Lecture 5: Simulation of OFDM communication systems

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Single carrier communication systems

- **Bandwidth:** $B$ (1 MHz)
- **Symbol period:** $T = 1/B$ (1 µs)
- If $\tau_{\text{max}} > T$, intersymbol interference (ISI)

**Equalization**
**OFDM communication system**

- Bandwidth is divided into $N$ sub-band
- Symbol period is $N$ time longer

Frequency

$B$

$\Delta f$

(1kHz)

Time

$T (1\text{ms})$

$\Delta f = \frac{B}{N}$

Symbol duration

$T = \frac{N}{B}; \quad T = \frac{1}{\Delta f}$
for the kth sub-carrier

\[
\begin{cases}
g(t,k) = \exp(j2\pi f_k t), & 0 \leq t < T, \\
g(t,k) = 0, & \text{otherwise.}
\end{cases}
\]

Orthogonal condition

\[
\begin{cases}
\int_0^T g(t,k) \cdot g^*(t,p) dt = 0, & k \neq p, \\
\int_0^T g(t,k) \cdot g^*(t,p) dt = \int_0^T |g(t,k)|^2 dt = T, & k = p.
\end{cases}
\]
OFDM-BPSK signal: 1, 1, -1, -1, 1

<table>
<thead>
<tr>
<th>data</th>
<th>Real part: $\cos(2\pi f_k t)$</th>
<th>Imaginary part: $\sin(2\pi f_k t)$</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td><img src="image1" alt="Graph" /></td>
<td><img src="image2" alt="Graph" /></td>
</tr>
<tr>
<td>1</td>
<td><img src="image3" alt="Graph" /></td>
<td><img src="image4" alt="Graph" /></td>
</tr>
<tr>
<td>-1</td>
<td><img src="image5" alt="Graph" /></td>
<td><img src="image6" alt="Graph" /></td>
</tr>
<tr>
<td>-1</td>
<td><img src="image7" alt="Graph" /></td>
<td><img src="image8" alt="Graph" /></td>
</tr>
<tr>
<td>1</td>
<td><img src="image9" alt="Graph" /></td>
<td><img src="image10" alt="Graph" /></td>
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<tr>
<td></td>
<td><img src="image11" alt="Graph" /></td>
<td><img src="image12" alt="Graph" /></td>
</tr>
</tbody>
</table>
OFDM spectrum

(a) Rectangular pulse

(b) Corresponding spectrum
\[ F(f(t)) \Rightarrow F(j2\pi f) \]
\[ F(f(t) \cdot e^{j2\pi \Delta f}) \Rightarrow F(j2\pi (f - \Delta f)) \]
Block diagram of OFDM systems

Binary data

Encoding (BPSK…)

Encoding (BPSK…)

IFFT

IFFT

Guard insertion

Guard deleting

Lowpass filtering

Lowpass filtering

Carrier modulation

Carrier demodulation

Channel

Channel estimation

OFDM symbol synchronisation

X(k) x(n) x'(n) s(t) y'(k) y(n) y'(n) r(t)
Using FFT and IFFT

\[
g(k) = \exp \left( j \frac{2\pi}{N} kn \right), \quad k = 0, 1, \ldots N - 1, \quad n = 0, 1, \ldots N - 1
\]

\[
\sum_{n=0}^{N-1} \exp \left( j \frac{2\pi}{N} pn \right) \exp \left( - j \frac{2\pi}{N} kn \right) = N, \quad k = p
\]

\[
\sum_{n=0}^{N-1} \exp \left( j \frac{2\pi}{N} pn \right) \exp \left( - j \frac{2\pi}{N} kn \right) = 0, \quad k \neq p
\]
OFDM signal expressions

Assume the information data be $X(k)$, then the transmitted signal is $s(n)$

$$s(n) = \sum_{k=0}^{N-1} X(k) \exp\left( j \frac{2\pi}{N} nk \right) \quad 0 \leq n \leq N - 1$$

The received signals

$$r(t) = s(t) + w(t)$$

$w(t)$: AWGN signal.
The signal on the kth subcarrier

\[ Y(k) = \frac{1}{N} \sum_{n=0}^{N-1} r(n) \exp \left( -j \frac{2\pi}{N} kn \right) \]

\[ = \frac{1}{N} \sum_{n=0}^{N-1} s(n) \exp \left( -j \frac{2\pi}{N} kn \right) + W(k), \quad 0 \leq k \leq N - 1 \]

\[ W(k) = \frac{1}{N} \sum_{n=0}^{N-1} w(n) \exp \left( -j \frac{2\pi}{N} kn \right), \quad 0 \leq k \leq N - 1 \]
About the guard interval

Multipath delay profile

\[ \tau_{\text{max}} (5\mu s, 17\mu s, \ldots) \]
OFDM Symbol $s(t)$

Guard interval $G > \tau_{\text{max}}$

$h(t)$

OFDM Symbol $s(t)$

Same signals
Problems in OFDM systems

- Peak to average power ratio (PAPR)
- Symbol synchronization
- Channel response estimation
- Impact of frequency error
- Impact of clock error
Peak to average power ratio (PAPR)
Pdf function of the OFDM samples

The peak to average power ratio (PAPR) is very large!
The impact of high PAPR to OFDM

Normal amplifier response

![Diagram showing normal amplifier response with input on the x-axis, output on the y-axis, and a linear range indicated.]
Truncation of OFDM time domain signals caused by amplifier
About the OFDM time domain signals

• It appears as Gaussian distribution
• The truncation of amplifier may cause distortion of the received signals
• The incorrect A/D conversion range may cause the distortion of the received signals
• The special time domain synchronization procedure should be introduced
Symbol synchronization

Why the synchronization is needed?

• To get the starting point of the FFT window
• The guard period is used for the symbol synchronization
• The guard period is not used for the signal demodulation
OFDM Symbols $(t)$

$S_{am e \ s i g n a l s}$

$h(t)$
Synchronization using guard interval

Input time domain signals

Delay $T$ \quad conjugate

Integral

Get the maximum

Get the phase

Frequen cy error
相关窗
The symbol synchronization of OFDM systems

Figure 2

The correlation function for OFDM synchronization (ideal condition)
The correlation function for OFDM synchronization
(SNR = 3dB, Multipath channel, carrier frequency error)
Frequency domain channel response

Figure 3

Note: when transmitted signal on all sub carriers are 1, you get the frequency domain channel response.
Pilot for OFDM frequency domain signals

$K_{\text{min}} = 0$

$K_{\text{max}} = 1704$ if $2K$
$K_{\text{max}} = 6816$ if $8K$

TPS pilots and continual pilots between $K_{\text{min}}$ and $K_{\text{max}}$ are not indicated

- boosted pilot
- data
Interpolation results

Zero order interpolation
Interpolation results

first order (linear) interpolation
Signal Constellation with different lowpass filters

\[ R = 0.5; \quad \text{% roll-factor} \]
\[ \text{delay} = 3; \quad \text{% number of side lobes} \]
Signal Constellation with different lowpass filters

R=0.2; % roll-factor
delay=3; % number of side lobes
Signal Constellation with different lowpass filters

\[ R = 0.2; \quad \text{% roll-factor} \]
\[ \text{delay} = 5; \quad \text{% number of side lobes} \]
Exercise 2

• Build OFDM system with 8 times upper sampling

• The channel are three types:
  – Ch-1: No AWGN and no multipath delay
  – Ch-2: AWGN channel
  – Ch-3: multipath delay without AWGN (channel parameters can be decided by yourself)
Exercise 2: Show the following figures and discuss the results

- Get time domain signal histograms for Ch-1, Ch-2, Ch-3, after down sampling, three figures (figure 1-3, see instruction 2-1)
- Perform time domain symbol synchronizations **before or after** down sampling for Ch-1, Ch-2, Ch-3, three figures, (Figure 4-6, see instruction 2-2)
- Modulate signal 1 on all sub-carriers, show the frequency domain response for Ch-1, Ch-2, Ch-3, in real and imaginary parts separately, three figures (figure 7-9, see instruction 2-3)
- Assume the pilot signals are given in page 32 and 33, using Ch-3 channel, perform interpolation for channel response estimation. Draw the figure together with Figure 9 see instruction 2-4)

- **The instructions are given in the following pages**
Instruction 2-1: for Figure 1-3

Binary data → 16QAM modulation → N-points IFFT → Guard insertion → Up sampling → Lowpass filtering

Binary data → 16QAM demodulation → N-points FFT → Guard deleting → Down sampling → Lowpass filtering A/D converting

X(k) → x(n) → x'(n) → s(t)

y'(k) → y(n) → y'(n) → r(t)

show Histogram

Suggestions and instruction are shows in the next page
Instruction 2-1: How to make OFDM system

- OFDM systems can be made simply by adding the yellow color blocks to your first exercise -16QAM communication system (Assume FFT point N=512; length of guard interval = 100)
- **Build OFDM transmitter**
  - Generate binary data sequence, for example length=N×4×10
  - Perform 16QAM modulation
  - Make 16QAM symbol as frames of length N, total 10 frames
  - Perform IFFT to each frame
  - Add guard interval to each frame, then, frame length = N+G
  - Make a vector consist of all frames, length of vector=(N+G) ×10
  - Up sampling and low pass filtering
- **Build OFDM Receiver**
  - Make received symbol (after down sampling) as frames of length N+G, total 10 frames
  - Remove guard interval for each frame
  - Perform IFFT to each frame
  - Make a vector consist of all frames, length of vector=N ×10
- In AWGN channel, the BER performance should be same as the original 16QAM system
- The histogram is shown in blue block for three channels
Instruction 2-2: synchronization for OFDM ---After down sampling

The instruction for the synchronization is shown in next page
Instruction 2-2: synchronization for OFDM ---After down sampling

FFT length = N; Guard length = G

In your report, you need to show y(n) value, where n is time domain index. y(n) can be expressed as (n=1,2,3…)

\[ y(n) = \sum_{i=0}^{G-1} x(n+i)x^*(n+i+N) \]
Instruction 2-2: synchronization for OFDM ---After down sampling

FFT length = N; Guard length = G
Instruction 2-2: synchronization for OFDM ---before down sampling

FFT length = N; Guard length = G; Up sampling rate = K

\[ y(n) = \sum_{i=0}^{G \times K - 1} x(n+i)x^*(n+i + N \times K) \]
Instruction 2-3: Show Channel response of OFDM systems

1,1,1,1… \rightarrow 16QAM modulation \rightarrow \text{N-points IFFT} \rightarrow \text{Guard insertion} \rightarrow \text{Up sampling} \rightarrow \text{Lowpass filtering} \rightarrow \text{Channel} \\

1,1,1,1… \leftarrow 16QAM demodulation \leftarrow \text{N-points FFT} \leftarrow \text{Guard deleting} \leftarrow \text{Down sampling} \leftarrow \text{Lowpass filtering A/D converting}

Show Channel response (real and Imaginary parts), for one frame ONLY

For figure 7,8,9
Instructions 2-4: channel response estimation of OFDM

1,0,0,0,1,0,0,0,1,0,0,0,1,0,0,0,1...

\[ X(k) \xrightarrow{\text{N-points IFFT}} x(n) \xrightarrow{\text{Guard insertion}} x'(n) \xrightarrow{\text{Up sampling}} s(t) \]

\[ \begin{array}{cccccc}
1,1,1,1 & \cdots & 1,0,0,0,1,0,0,0,1,0,0,0,1,0,0,0,1 & \cdots & 1,1,1,1 & \cdots \\
\end{array} \]

\[ y'(k) \xrightarrow{\text{N-points FFT}} y(n) \xrightarrow{\text{Guard deleting}} y'(n) \xrightarrow{\text{Down sampling}} r(t) \]

Interpolation to get channel response (real and imaginary), show one frame ONLY

Detail explanation is given in next page
Instructions 2-4: channel response estimation of OFDM systems

• Assume you have pilot subcarriers with carrier number 1, 5, 9, 13…,

• The pilot subcarriers are modulated by signal value 1 and rest of subcarriers are empty

• You get received signals at those pilot subcarriers (in fact it is frequency domain channel response)

• You need to get the frequency domain channel response of the empty subcarriers, you can do the interpolation as shown in the next page
Solid line: accurate channel response of your figure 9
Dotted line: Linear interpolation results