matrix for a code over $\mathrm{GF}(3)$. Construct a parity-check matrix for it. What is the minimum distance of this code? Construct a syndrome table including all error patterns with a single error. Are the following words codewords (if not, correct them): 2122, 2012?

2. (6p.) Define or explain briefly the following concepts:

   (a) constraint length of a convolutional code

   (b) throughput of an ARQ protocol

   (c) perfect block code

   (d) ideal (in algebra)

   (e) narrow-sense BCH code

   (f) conjugates of a field element

3. (a) (3p.) Describe the *systematic* encoding technique for an (n, k) cyclic code C defined by the generator polynomial $g(x)$, when the k-bit message block is given by the message polynomial $m(x)$. Apply this technique to obtain the systematic codeword when the (15,10) code generated by $g(x) = x^5 + x^4 + x^2 + 1$ is used and $m(x) = x^8 + x^7 + x^2 + 1$.

   (b) (3p.) Find a generator matrix for the Reed-Muller code $\mathcal{R}(1, 3)$. What can you say about the dual code of this code?

4. (6p.) Consider the convolutional code defined by the transfer function matrix

$$\mathbf{G}(D) = \begin{bmatrix} 1 + D & 1 + D^2 & 1 + D + D^2 \end{bmatrix}.$$

   A message is encoded, using this code, and transmitted over a binary symmetric channel. The received word is $\mathbf{r} = 010, 111, 110, 110, 010, 110, 011$. Decode the message using the hard-decision Viterbi algorithm.